Nonlinear Diffusion Noise Reduction in CT Using Correlation Analysis

M. Mayer, A. Borsdorf, H. Köstler, J. Hornegger and U. Rüde

Lehrstuhl für Mustererkennung and Lehrstuhl für Systemsimulation,
Friedrich-Alexander Universität Erlangen

Abstract. Noise reduction in CT images gains more and more attention. It provides a possibility to increase signal-to-noise ratio, hence giving more space for a further reduction of radiation dose. Nevertheless, a reduction of noise also bears the risk of suppressing medical relevant information. We propose a new noise reduction method that tries to minimize this risk by estimating the real image structure out of the correlations of two input data sets affected with uncorrelated noise. Such data sets can be achieved by reconstructing a CT scan with only the odd and the even numbered projections respectively. Furthermore, the method adapts itself to the spatially changing behavior of noise on CT images by estimating the local noise variance out of the difference of the input images. It can be applied to 2D and 3D data, with the latter giving better results due to the fact that more pixels are available for correlation computation and variance estimation. Examples show that the new method easily surpasses standard approaches and leads to noise suppression rates of about 66%.

1 Introduction

Common noise reduction methods often fail to produce convincing results when dealing with CT images. The reason for this lies in the unknown distribution of the noise in the reconstructed data. The intensity of the noise is spatially varying and directed noise structures appear. We present a new denoising method based on nonlinear isotropic diffusion that adapts itself to changing noise variance in different image regions and reduces oriented noise without noticeable loss of resolution by taking correlations of input images with uncorrelated noise into account. The approach is suitable for both 2D and 3D data.

2 Previous Work

In [1] A. Borsdorf proposed a Wavelet based denoising method for CT images. By separately reconstructing the odd and even numbered projections of a CT scan two sets of slices are obtained which include the same information but noise between the data is uncorrelated. By using correlation analysis in the wavelet domain combined with an orientation and position dependent noise estimation [2] only those wavelet coefficients containing image structure are kept for reconstruction of a noise suppressed image. In this work, the idea of this approach, e.g. using two data sets with uncorrelated noise, is picked up and transfered to the spatial domain and nonlinear diffusion methods.

3 Method

Noise is removed by minimizing an energy functional, which results in the following Euler-Lagrange equation:

$$u - u_0 = \tau \mathrm{div}(g(\|\nabla u\|)\nabla u) \tag{1}$$

This is equivalent to solving a Perona and Malik nonlinear isotropic diffusion equation [3] for a fixed artificial timestep τ. The initial image u_0 is set to the average of the two input images u_1 and u_2. The sought-after solution is u. At the image boundary a homogeneous Neumann condition is applied. $g(\|\nabla u\|)$ is called an edge-stopping function regulating the diffusion process. Numerous edge-stopping functions have been proposed by the researching community, we have chosen to use the Tukey edge-stopping function introduced in [4] because of its good edge preserving properties.

For denoising CT images we have to modify $g(\|\nabla u\|)$ to achieve adequate results. Two ways of exploiting the availability of two input images with uncorrelated noise are to compute the correlation between both and to estimate noise variance. Because of the spatially varying noise properties in CT images these analysis is done locally in a neighborhood $\Omega_{\mathbf{x}}$ around a pixel $\mathbf{x}$. Additionally the influence of the neighboring pixels $\mathbf{i}$ is weighted with gaussian weights $w(\mathbf{i}, \mathbf{x})$ depending on the distance between pixel $\mathbf{i}$ and $\mathbf{x}$.

A local estimate for the correlation of two image regions can be computed by:

$$c_w(\mathbf{x}) = \frac{\sum_{\Omega_{\mathbf{x}}}(u_1(\mathbf{i}) - \bar{u_1})(u_2(\mathbf{i}) - \bar{u_2})w(\mathbf{i}, \mathbf{x})}{\sqrt{\sum_{\Omega_{\mathbf{x}}}(u_1(\mathbf{i}) - \bar{u_1})^2 w(\mathbf{i}, \mathbf{x}) \cdot \sum_{\Omega_{\mathbf{x}}}(u_2(\mathbf{i}) - \bar{u_2})^2 w(\mathbf{i}, \mathbf{x})}}, \tag{2}$$

$$C_w(\mathbf{x}) = \begin{cases} 1 & c_w(\mathbf{x}) > 0, \\ 0 & c_w(\mathbf{x}) \leq 0; \end{cases} \tag{3}$$

Because in our case only the amount of similarity between image regions is interesting, the values below 0 of the weighted correlation coefficient c_w, denoting anticorrelation, are set to 0, yielding in a local similarity measure C_w.

Two input images give us the possibility to estimate the local noise variance of the average of the input images by:

$$V(\mathbf{x}) = \frac{\sum_{\Omega_{\mathbf{x}}} w(\mathbf{i}, \mathbf{x})(u_1(\mathbf{i}) - u_2(\mathbf{i}))^2}{4\sum_{\Omega_{\mathbf{x}}} w(\mathbf{i}, \mathbf{x})}; \tag{4}$$

Based on the Tukey edge-stopping function we now derive a new function taking into account V and C_w. The fixed parameter for the noise standard derivation of the Tukey edge-stopping function is replaced by $V(\mathbf{x})$, with a parameter β serving as an additional weighting factor:

$$g_V(x) = \begin{cases} \left(1 - (\frac{x^2}{V(\mathbf{x})})\right)^2, & |x| \leq \beta\sqrt{V(\mathbf{x})}, \\ 0, & |x| > \beta\sqrt{V(\mathbf{x})}. \end{cases} \tag{5}$$

The square root of the product of the gradients on the input images is taken as the input for the edge-stopping function. It is further linearly scaled by the local similarity measure with a parameter λ. The idea behind this is to weaken high gradients in image regions with small similarity, e.g. in homogeneous regions, and to enlarge the gradient where similarity is high, i.e. when image structure is present.

$$g(\|\nabla u_{1,2}\|) = \begin{cases} g_V(x)(\|\nabla u_{1,2}\| \cdot W(\mathbf{x})) & \text{if } W(\mathbf{x}) > 0, \\ g_V(x)(0) & \text{else;} \end{cases} \tag{6}$$

where $\|\nabla u_{1,2}\| = \sqrt{\|\nabla u_1\| \cdot \|\nabla u_2\|}$ and

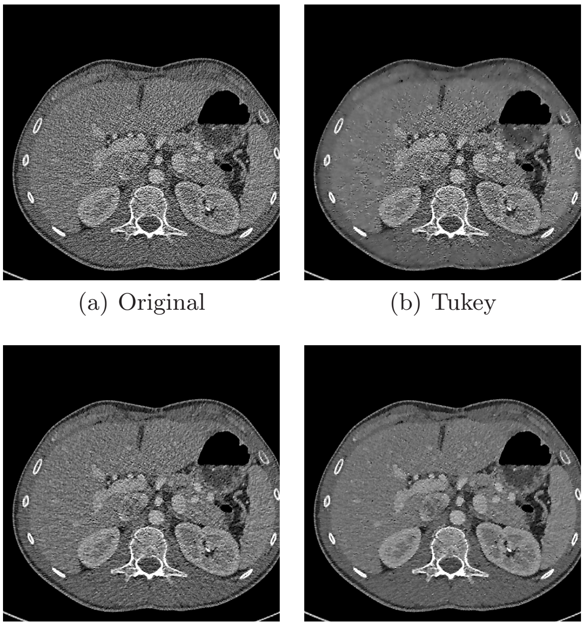
$$W(\mathbf{x}) = 1 + \lambda(2C_w(\mathbf{x}) - 1)). \tag{7}$$

This edge-stopping function is used in the Perona and Malik diffusion equation as presented in equation (1) to denoise CT images.

The diffused images of the two input images u_1 and u_2 and the average u_A are calculated for a fixed timestep τ. All diffusion processes are regulated by the same edge-stopping function $g(\|\nabla u_{1,2}\|)$. The gradients are discretized by finite differences and the equation system by finite volumes. The method is implemented both in 2D and 3D. The partial differential equation system is solved by a nonlinear multigrid solver [5,6]. The solver of the diffusion equation updates u_A, u_1 and u_2 simultaneously, calculating $g(\|\nabla u_{1,2}\|)$ from the current images u_1 and u_2 to preserve nonlinearity. The output of the denoising method is the diffused image u_A.

4 Results

Fig. 1 shows results from the proposed method and one standard nonlinear diffusion method applied to a thin reconstructed slice (0.8 mm) of a liver CT scan compared to the average of the input images, which reflects the result of a reconstruction of all projections. It is referred to as the original image. In Fig. 2 the difference images to the original image are shown, providing an impression of the denoising behavior of the different approaches. Fig. 1(b) clearly shows that a standard nonlinear diffusion method fails to denoise a CT image with spatially varying noise power in an adequate manner. While noise in the center of the image is nearly unchanged, the outer regions are already blurred. Using the



(a) Original (b) Tukey

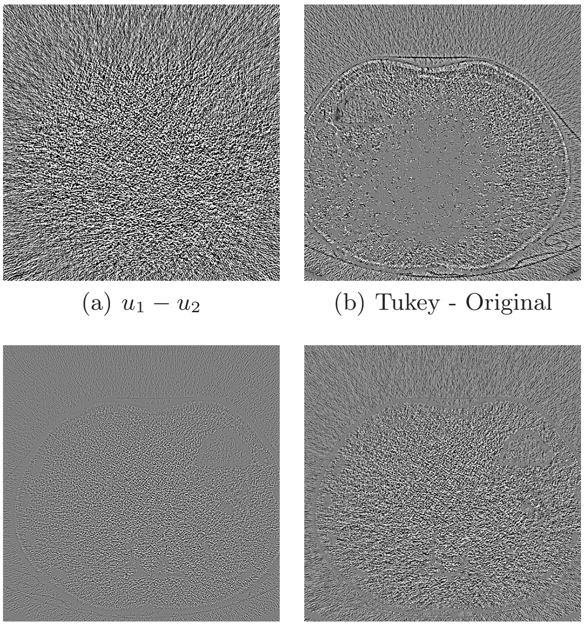
(c) 2D Proposed (d) 3D Proposed

Fig. 1: Denoising results for a CTA of a liver, displayed with $c = 200$ and $w = 700$.

proposed method in 2D, Fig. 1(c) shows that this method is capable of adapting itself to the local noise variance, thus removing noise more uniformly. A noise reduction of 45% is achieved throughout the image. To get a proper estimate of the correlation of the input images a gaussian window with a standard deviation of 2 was used in a 9×9 neighbourhood. Because image structures like edges have influence on the correlation value of distant pixels in their neighbourhood, unfortunately noise remains around high contrast edges. Hence, if a natural look of the image should not be sacrificed the noise suppression must be kept weak. Using 3D data reduces this problem, because pixels for estimating the noise variance and correlation can be taken from the neighbourhood in all three dimension. Fig. 1(d) shows the result using a gaussian window of standard deviation 1.5 in a $5 \times 5 \times 5$ neighbourhood. It can be seen clearly that a strong noise suppression of about 66% is achieved while image structure nearly remains unharmed.

5 Conclusions

A modified Perona-Malik diffusion process was presented that is able to deal with the special noise characteristics of CT data. The method surpasses standard diffusion methods due to its adaption on local noise variations and its regularizing of the diffusion depending on an estimation of the real image structure by calculating correlations between two input images with uncorrelated noise. A noise suppression rate of about 66% can be achieved.



(a) $u_1 - u_2$ (b) Tukey - Original

(c) 2D Proposed - Original (d) 3D Proposed - Original

Fig. 2: Difference images, displayed with $c = 0$ and $w = 200$.

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