FDK-Type Image Reconstruction from Cone Beam Projections for Reverse Helical Trajectory

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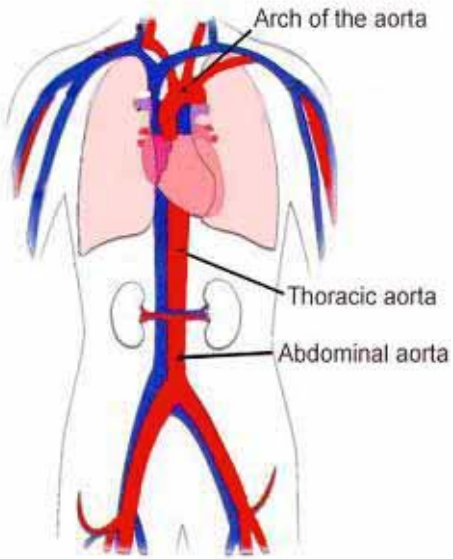
Outline

- Introduction
- Reverse Helical Trajectory
- 2D Fan-Beam Reconstruction
- 3D reconstruction
  - Scan configuration
  - Reconstruction method I: FFDK
  - Reconstruction method II: SSFDK
- Results
- Summary and Outlook

Introduction



- Topic: long object imaging using C-arm
- Problem: rotation angle limited
- Proposal: reverse helical trajectory(1,2,3)

(1): Exact Image Reconstruction in Reverse Helical Cone-Beam CT, Cho. et.al., Fully 3D, 2007
(2): On the problem of axial data truncation in the reverse helix geometry, Noo. et.al., Fully 3D, 2009
(3): A BPF-FBP tandem algorithm for image reconstruction in reverse helical cone-beam CT, Cho. et.al., Med. Phys., 2010

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Reverse helical trajectory

Reverse Helix

Pi-line surface: Σ_1^+

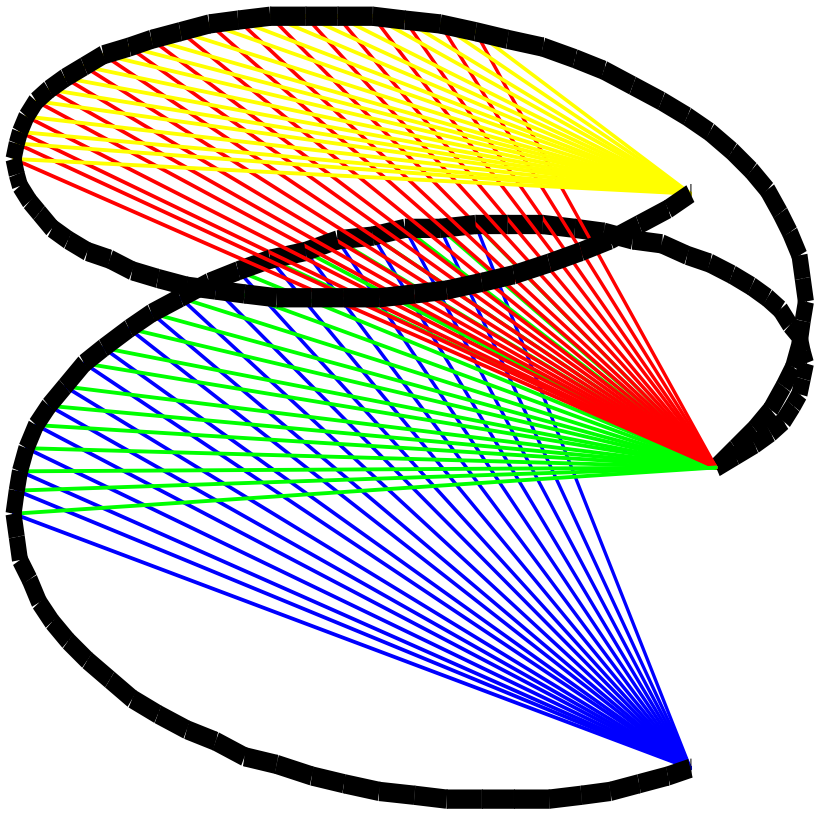
Pi-line surface: Σ_0^+

Pi-line surface: Σ_0^-

Pi-line surface: Σ_1^-

$$\pi - \text{line region}: [\Sigma_0^+, \Sigma_1^+] \,\&\, [\Sigma_1^-, \Sigma_0^-]$$

$$\text{core of missing } \pi\text{-line}: \quad [\Sigma_0^-, \Sigma_0^+]$$



Ref: A solution to the long-object problem in helical cone-beam tomography, Defrise. et.al, PMB, 2000

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2D Fan-Beam Reconstruction I

- Conventional FBP reconstruction(FBP)

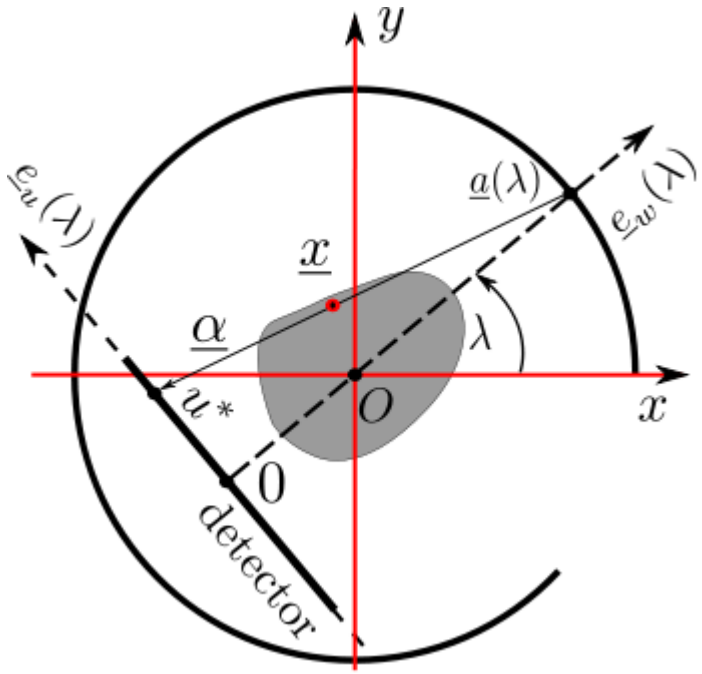
$$f(x) = \int_{\Lambda} d\lambda \frac{RD}{(R - \underline{x} \cdot \underline{e}_w(\lambda))^2} \int_{-u_m}^{u_m} du \frac{D}{\sqrt{u^2 + D^2}} m(\lambda, u) g(\lambda, u) h_F(u^* - u)$$

Reconstruction Steps:

1: weighting

2: filtering(ramp)

3: back projection



2D Fan-Beam Reconstruction II

- Super short scan FBP reconstruction(SSFBP)

$$f(x) = \frac{1}{2\pi}\int_{\Lambda} d\lambda \frac{1}{R - \underline{x}\cdot\underline{e}_w(\lambda)} m(\lambda, u^*) g_F(\lambda, u^*)$$

$$\text{where: } g_F(\lambda, u^*) = \int_{-u_m}^{u_m} du \frac{D}{\sqrt{u^2 + D^2}} h_H(u^* - u) g'\left(\lambda, \underline{\alpha}(\lambda, u)\right)$$

Reconstruction steps:

1: differentiation over λ

2: filtering(Hilbert)

3: weighting

4: back projection

Ref: Image reconstruction from fan-beam projections on less than a short scan, Noo. et.al, PMB,2002

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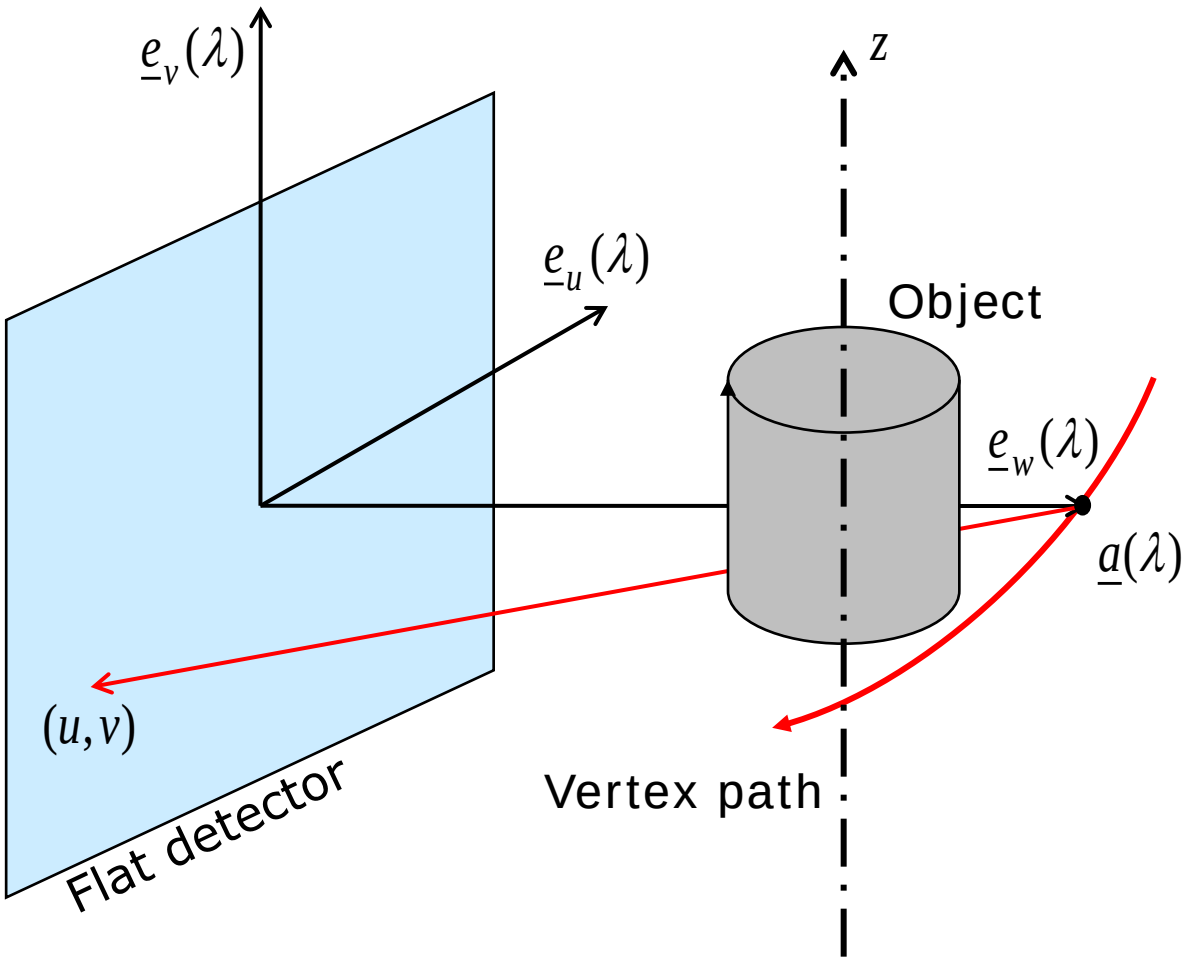
Scan geometry



Common parameters
detector resolution(cm): (du,dv) = (0.0616, 0.0616)
focus to detector distance: D=119.9cm
scan radius: R=78.5cm
Angular range =1.2pi
Projection: 2 projections per degree

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FDK

$$f(x) = \int_{\Lambda} d\lambda \frac{RD}{\left(R - \underline{x} \cdot \underline{e}_w(\lambda)\right)^2} g_F\left(\lambda, u^*, v^*\right)$$

$$\text{where}: g_F(\lambda, u, v) = \int_{-u_m}^{u_m} du' \frac{Dh_F(u - u')}{\sqrt{u'^2 + v^2 + D^2}} m(\lambda, u') g(\lambda, u', v)$$

$$\text{with}: \mathrm{u}^* = \frac{\mathrm{D}(\underline{\mathrm{x}} - \underline{\mathrm{a}}(\lambda)) \cdot \underline{\mathrm{e}}_u(\lambda)}{R - \underline{x} \cdot \underline{e}_w(\lambda)} \quad \text{and} \quad \mathrm{v}^* = \frac{\mathrm{D}(\underline{\mathrm{x}} - \underline{\mathrm{a}}(\lambda)) \cdot \underline{\mathrm{e}}_v(\lambda)}{R - \underline{x} \cdot \underline{e}_w(\lambda)}$$

Steps:

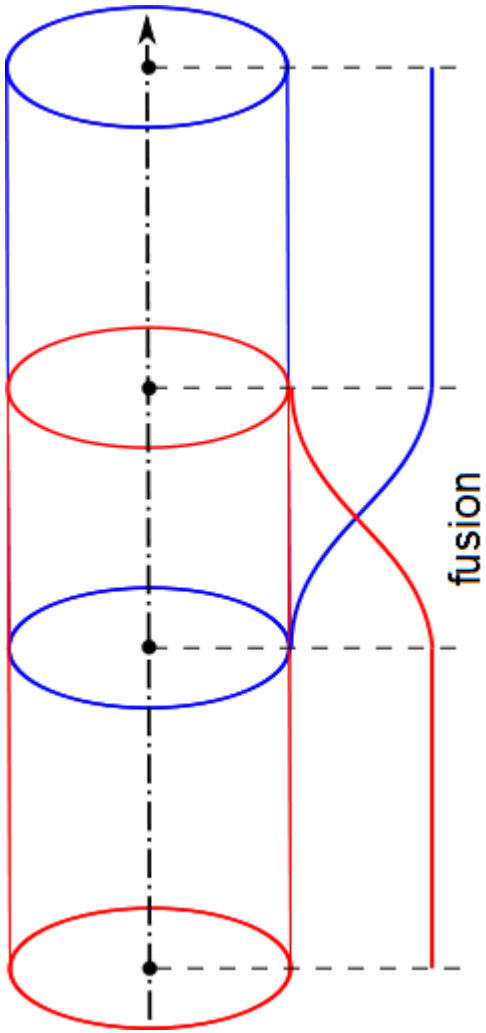
1: weighting

2: filtering along the horizontal line

3: back-projection within Λ (the angle position of the helical path)

Ref: Practical cone-beam algorithm, Feldkamp. et.al, J.Opt.Soc.,1984

Reconstruction method I: FFDK(Fusion-FDK)



Reconstruct the blue volume using the projection data from the top helix only

Reconstruct the red volume using the projection data from the bottom helix

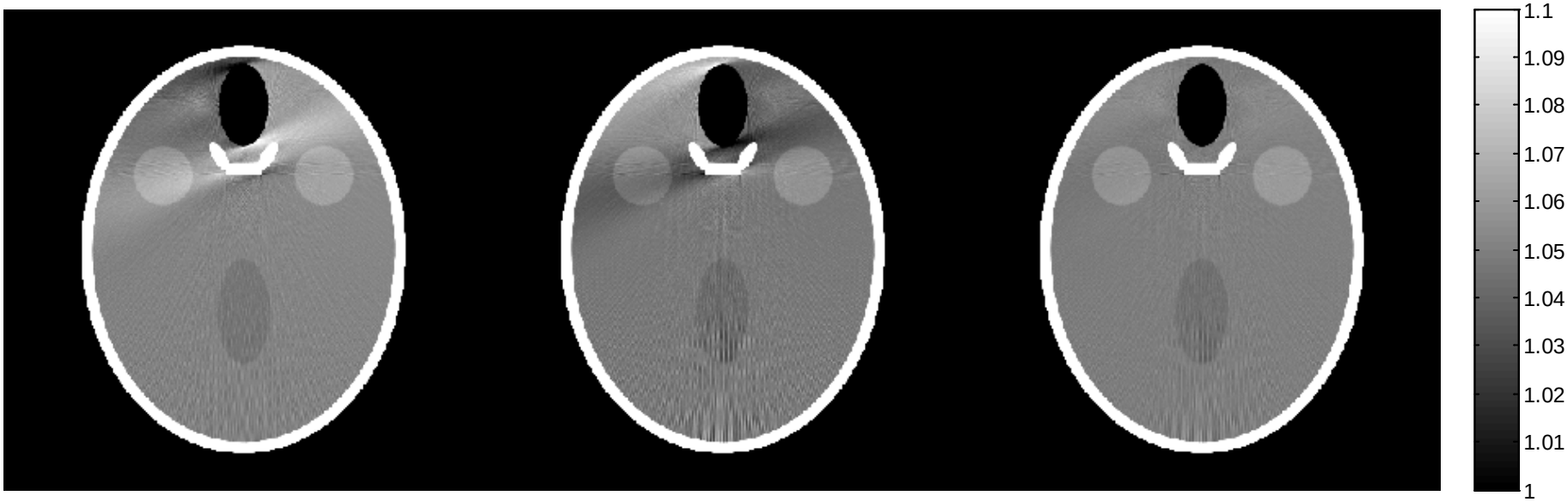
Combine the blue and red volume by feathering along the overlapped region

Pros: --smooth the reconstruction of the fusion region
--cooperate more data into the reconstruction in fusion region

Cons: --reconstruction using data not satisfying the Tuy's condition
--fusion increases the detector requirement

Example

$$z = 1.96cm @ \mathrm{EXP003}$$



from top half helix from bottom half helix after fusion

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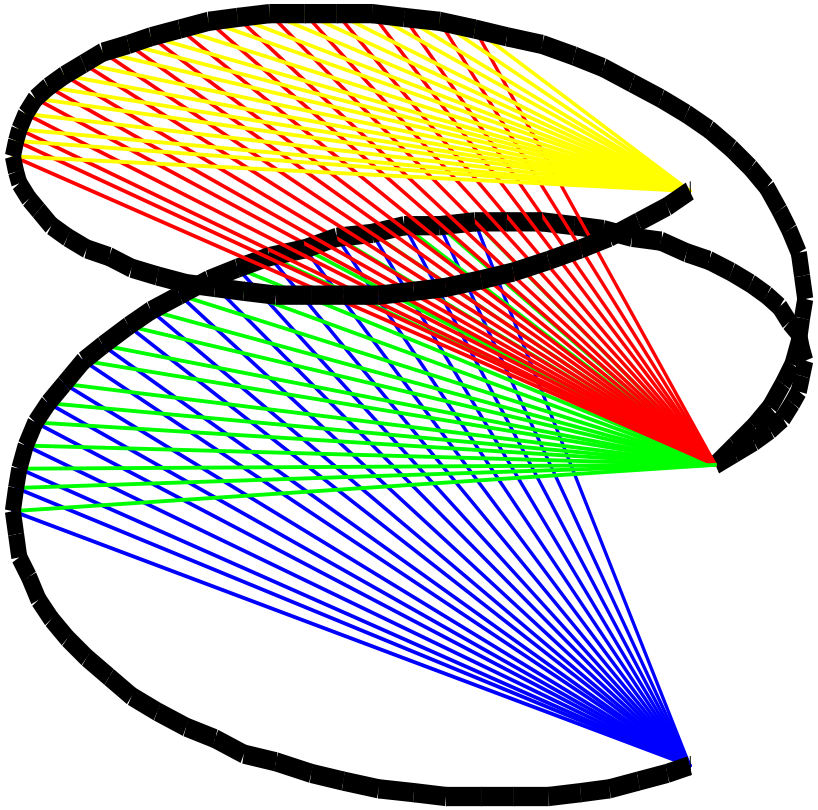
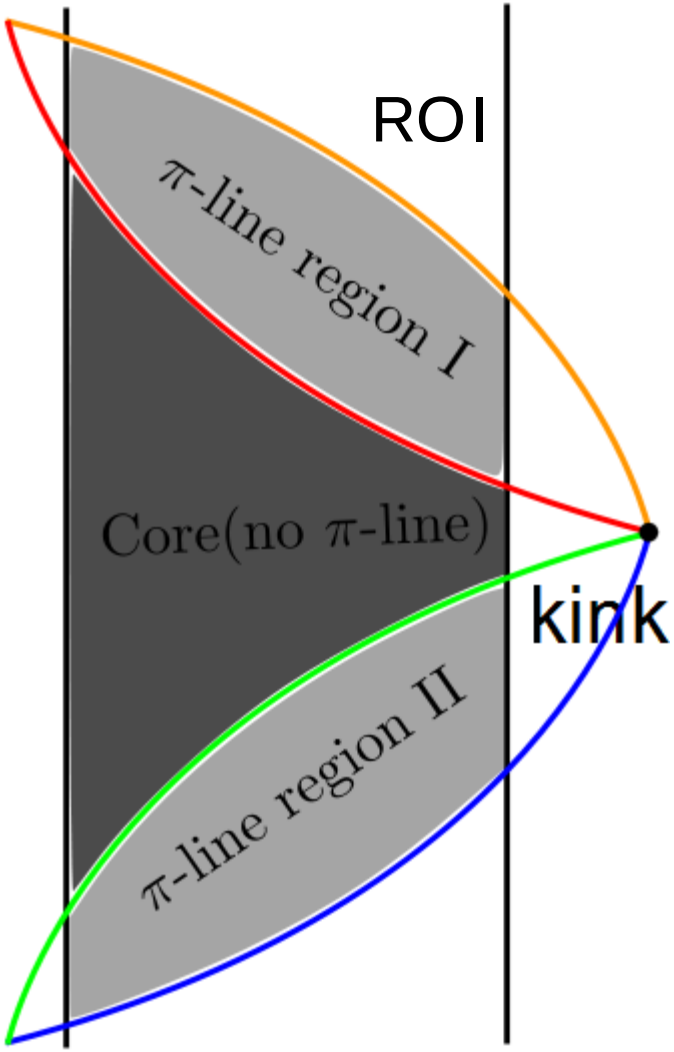
Reconstruction method II—SSFDK(Super Short-FDK)





Reconstruction method II: SSFDK

$$f(x) = \frac{1}{2\pi}\int_{\Lambda} d\lambda \frac{m(\lambda, \underline{x})}{R - \underline{x}\cdot\underline{e}_w(\lambda)} g'_F(\lambda, u^*, v^*)$$

$$\text{where}: g_F(\lambda, u, v) = \int_{-u_m}^{u_m} du' \frac{\sqrt{D^2 + v^2}}{\sqrt{u'^2 + v^2 + D^2}} g'(\lambda, \underline{\alpha}(\lambda, u, v)) h_H(u - u')$$

$$\text{with}: \mathrm{u}^* = \frac{\mathrm{D}(\underline{\mathrm{x}} - \underline{\mathrm{a}}(\lambda))\cdot\underline{\mathrm{e}}_u(\lambda)}{R - \underline{x}\cdot\underline{e}_w(\lambda)} \quad \text{and} \quad \mathrm{v}^* = \frac{\mathrm{D}(\underline{\mathrm{x}} - \underline{\mathrm{a}}(\lambda))\cdot\underline{\mathrm{e}}_v(\lambda)}{R - \underline{x}\cdot\underline{e}_w(\lambda)}$$

Steps:

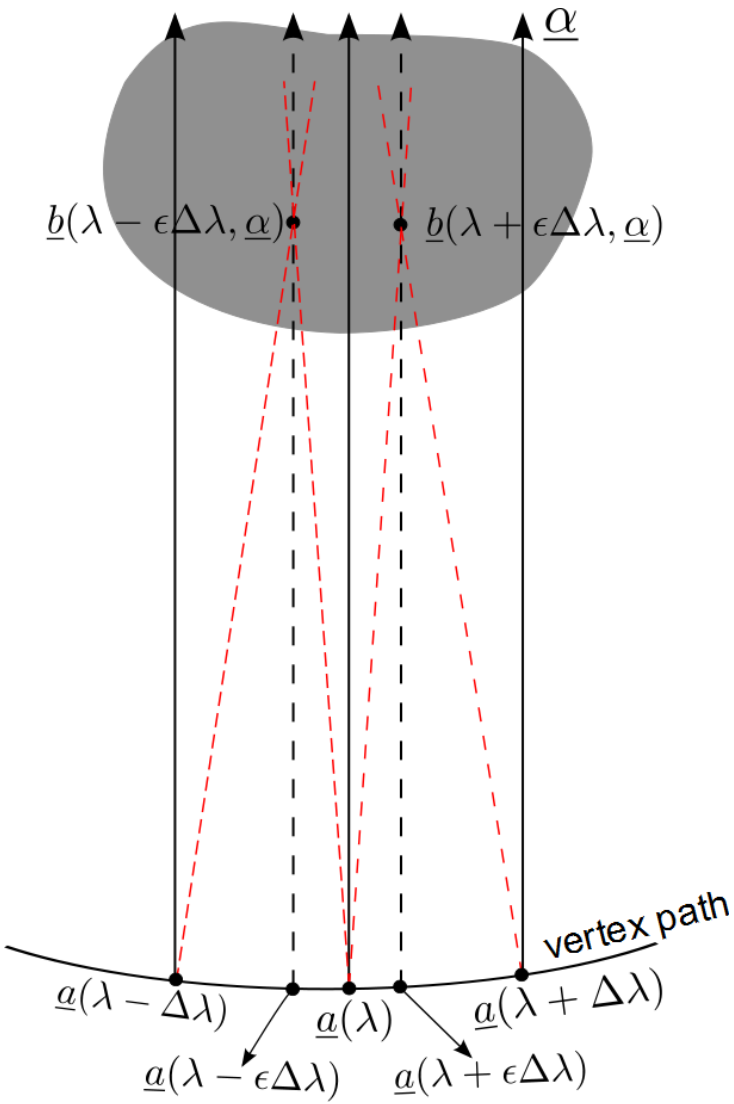
1: differentiation
2: filtering
3: weighting
4: back-projection

SSFDK--Derivation

$$g'(\lambda,\underline{\alpha}) = \partial g(\lambda,\underline{\alpha})/\partial\lambda \ \text{ for fixed } \underline{\alpha}$$

1: blended chain-rule (1)
2: split chain-rule(1)
3: scheme from Faridani (2)
4: new scheme from Noo 2007 (1)

$$g'(\lambda,\underline{\alpha}) = \frac{g(\lambda+\varepsilon\Delta\lambda,\underline{\alpha}) - g(\lambda-\varepsilon\Delta\lambda,\underline{\alpha})}{2\varepsilon\Delta\lambda}$$

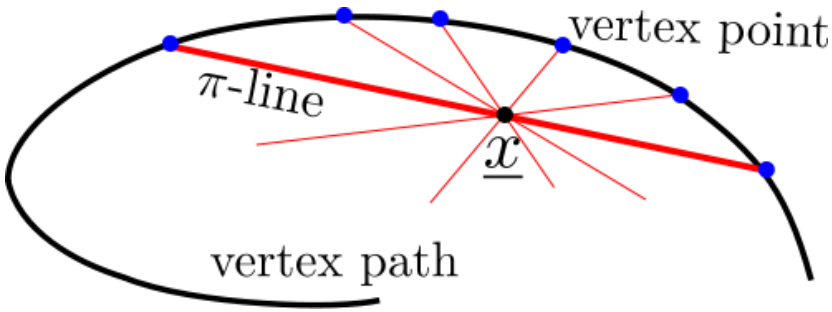


(1): A new scheme for view-dependent data differentiation in fan-beam and cone-beam computed tomography, Noo. et.al, PMB, 2007

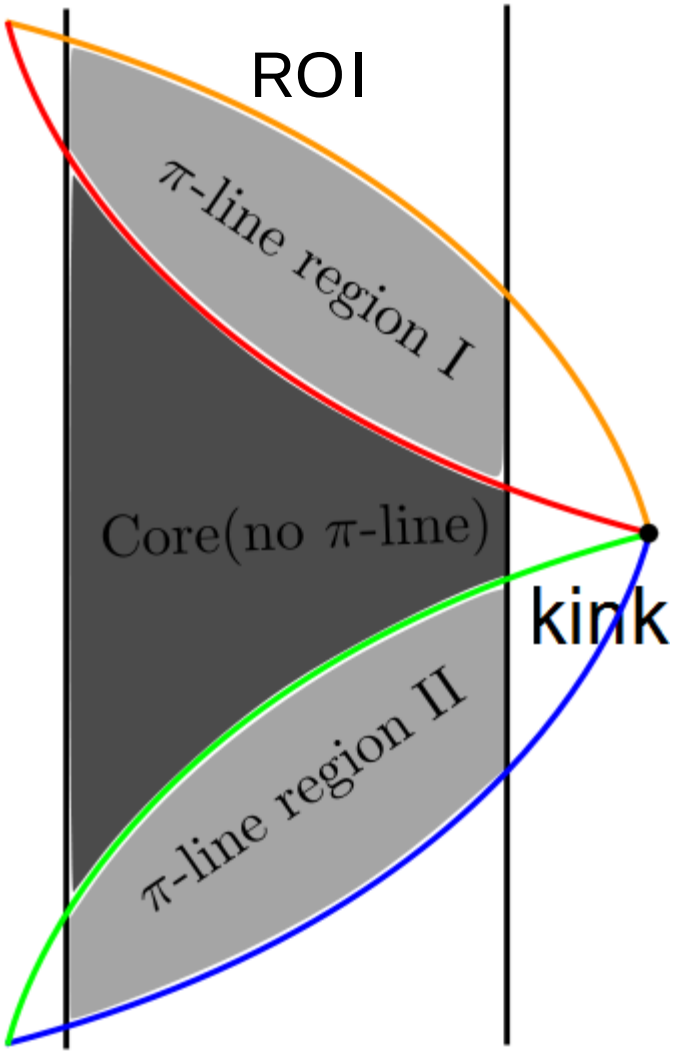
(2): Numerical and theoretical explorations in helical and fan-beam tomography, Faridani. et.al, Journal of physics, conference series 124, 2008

SSFDK--weighting

weighting for the π - line region

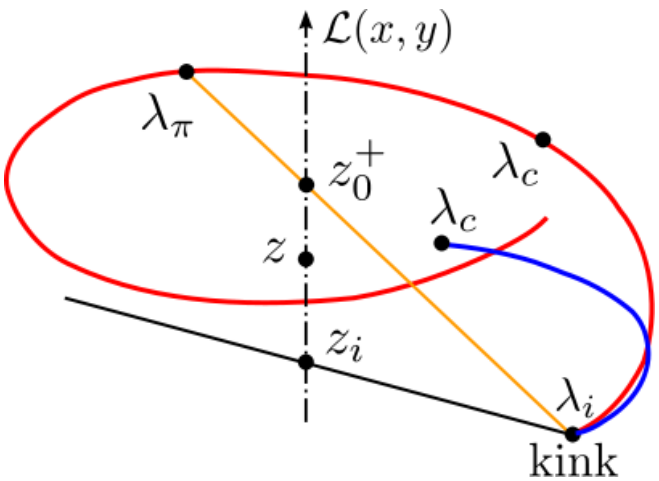


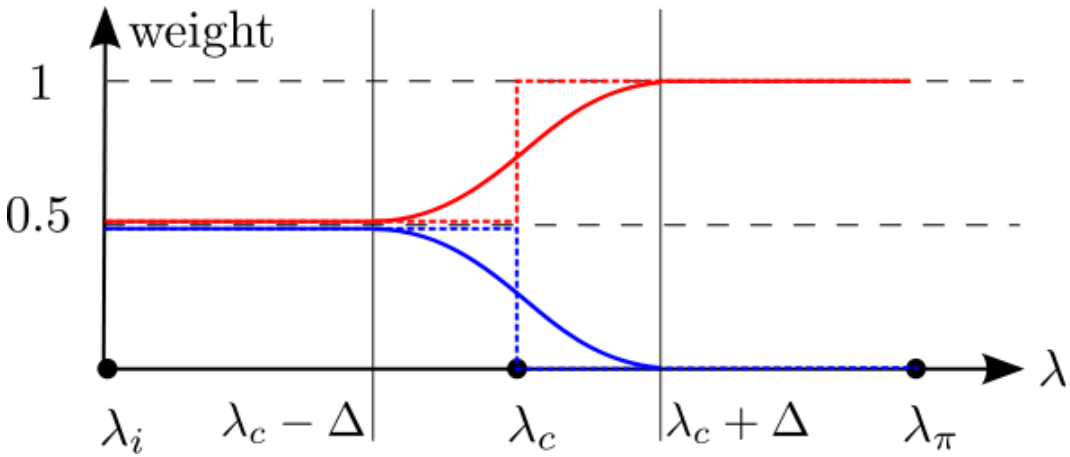
Each point in region I and II is covered by one and only one π - line . Therefore, in these two regions, we reconstruct each point from projections within its π – line.



SSFDK--weighting

weighting for the core





$$\lambda_c = \frac{z_0^+ - z}{z_0^+ - z_i} \times (\lambda_\pi - \lambda_i) + \lambda_i$$

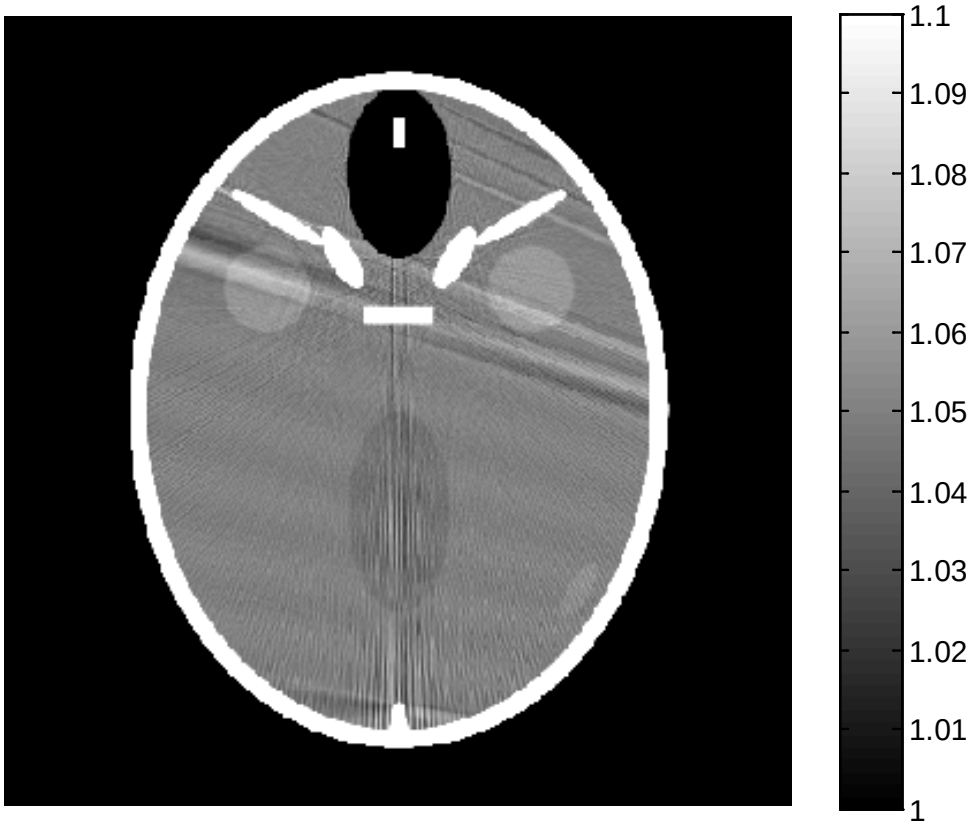
$$\Delta = \beta \min(\lambda_\pi - \lambda_c, \lambda_c - \lambda_i)$$

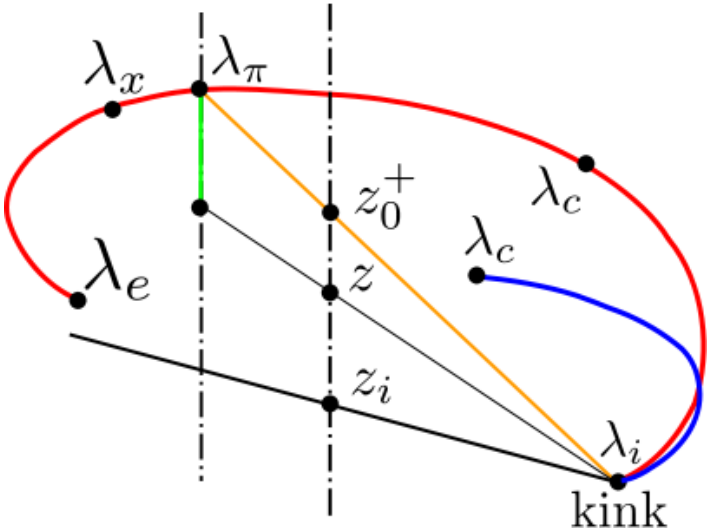
$$\text{with}: \beta \in (0, 0.5)$$

$$w(\lambda) = \begin{cases} 1 & \lambda_c + \Delta \le \lambda \le \lambda_\pi \\ \frac{1}{2} + \frac{\sin^2}{2}\left(\frac{(\lambda + \Delta - \lambda_c)\pi}{4\Delta}\right) & \lambda_c - \Delta \le \lambda < \lambda_c + \Delta \\ \frac{1}{2} & \lambda_i \le \lambda < \lambda_c - \Delta \\ 0 & \text{otherwise} \end{cases}$$

SSFDK--weighting

Problem shooting

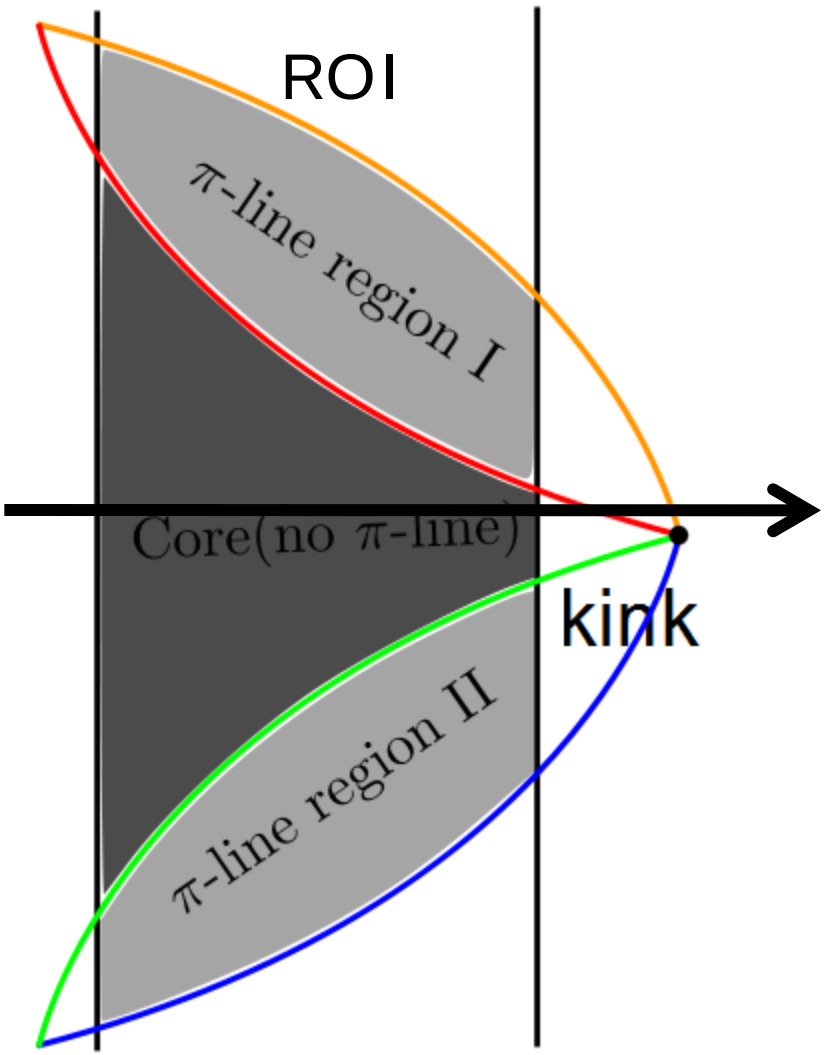


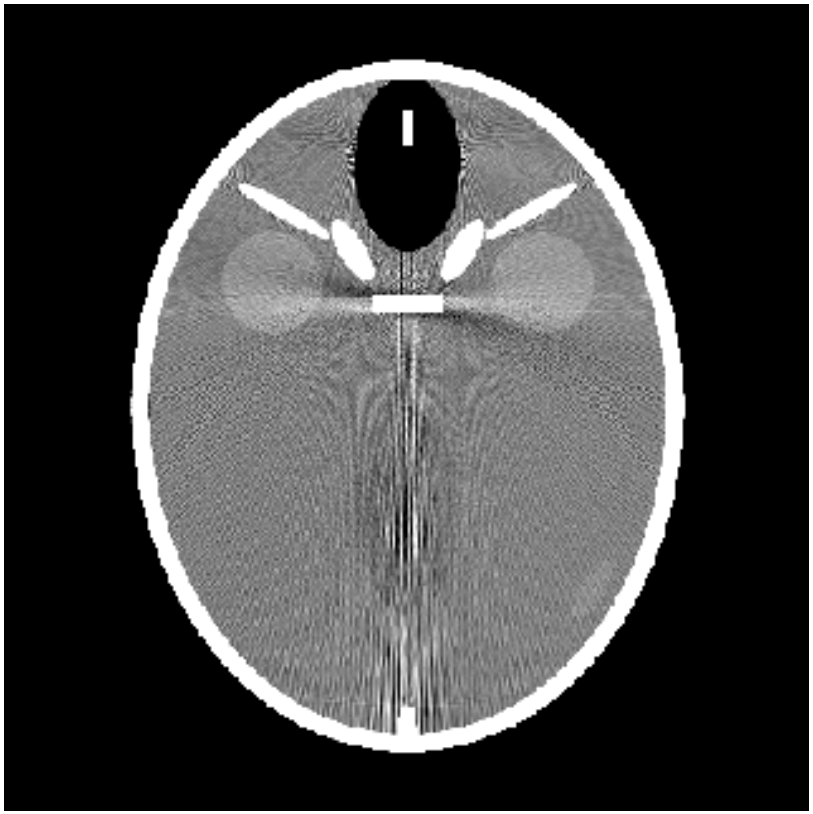


$$\lambda_x = \frac{z_0^+ - z}{z_0^+ - z_i} \times (\lambda_e - \lambda_\pi) + \lambda_\pi$$

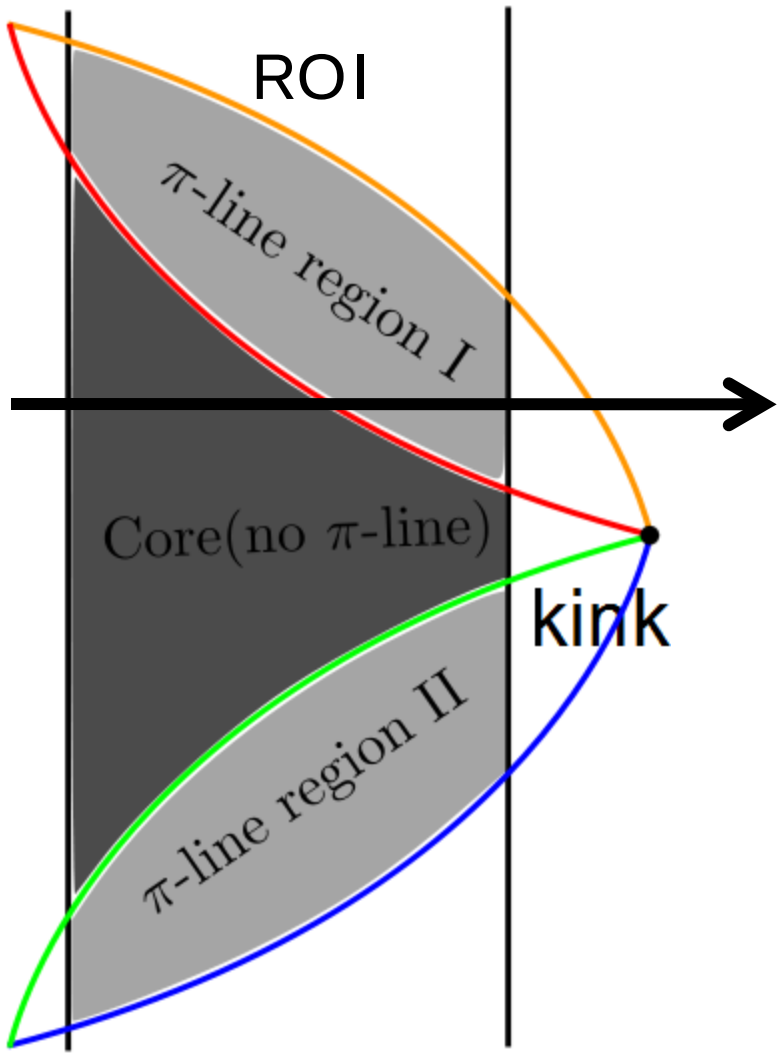
Parker's weighting for $[\lambda_i, \lambda_x]$

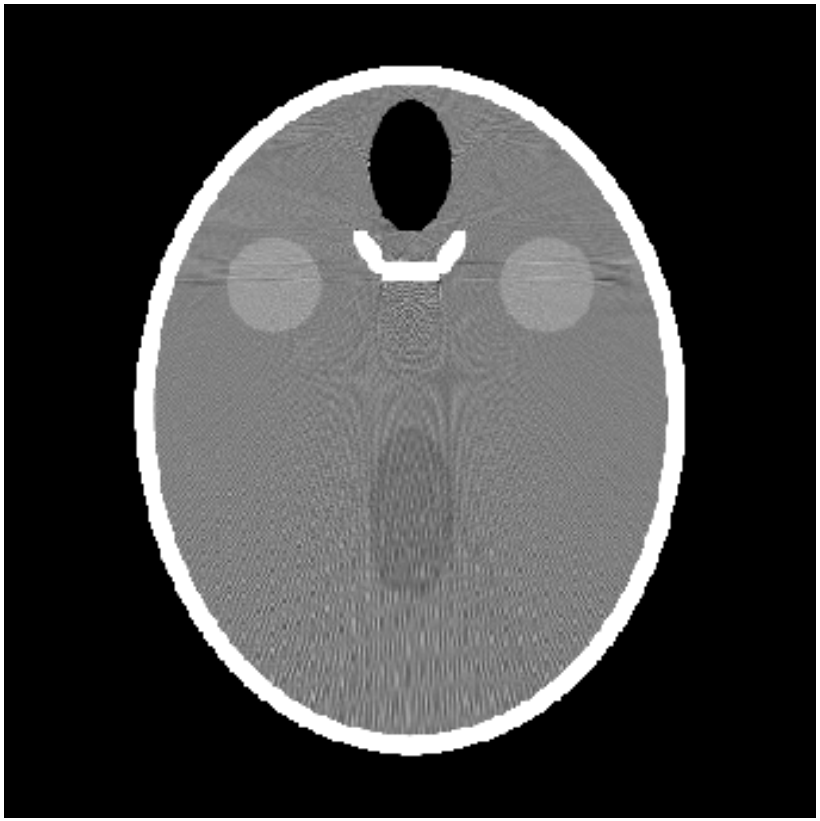
Example



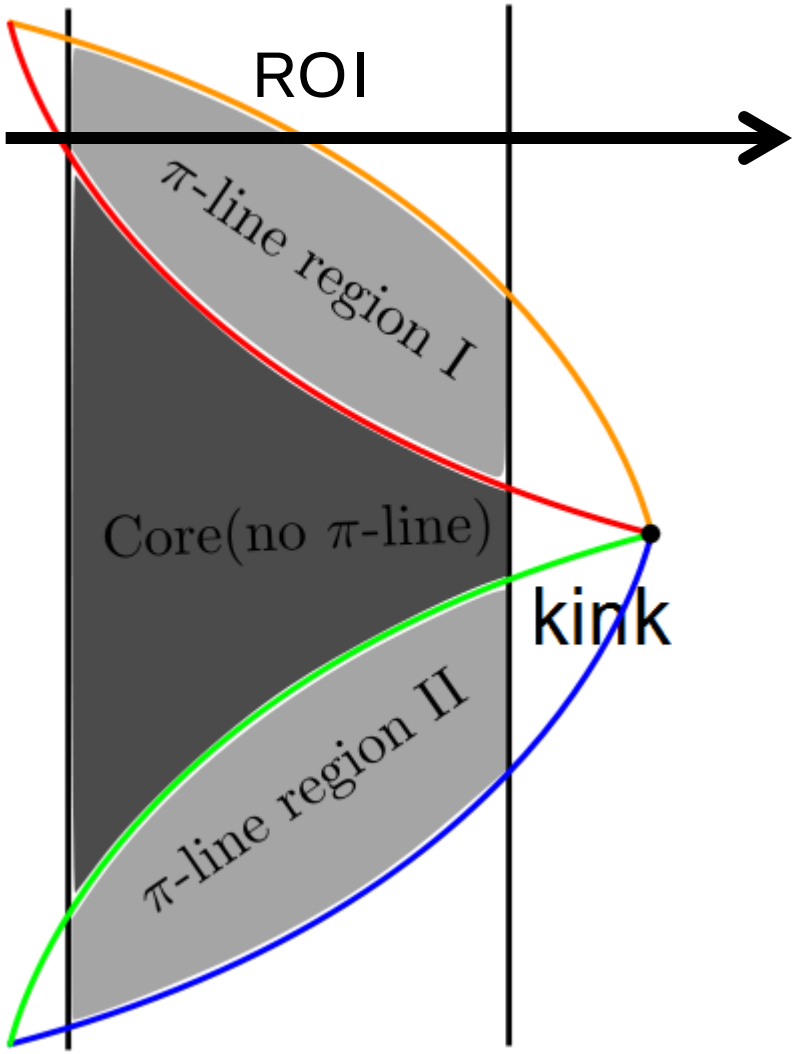


Example





Example



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Experiments overview

EXP001: HEAD	
pitch	10cm
kink	-0.44cm
detector	38*38cm
fusion	-2.8~2.8cm

EXP002: HEAD	
pitch	20cm
kink	-0.88cm
detector	38*38cm
fusion	-2.8~2.8cm

EXP003: HEAD	
pitch	20cm
kink	2cm
detector	48*48cm
fusion	-0.84~4.76cm

EXP004: THORAX	
pitch	10cm
kink	-0.44cm
detector	89*89cm
fusion	-5.4~4..6cm

EXP005: THORAX	
pitch	20cm
Kink	-0.88cm
Detector	89*89cm
Fusion	-5.9~5.1cm

Results: Experiment 001

y

x

z=-0.42cm FFDK

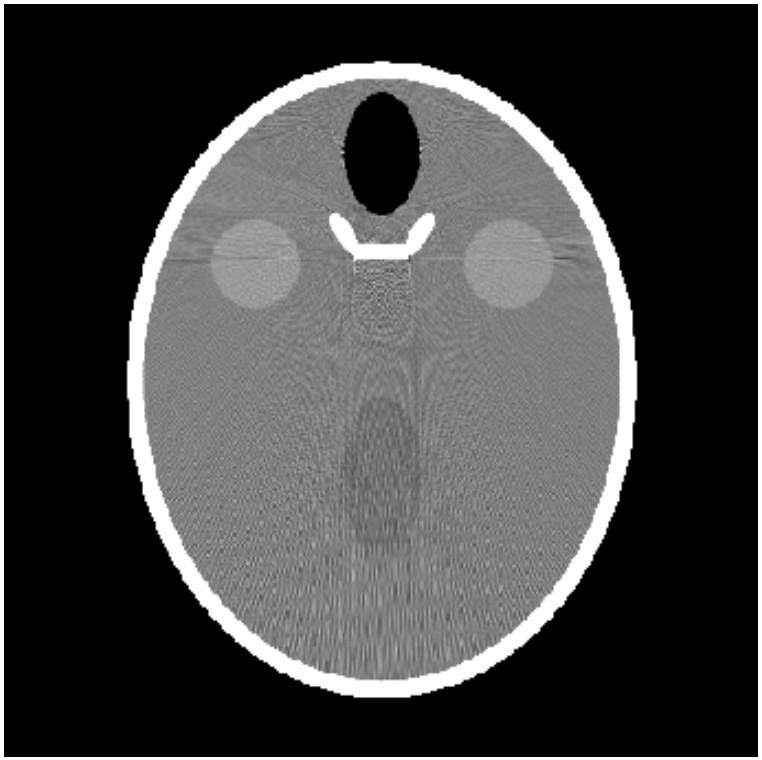
z=-0.42cm SSFDK

Angular range : 1.2π; Kink : -0.44cm; Pitch : 10cm;

Fusion : $z \in$ [-2.8cm, 2.8cm]; No truncation : [-5.5cm 4.6cm]

Results: Experiment 001

z=2.03cm FFDK

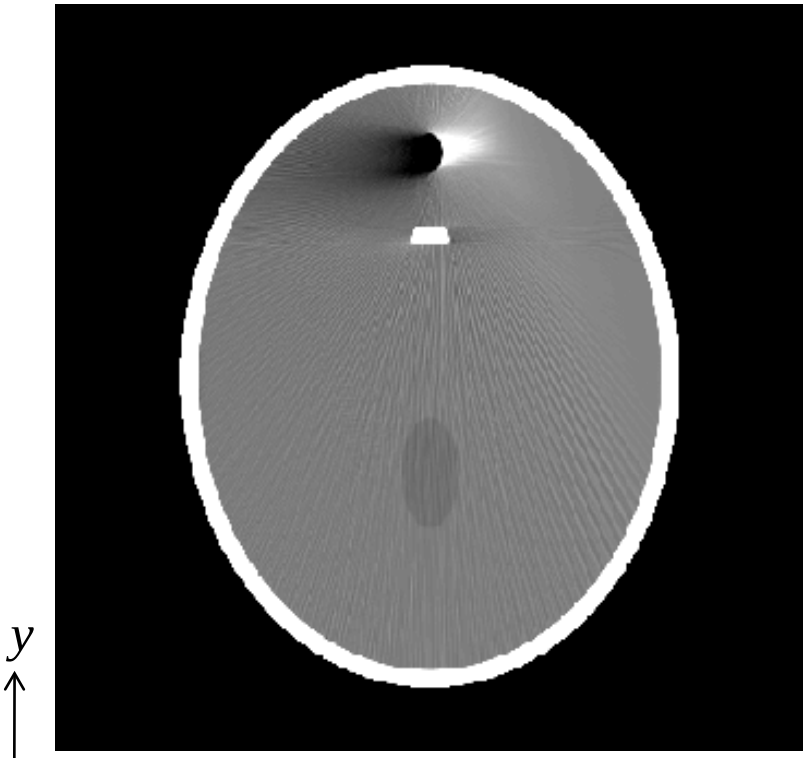


z=2.03cm SSFDK

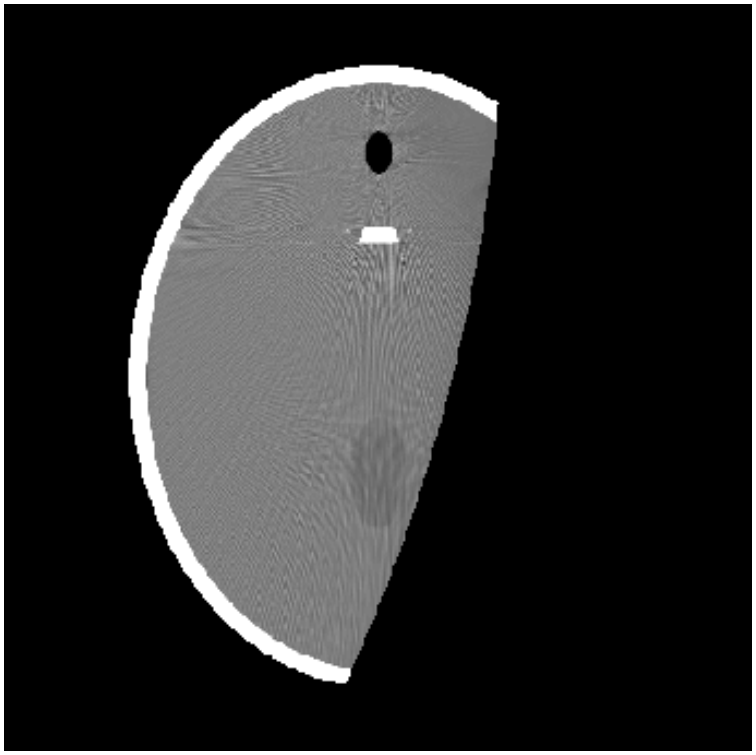
Angular range : 1.2π; Kink : - 0.44cm; Pitch : 10cm;

Fusion : $z \in$ [-2.8cm, 2.8cm]; No truncation : [-5.5cm 4.6cm]

Results: Experiment 001



z=2.94cm FFDK



z=2.94cm SSFDK

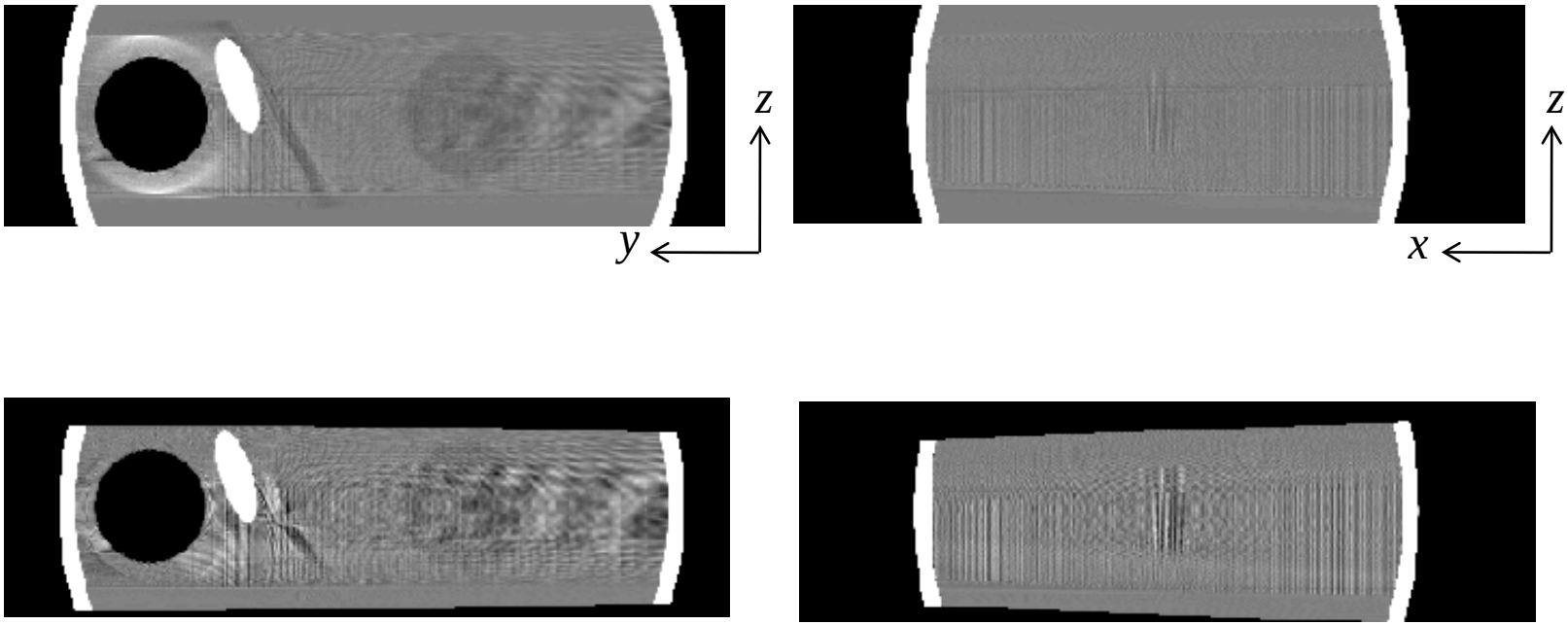
Angular range : 1.2π; Kink : - 0.44cm; Pitch : 10cm;

Fusion : $z \in$ [-2.8cm, 2.8cm]; No truncation : [-5.5cm 4.6cm]

Results: Experiment 001

FFDK

SS FDK

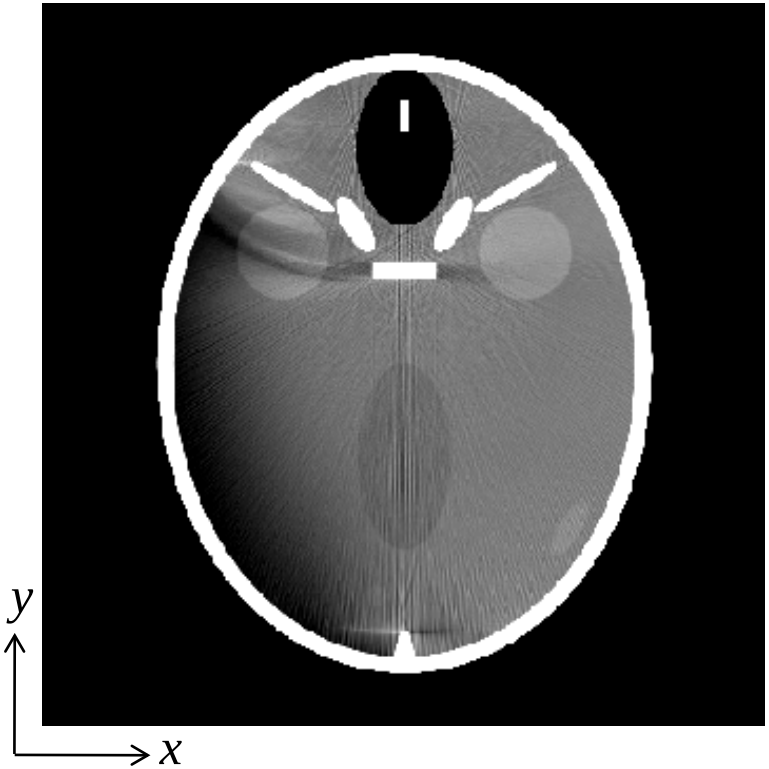


(a) $x = 1.44cm \; z \in [-4.6cm, 3.8cm]$ (b) $y = 0 \quad z \in [-4.6cm, 3.8cm]$

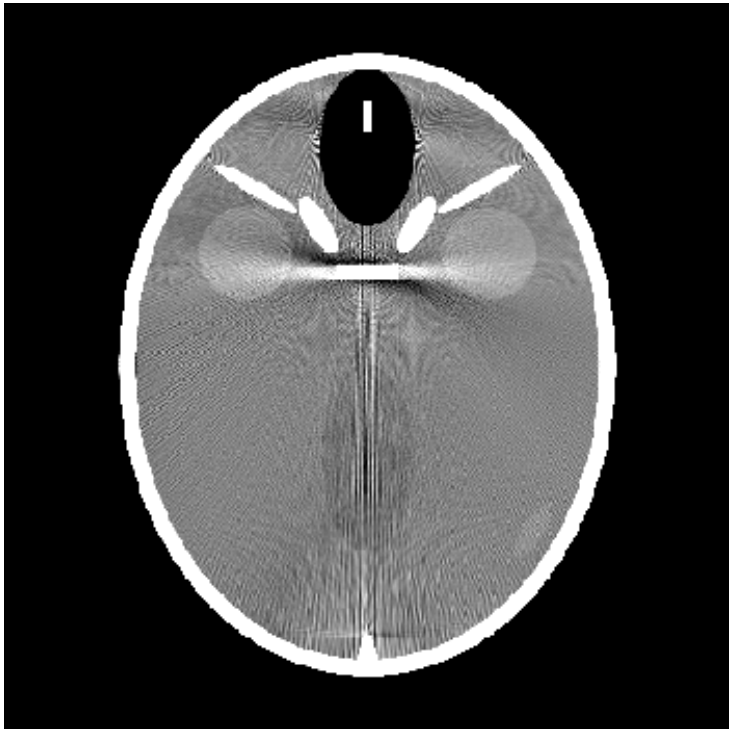
Angular range : 1.2π; Kink : - 0.44cm; Pitch : 10cm;

Fusion : z $\in$ [-2.8cm, 2.8cm]; No truncation : [-5.5cm 4.6cm]

Results: Experiment 002



z = 0, FFDK



z = 0, SSFDK

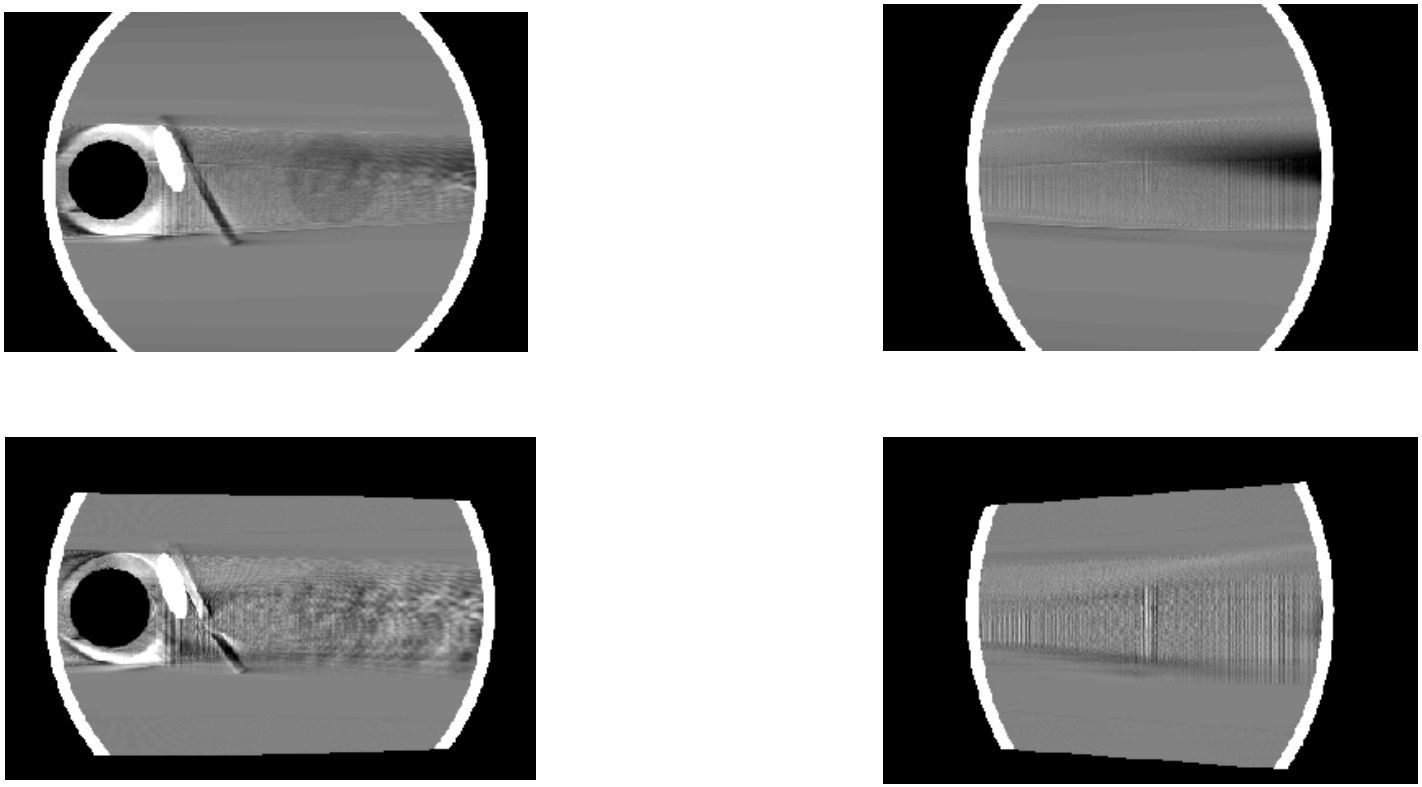
Angular range : 1.2π; Kink : $z = -0.88$cm; Pitch : 20cm;

Fusion : $z \in [-2.8\text{cm}\ 2.8\text{cm}]$; Fusion zone is only visible by one half helix

Results: Experiment 002

FFDK

SSFDK



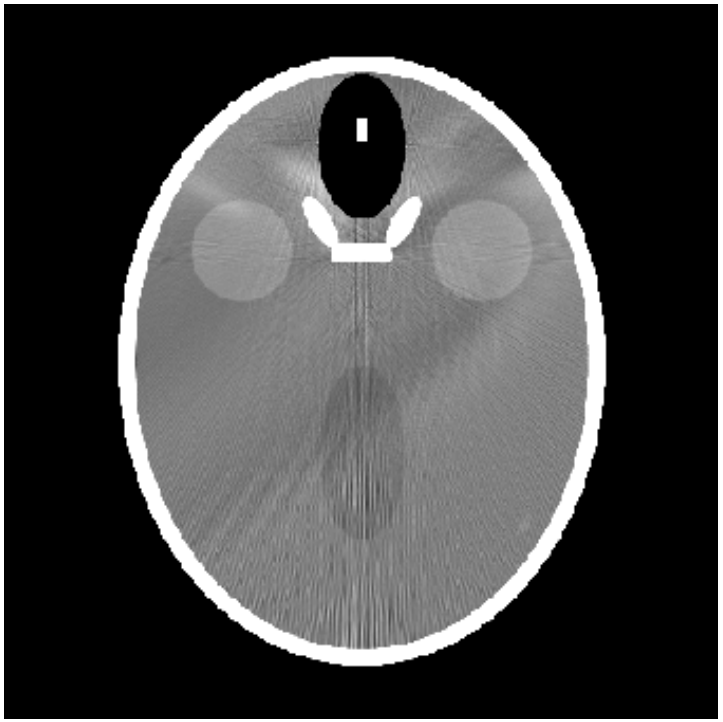
(a) $x = 1.44cm \; z \in [-9.1cm, 9.1cm]$

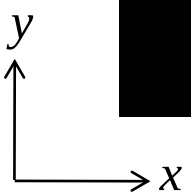
(b) $y = 0 \quad z \in [-9.1cm, 9.1cm]$

Angular range : 1.2π; Kink : z = - 0.88cm; Pitch : 20cm;

Fusion : z ∈ [-2.8cm 2.8cm]; Fusion zone is only visible by one half helix

Results: Experiment 003





z=1.12cm, FFDK

z=1.12cm, SSFDK

Angular range : 1.2π; Kink : $z = 2$cm; Pitch : 10cm;

Fusion : $z \in$ [-0.84cm 4.76cm]; No truncation : [-6cm 10cm]

Results: Experiment 003

z=1.68cm, FFDK

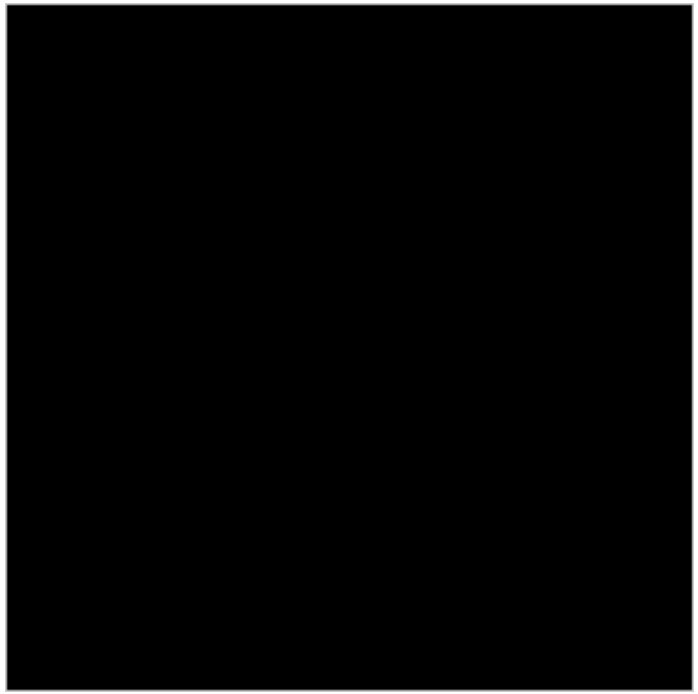
z=1.68cm, SSFDK

Angular range : 1.2π; Kink : $z = 2$cm; Pitch : 10cm;

Fusion : $z \in$ [-0.84cm 4.76cm]; No truncation : [-6cm 10cm]

EXP004

FFDK



SSFDK

Angular range : 1.2π; Kink : $z = -0.44$cm; Pitch : 10cm;

Fusion : $z \in [-5.4\text{cm}\ 4.6\text{cm}]$; No truncation : $[-15.1\text{cm}, 14.2\text{cm}]$

EXP005

FFDK

SSFDK

Angular range : 1.2π; Kink : $z = -0.88$cm; Pitch : 20cm;

Fusion : $z \in [-5.9\text{cm}\ 4.1\text{cm}]$; No truncation : $[-9.5\text{cm}, 7.7\text{cm}]$

Summary and Outlook

Summary

- two approximate FDK-type reconstruction methods
- five experiments using HEAD and THORAX phantoms
- reconstruction around the kink plane, SSFDK is similar to FFDK
- reconstruction between the kink plane and the pi-surface, different artifacts
- pi-line region: SSFDK has much less cone-beam artifacts

Outlook

- Filtering along a tilted line in the detector
- Reconstruction for the torsion free reverse helix

Thank you for your attention!