Partial Image Data Registration using Stochastic Optimization

Jingfeng Han* Min Qiao † Joachim Hornegger ‡

Torsten Kuwert § Werner Bautz ¶ Wolfgang Römer ‖

August 11, 2005

Abstract

This paper proposes a novel method that uses probabilistic optimization to solve the rigid partial data registration problem. The fundamental task of partial data registration is to align the incomplete image data with the entire image data. We propose a novel registration algorithm based on the idea of adaptive random searching. The proposed method works more reliably than the existing methods for the partial data registration because it successfully overcomes the local optimum problem. With appropriate similarity measures, this framework is applicable to both mono–modal and multi–modal registrations with partial data. An evaluation on real 2D medical images and a comparison of different rigid registration methods show the feasibility of the proposed method.

1 Introuduction

Image registration is one of the most challenging image processing problems. The fundamental task of registration is to find the spatial transformation such that two images are best aligned. A registration problem of high importance is the so–called *partial data registration* that seeks to locate a small image (*partial data*) within the space of a larger image (*full data*). In this paper, we address the problem of rigid partial data registration. An illustrative example of rigid partial data registration is presented in Fig.1. For partial data registration, a key challenge is to find an appropriate way to avoid the local convergence. Most of the standard techniques, like Iterative Closest Point algorithm (ICP) [RL01] and various gradient–based algorithms, are locally convergent schemes, which require close initializations.

The contribution of this paper is that we propose a novel registration algorithm based on the idea of adaptive random searching [HN00] to overcome the local optimum problem. Different from the previous registration methods, the parameter set is searched randomly

*Institute of Computer Science, University of Erlangen–Nürnberg, Erlangen, Germany

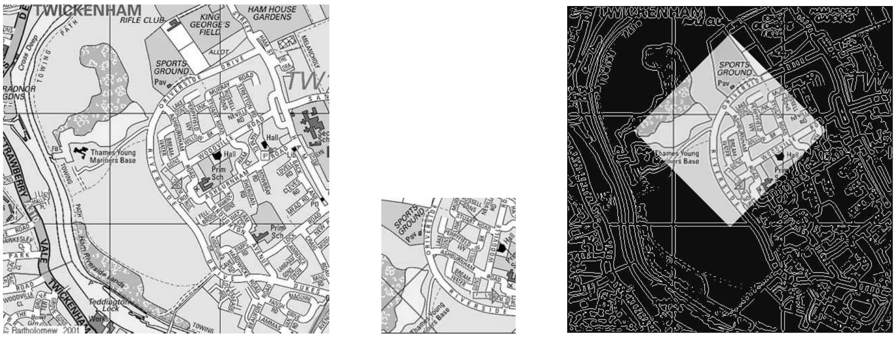
†Institute of Computer Science, University of Erlangen–Nürnberg, Erlangen, Germany

‡Institute of Computer Science, University of Erlangen–Nürnberg, Erlangen, Germany

§Clinic of Nuclear Medicine, University of Erlangen–Nürnberg, Erlangen, Germany

¶Institute of Radiology, University of Erlangen–Nürnberg, Erlangen, Germany

‖Clinic of Nuclear Medicine, University of Erlangen–Nürnberg, Erlangen, Germany



(a) (b) (c)

Figure 1: An example of partial data registration, locating a community map among a city map. (**a**): The city map is the full data. (**b**): The community map is the partial data. (**c**): The registered community map.

and globally in this method. The search process could be guided by an adaptive random process that accumulates the history of previous similarity evaluation as well. In principle, the parameter regions yielding higher similarity measure are the more promising parts to be further searched.
In the following sections we give the mathematical description of partial data registration problem. Then we present the probabilistic search based registration algorithm. Finally, we evaluate this method on real 2D medical images and compare it with the gradient descent method.

2 Methods

2.1 Background

Given a partial data $P(\vec{x})$ and a full data $Q(\vec{x})$, the goal of partial data registration is to find an optimal spatial transform $\mathbf{T}$ such that data $P(\vec{x})$ can be correlated with data $Q(\vec{x})$ with a high amount of accuracy. The mathematical definition of this problem can be formulated as

$$\mathbf{T} = \min_{\mathbf{T}} D(Q(\vec{x}), P(\mathbf{T}(\vec{x}))), \tag{1}$$

where the partial data $P(\vec{x})$ is transformed by $\mathbf{T}$ to align with the full data $Q(\vec{x})$. The distance measure D is an indicator for the dissimilarity between the two data sets.
For the rigid registration, the spatial transform is defined by a vector of translations $(\vec{t})$ and a vector of rotation angles $(\vec{\theta})$. See the example of Fig.1. There are various distance measures have been proposed to evaluate the matches of data. [ZF03][HBHH01] In this work, we heavily used *Sum of Squared Distance* (SSD) in the experimental evaluation on

Table 1: Random searching method for partial data registration

1. Input the partial data $P(\vec{x})$ and the full data $Q(\vec{x})$.
2. Define a covariance matrix Σ and a contraction factor $\gamma < 1$.
3. Distribute a points in the search space uniformly.
4. Select the best b ($b < a$) points in the search space with lowest distance measure D and sort these b points into a list.
5. The points of the list are used to generate the other b new random points in the following way.

   Build up b normal distributions, where the mean vectors correspond to the computed list elements and the covariances are given by Σ. Compute one new random point using each normal distribution.

6. Decrease the covariance matrix Σ by multiplication with the contraction factor γ.
7. Stop the algorithm if stop criteria is fulfilled. Otherwise, return to step 4.

the mono–modal image data. Nevertheless, the proposed algorithm in this work is a general registration method that is applicable for any distance measures.

2.2 Probabilistic search based registration

The rigid partial data registration problem can also be interpreted as searching for an optimal parameter set in a bounded parameter space $(\vec{t}, \vec{\theta})$, the so–called *search space*. The search process can be either deterministic, like a gradient decent search, or probabilistic, like a adaptive random search [HN00].
The most striking property of the adaptive random searching is that the optimization can easily jump over local optimums. In the initialization, plenty of test points are uniformly distributed within the entire search space, so that no close initialization is required. In each iteration, the algorithm fulfills the following two tasks. The first task is to evaluates every test points and reserve the reasonable ones with low distance measures. The second task is to generate new test points via a mixed normal distribution. At the beginning, these normal distributions are given large variances, which allow the generated points to largely deviate from the previous test points. In the following iterations, the variances of normal distributions are decreased by a *contraction factor*, so the new test points are more likely to be confined in the neighboring of the previous ones. With this random generation mechanism, the parameter searching is more global and more random in the early stage, so that the local optimums can be jumped over. In the later stage, the parameter searching become more local and deterministic, in order to converge to the correct optimum. Now we adapt this method to the partial data registration problem and summarize the algorithm in table 1.

Table 2: The table of success rates

Method	*GD*	*PS*
succ. rate	23.7%	99.4%

If the global optimal parameter set is unique, the stop criterion can be the following: the absolute difference of the highest and lowest distance measures in the sorted list is bounded by a given constant δ.

$$|D_s^{\text{highest}} - D_s^{\text{lowest}}| < \delta$$

The best parameter set, which is associated with lowest distance measure, is selected as the registration result.

3 Experiments

In order to analyze the reliability and reproducibility of the proposed method, the following experiments on the 2D MRI data is designed. We seek to register the image of the segmented brain (partial data) to the image of entire head (full data). See Fig.2. In this experiment, we compare the performance of the proposed Probabilistic Search based method (PS) method with the standard Gradient Decent method (GD). Since the ideal transformation between two data sets is known, we compare the computed parameters ($\vec{T} = [\vec{t}, \theta]$) with this ground truth ($\vec{T}^0 = [0, 0, 0]^T$) to evaluate two methods. If $t_{\{1,2\}} < 1$ pixel and $\theta < 0.2°$ we define that the registration experiment successes, otherwise it fails. The experiments are performed for 1000 times for each methods. The search space is confined in a bounded domain that $\|\vec{t}\| < 50$ pixels and $\|\theta\| < 45°$. For the gradient decent method, the initialization points are generated via an uniform distribution. The experiment shows that the probabilistic based method appears more reliable than the standard gradient decent method for this partial data registration problem. (See table 2.) The parameters are set as following, $a = 1000, b = 20, \gamma = 0.7, \Sigma = \text{diag}(5, 5, 5)$. We also plots the intermediate test points of PS methods. (Fig.3) It shows that the convergence of probabilistic searching processes.

4 Conclusion and Future work

This paper presents the algorithm that makes use of probabilistic search techniques to handle rigid registration problem. The comparison of the proposed algorithm to the well–established algorithms shows that our approach turns out to be more reliably, especially for partial data registration problem. A potential drawback of this algorithm is the problem of so–called "curse of dimensionality". For instance, in the 3D rigid registration problem, the search space is 6-dimensional (3 rotation and 3 translation). One future work is to develop a "marginalization" method to decompose the search space, so that the search process can be further sped up.

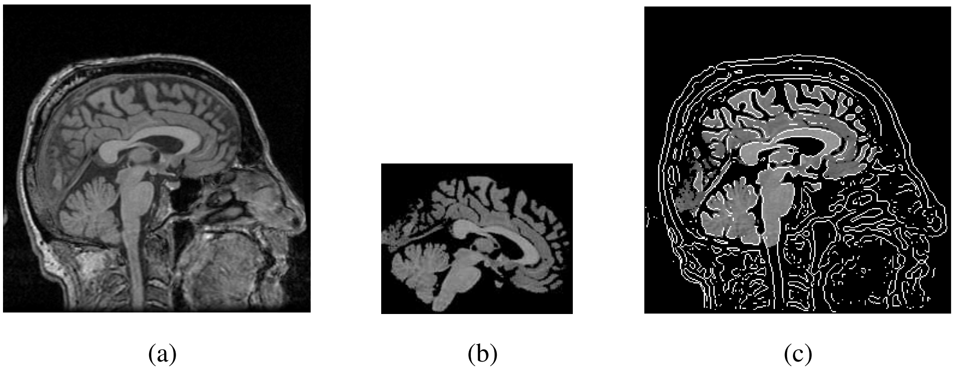


Figure 2: Partial data registration experiment on MRI data. (**a**): The full MRI data. (**b**): The segmented data. (**c**): The partial data is registered by the probabilistic based registration. (The size of full image is 256×256.)

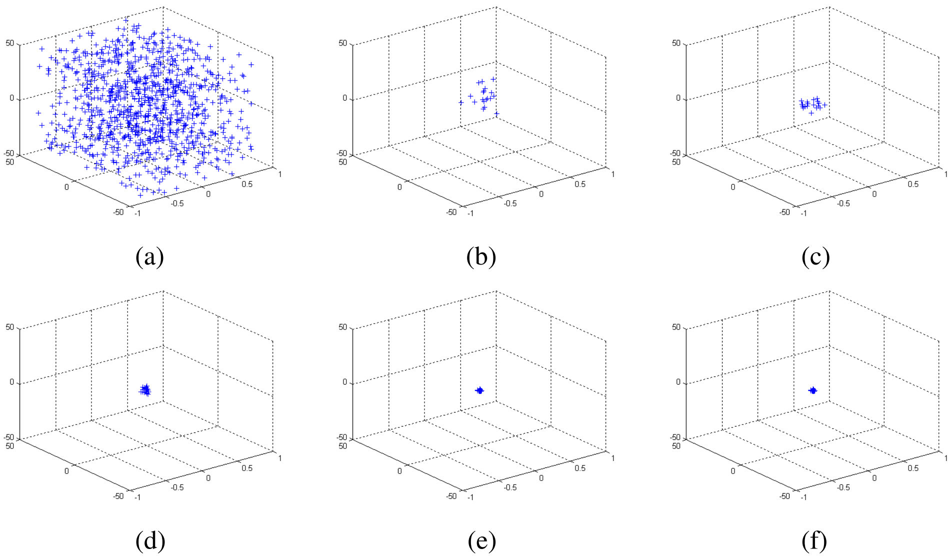


Figure 3: The plots of test points of PS method among the parameter space. (**a**): The initialization step ($a = 1000$). (**b–f**): The plots of the intermediate test points in the step: $1, 5, 8, 15, 20 (b = 20)$.

References

[HBHH01] Derek LG Hill, Philipp G Batchelor, Mark Holden, and David J Hawkes. Medical image registration. *Physics in Medicine and Biology*, 46:R1–R45, 2001.

[HN00] Joachim Hornegger and Heinrich Niemann. Probabilistic modeling and recognition 3-d objects. *Internaltional Journal of Computer Vision*, 39(3):229–251, 2000.

[RL01] Szymon Rusinkiewicz and Marc Levoy. Efficient variants of the icp algorithm. In *Proceedings of 3D Digital Imaging and Modelling*, pages 145–152. IEEE Computer Society Press, 2001.

[ZF03] Barbara Zitová and Jan Flusser. Image registrtaion methods: a survey. *Image and Vision Computing*, 21:977–1000, 2003.