Detecting Elliptic Objects Using Inverse Hough–Transform

Joachim Hornegger, Dietrich W. R. Paulus

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Joachim Hornegger *and* Dietrich W. R. Paulus

Lehrstuhl für Informatik 5, (Mustererkennung), Martensstraße 3, Universität Erlangen–Nürnberg, Germany

Abstract
We describe a fast method for detecting one circular or elliptical object in an image. Based on the well known Hough–Transform for lines a new method with low complexity is developed to compute the centre of gravity and the focal points of an ellipse without knowing the exact contour. The experiments yield satisfactory results both with synthetical images and real scenes like an image of a gastric ulcer. We will also study the robustness of our method with regard to noise. The algorithms are integrated in an object–oriented programming environment for image analysis.

1 Introduction

Objects of circular or elliptical shape will yield an approximately elliptical object when projected on the two dimensional image plane. The estimation of the parameters of the ellipse can be a valuable feature for the recognition and localization of these objects. Examples are medical images of tumors [1] or the problem of finding cylinders in range images. Classical approaches to this problem use the Hough–technique with a five–dimensional parameter [1]; others fit a general second–order curve with a subset of the given data and determine whether or not the result is an ellipse or choose five given points and calculate a conic section fitting to these points [2]. These methods have their own advantages and disadvantages and they are fairly time consuming. Our new approach has the property that the positional parameters of one object with fuzzy contours and roughly elliptical shape can be estimated using only edge elements by applying ideas of the Hough–Transform for lines. The experimental results show that this approach to the given problem offers an efficient and robust solution.

2 Hough–Transform for Lines from Edges

The following methods for detecting elliptic objects are based on edge–images. For each pixel in the graylevel–image the discrete values for orientation and absolute values of the gradients can be computed using an edge operator, e.g. the Sobel–operator. The orientation computed from edge masks (e.g. the operator of Nevatia and Babu) is perpendicular to the gradient. Both methods result in a uniform edge image object in the object–oriented programming environment [4], with edge orientation aligned parallel to the edge.

A straight line in a (x, y)–coordinate system, non–parallel to the y–axis, can be represented using the formula: $y = a\,x + b$, where the parameter a is called the *slope* and b the *translation* of the given line. Obviously, we can associate with each line in the (x, y)–plane one point in the (a, b)–parameter space. The detection of lines based on a gradient image is done in the following manner: For each pixel p_i we compute the slope a_i and the translation b_i of the line using the given information of the edge image. In the (a, b)–array, the *accumulator*, we increment the entry (a_i, b_i) which is initialized with zero. After all pixels of the image are visited, we utilize the values of the entries in the accumulator and conclude which lines occur in the given image. Using the strength of the edges and some given thresholds for the entry in the accumulator, the strength and the length of the lines we get lines corresponding to edges. We do not treat lines of infinite slope here (see for instance [1] on this topic).

3 Inverse Hough–Transform for Circles

The new idea of the *Inverse Hough–Transform* is to use the classical Hough–Transform the other way round. With a given line in the parameter space $b = -x\,a + y$ we can associate a point in the (x, y)–plane in a unique manner, i.e. the point (x, y).

Lemma 1 *For all straight lines $g_i : y = a_i\,x + b_i$ $(1 \le i \le N)$ which intersect in one point $M = (x_M, y_M)$, we can associate the points (a_i, b_i) $(1 \le i \le N)$ in the parameter space. All points (a_i, b_i) $(1 \le i \le N)$ are element of a straight line s in the (a, b)–parameter space satisfying the following equation:*

$$s: \; b = -x_M\,a + y_M. \tag{1}$$

Proof:
Let $M = (x_M, y_M)$. All straight lines g_i $(1 \le i \le N)$ intersecting in M fulfil the following equations $g_i : \; y = a_i\,(x - x_M) + y_M$. Therefore we associate the point $(a_i, -a_i\,x_M + y_M)$ in the parameter space with each line g_i. Obviously, all such points in the parameter space are elements of the line $s : \; b = -x_M\,a + y_M$. □

The lemma can be used to compute the centre of an image containing one circle. The image is segmented into a edge–image for this purpose. Using the edge information and the coordinates of each point we calculate the slope and the translation of the straight line, which is perpendicular to the tangent line. Ideal conditions assure that all points in the parameter space associated with the slopes and translations of the line bundle satisify one linear equation of the form: $b = -x_M\,a + y_M$, where $M = (x_M, y_M)$ is the centre of the circle. Suppose that a noisy image of a circle is given. Analogously, we can compute the gradient image and finally the lines perpendicular to tangent lines. Certainly the majority of these lines will intersect close to the centre of the circle. Consequently the associated points in the parameter space will not fit with exactly one straight line. Using linear regression analysis we fit a line $b = -x_R\,a + y_R$ through these points (a_i, b_i) with minimal quadratic error, where

$$x_R = -\frac{n\,\sum_i a_i b_i - \sum_i a_i \sum_j b_j}{n\,\sum_i {a_i}^2 - \sum_i a_i \sum_j a_j} \quad \text{and} \quad y_R = \frac{\sum_i {a_i}^2 \sum_i b_i - \sum_i a_i \sum_j a_j b_j}{n\,\sum_i {a_i}^2 - \left(\sum_i a_i\right)^2}. \tag{2}$$

The associated point $R = (x_R, y_R)$ in the image plane will be an approximation of the centre (x_M, y_M) (see Figure 1) The average distance between the centre and the edge–points determines the radius of the circle. We may remark, that one normal line intersects the circular arc twice. Both points have identical normals. Consequently half of the circle yields the same set of normals as the complete circle. The described method is therefore very robust with respect to partial occlusion.

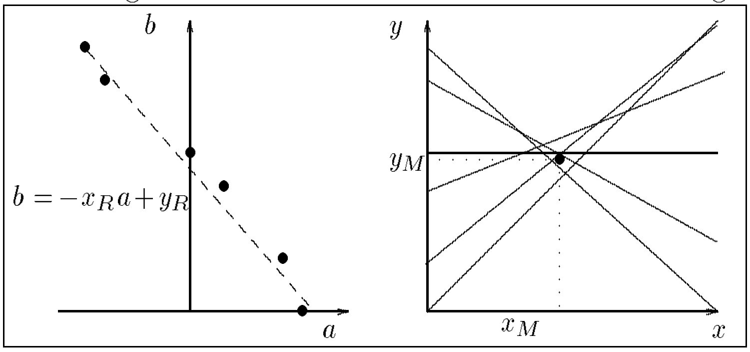

Figure 1: (Inverse) Hough–Transform

The equation 2 can be computed during the analysis of the image with a computational effort depending linearly on the number of edge points. In contrast to the classical Hough–Transform, neither storage space for an accumulator array nor computationally expensive search for local maxima are required.

4 Elliptical Shapes

The lemma 1 can also be used for computing the features of ellipses in general. In the ideal case, where one principal axis is parallel to the x–axis, the method follows immediately from the twofold application of the lemma:

Let the point (x_0, y_0) be the centre of gravity of a given ellipse with the mentioned restriction. Then, the ellipse can be specified by four parameters using the equation

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1. \tag{3}$$

All normals of the elliptic line intersect close to the centre of gravity (x_0, y_0). If the equations of the normals are known, and indeed they are, the centre will be calculated using linear regression and the lemma 1. Consequently, the squares $(x - x_0)^2$ and $(y - y_0)^2$ can be determined for each contour point. The equation for the ellipse can be written in the following manner:

$$\frac{1}{a^2} = -\frac{(y - y_0)^2}{(x - x_0)^2} \cdot \frac{1}{b^2} + \frac{1}{(x - x_0)^2}. \tag{4}$$

This term can be interpreted as an affine function in the variables $\alpha := \frac{1}{a^2}$ and $\beta := \frac{1}{b^2}$ with the slope $-\frac{(y-y_0)^2}{(x-x_0)^2}$ and the translation $\frac{1}{(x-x_0)^2}$. Obviously, the above equation is statisfied for each point of the elliptic line. If α and β are unknown parameters, we will associate with each contour point (x, y) one straight line (4) and finally one point in the paramter space assigned to slope and translation. All these lines should intersect in one point $(\frac{1}{a^2}, \frac{1}{b^2})$ determined by the length of principal axis. In noisy images these lines will just intersect close to this point. Using linear regression we can approximately compute this point in the parameter space; the principal axis, focal points and the radius of the ellipses follow directly.

Our issue is to detect a gastric ulcer in medical images. Due to experts in medicine those tumors have approximately elliptical shape. The above ideas can be useful to find an ellipse enclosing the tumor. Indeed the ellipse will not have principal axis parallel to the axis of the underlying coordinate system. First of all, the centre of gravity can be calculated with linear regression. A shift of the origin of the coordinate system into the centre of gravity splits the image into four areas. For each region, the lemma can be used to compute a point. The centres of gravity in the second and the third quadrant are weighted with the number of edge points used for their calculation. The average weighted sum over the x– and y–coordinates results in one focal point. The second focal follows by reflection at the origin of the new coordinate system.

5 Experimental Results

Focal Points	(calc.)	Centre of Gravity	(calc.)	Radius	(calc.)
$(50, 100)$, $(150, 100)$	$(50, 100)$, $(149, 99)$	$(100, 100)$	$(99, 99)$	160	159
$(50, 130)$, $(100, 180)$	$(57, 149)$, $(92, 149)$	$(75, 155)$	$(75, 155)$	100	94

Table 1: Parameters for synthetic ellipses and calculated values

The results of the algorithm on synthetic images are shown in Table 1. The first values are used for the generation of the images, the second values are calculated. These methods for the calculation of features of elliptical objects can be used for applications in medical imaging. Figure 2 shows the computed elliptic line overlayed to the red channel of a colour endoscopic image. In a further processing step, the edges close to the elliptic line are used to compute the fractal dimension of the border line to decide, whether the ulcer is malicious or not ([3]).

Figure 2: Endoscopic Image of Gastric Ulcer

References

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