Noise Adaptive Bilateral Filtering in Computed Tomography

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I. Introduction

Precise knowledge of the local image noise is important for the efficient application of post-processing methods such as bilateral filtering to computed tomography (CT) images. The non-stationary, object dependent noise in CT images directly results from the noise present in the projection data. Since quantum and electronics noise are the main noise sources, comparably simple physical models can be used for noise estimation in the individual projections.

II. Method

We developed a fast approximate method for analytic propagation of these noise estimates through indirect fan-beam filtered backprojection (FBP) reconstruction [1]. Mainly, all single steps of the reconstruction pipeline (interpolations, convolution and backprojection) can be expressed as linear combinations of noisy data or random variables. Starting from the estimated noise variance in the fan-beam projections the variance of a linear combination of random variables is computed. Furthermore, we approximate at every step the influence of the processing to the correlation of the data and take this into account for the covariance computation. Finally, we get for every pixel position $\mathbf{x} = (x, y)$ of the reconstructed CT image $f(\mathbf{x})$ an estimate of the noise variance $\sigma^2(\mathbf{x})$.

Especially for objects with directed noise due to strong attenuation along certain directions we would like to have separate noise estimates for the horizontal (H) and vertical (V) directions, such that $\sigma^2(\mathbf{x}) = \sigma^2_{\mathrm{H}}(\mathbf{x}) + \sigma^2_{\mathrm{V}}(\mathbf{x})$. This is achieved by computing the contribution of noise for every ray of the fan-beam projections to the horizontal and vertical directions and separate noise propagation through the reconstruction algorithm as described above.

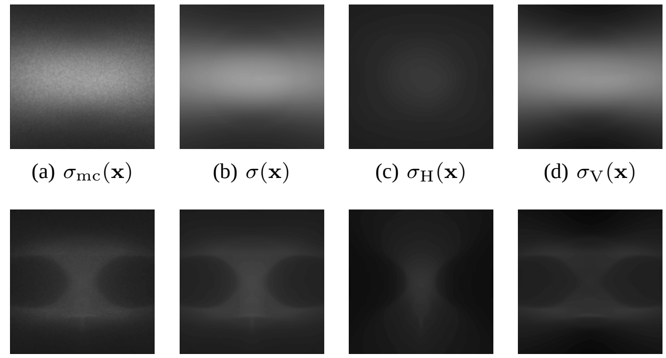
The orientation dependent noise estimates can be used for adapting post-processing methods, e.g. bilateral filtering [2] to the non-stationary and directed noise in CT. The filtered CT image $\tilde{f}(\mathbf{x})$ is computed as follows:

$$\tilde{f}(\mathbf{x}) = \frac{1}{k(\mathbf{x})} \sum_{\mathbf{x}'} f(\mathbf{x}') \cdot c(\mathbf{x}, \mathbf{x}') \cdot s(f(\mathbf{x}), f(\mathbf{x}')), \quad (1)$$

where $k(\mathbf{x})$ is needed for normalization. The domain-filter $c(\mathbf{x}, \mathbf{x}')$ takes into account the geometric closeness of pixels $\mathbf{x}$ and $\mathbf{x}'$, and the range-filter $s(f(\mathbf{x}), f(\mathbf{x}'))$ considers the photometric closeness of the intensity values during averaging. Instead of using simple Gaussian filters as in [2], we now propose to adapt both parts to the local, orientation dependent noise estimates: We define the domain-filter to be a multivariate Gaussian filter and the range-filter is built as a separable Gaussian, that takes into account the horizontal and vertical noise estimates in order to allow stronger smoothing in the direction of stronger noise.

III. Results

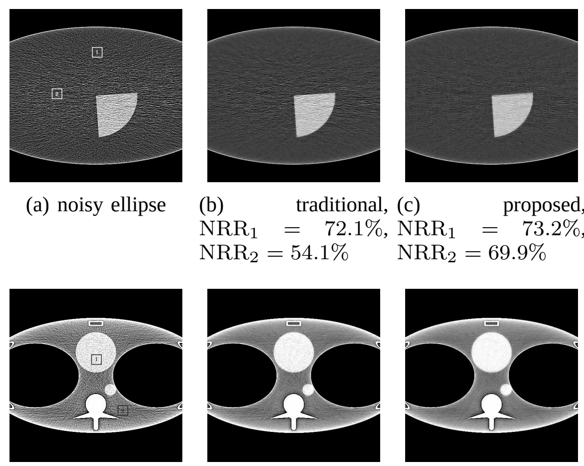
In Fig. 1 the results of the noise propagation are displayed for two examples. In Fig. 2 the corresponding two noisy phantoms are displayed together with their denoising results from traditional and our noise adaptive bilateral filtering method. We measured the standard deviation of noise in two pixel regions, the first region with more homogeneous noise and the second one with strong streaks (see Fig. 2(a) and 2(d)). The noise reduction rates (NRR) in percent are also presented in Fig. 2. It can be seen that especially in regions with streaks the proposed method clearly outperforms the traditional filtering approach at comparable resolution.



(a) $\sigma_{\mathrm{mc}}(\mathbf{x})$ (b) $\sigma(\mathbf{x})$ (c) $\sigma_{\mathrm{H}}(\mathbf{x})$ (d) $\sigma_{\mathrm{V}}(\mathbf{x})$

(e) $\sigma_{\mathrm{mc}}(\mathbf{x})$ (f) $\sigma(\mathbf{x})$ (g) $\sigma_{\mathrm{H}}(\mathbf{x})$ (h) $\sigma_{\mathrm{V}}(\mathbf{x})$

Fig. 1. Analytic noise propagations (top: ellipse, bottom: thorax) compared to Monte Carlo results (σ_{mc}) from 100 noisy CT images. Additionally, the noise contribution to the horizontal and vertical direction are shown (displayed with $c = 50$ and $w = 100$).



(a) noisy ellipse (b) traditional, $\mathrm{NRR}_1 = 72.1\%$, $\mathrm{NRR}_2 = 54.1\%$ (c) proposed, $\mathrm{NRR}_1 = 73.2\%$, $\mathrm{NRR}_2 = 69.9\%$

(d) noisy thorax (e) traditional, $\mathrm{NRR}_1 = 61.4\%$, $\mathrm{NRR}_2 = 54.2\%$ (f) proposed, $\mathrm{NRR}_1 = 76.0\%$, $\mathrm{NRR}_2 = 69.6\%$

Fig. 2. Noisy examples (top:ellipse, bottom:thorax) and denoised results from traditional bilateral filtering compared to the proposed noise adaptive bilateral filtering (displayed with: top $c = 50$ and $w = 200$, bottom $c = 50$ and $w = 100$).The regions for noise evaluation are marked in (a) and (d).

IV. Conclusions

We proposed a method for orientation dependent noise estimation in CT images. These noise estimates are then used for adapting bilateral filters to the non-stationary and non-isotropic noise in the CT image. This adaptation leads to improved noise suppression at comparable resolution.

References

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[2] C. Tomasi and R. Manduchi, "Bilateral filtering for gray and color images," in *IEEE International Conference on Computer Vision*, Bombay, India, 1998, pp. 836–846, http://www.cse.ucsc.edu/˜manduchi/Papers/ICCV98.pdf.

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