FDK-Type Image Reconstruction from Cone Beam Projections for Reverse Helical Trajectory

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Outline

- Introduction
- Reverse Helical Trajectory
- 2D Fan-Beam Reconstruction
- 3D reconstruction
  - Scan configuration
  - Reconstruction method I: FFDK
  - Reconstruction method II: SSFSDK
- Results
- Summary and Outlook
Introduction

- Topic: long object imaging using C-arm
- Problem: rotation angle limited
- Proposal: reverse helical trajectory(1,2,3)

(1): Exact Image Reconstruction in Reverse Helical Cone-Beam CT, Cho. et.al., Fully 3D, 2007
(2): On the problem of axial data truncation in the reverse helix geometry, Noo. et.al., Fully 3D, 2009
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Reverse helical trajectory

- Reverse Helix
- Pi-line surface: $\Sigma_1^+$
- Pi-line surface: $\Sigma_0^+$
- Pi-line surface: $\Sigma_0^-$
- Pi-line surface: $\Sigma_1^-$

$\pi$-line region: $[\Sigma_0^+, \Sigma_1^+] \& [\Sigma_1^-, \Sigma_0^-]$

Core of missing $\pi$-line: $[\Sigma_0^-, \Sigma_0^+]$

Ref: A solution to the long-object problem in helical cone-beam tomography, Defrise. et.al, PMB, 2000
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2D Fan-Beam Reconstruction I

- Conventional FBP reconstruction (FBP)

\[
f(x) = \int_\Lambda d\lambda \frac{RD}{(R - x \cdot e_w(\lambda))^2} \int_{-u_m}^{u_m} du \frac{D}{\sqrt{u^2 + D^2}} m(\lambda, u) g(\lambda, u) h_F(u^* - u)
\]

Reconstruction Steps:
1: weighting
2: filtering (ramp)
3: back projection
2D Fan-Beam Reconstruction II

- **Super short scan FBP reconstruction (SSFFBP)**

\[
f(x) = \frac{1}{2\pi} \int_{\Lambda} d\lambda \frac{1}{R - x \cdot e_w(\lambda)} m(\lambda, u^*) g_F(\lambda, u^*)
\]

where: \( g_F(\lambda, u^*) = \int_{-u_m}^{u_m} du \frac{D}{\sqrt{u^2 + D^2}} h_H(u^* - u) g'(\lambda, \alpha(\lambda, u)) \)

Reconstruction steps:

1: differentiation over \( \lambda \) 
2: filtering (Hilbert)
3: weighting 
4: back projection

Ref: Image reconstruction from fan-beam projections on less than a short scan, Noo. et.al, PMB, 2002
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Scan geometry

Common parameters

- Detector resolution (cm): \((du, dv) = (0.0616, 0.0616)\)
- Focus to detector distance: \(D = 119.9\, \text{cm}\)
- Scan radius: \(R = 78.5\, \text{cm}\)
- Angular range: \(= 1.2\pi\)
- Projection: 2 projections per degree
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FDK

\[
f(x) = \int_{\Lambda} d\lambda \frac{RD}{\left(R - x \cdot \underline{e}_w(\lambda)\right)^2} g_F(\lambda, u^*, v^*)
\]

where:
\[
g_F(\lambda, u, v) = \int_{-u_m}^{u_m} du' \frac{Dh_F(u - u')}{\sqrt{u'^2 + v^2 + D^2}} m(\lambda, u') g(\lambda, u', v)
\]

with:
\[
u^* = \frac{D(x - a(\lambda)) \cdot \underline{e}_u(\lambda)}{R - x \cdot \underline{e}_w(\lambda)} \quad \text{and} \quad v^* = \frac{D(x - a(\lambda)) \cdot \underline{e}_v(\lambda)}{R - x \cdot \underline{e}_w(\lambda)}
\]

Steps:
1: weighting
2: filtering along the horizontal line
3: back-projection within \(\Lambda\) (the angle position of the helical path)

Reconstruction method I: FFDK (Fusion-FDK)

Reconstruct the blue volume using the projection data from the top helix only

Reconstruct the red volume using the projection data from the bottom helix

Combine the blue and red volume by feathering along the overlapped region

**Pros:**
-- smooth the reconstruction of the fusion region
-- cooperate more data into the reconstruction in fusion region

**Cons:**
-- reconstruction using data not satisfying the Tuy’s condition
-- fusion increases the detector requirement
Example

$z = 1.96 cm \text{ @ EXP003}$

from top half helix \quad from bottom half helix \quad after fusion
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Reconstruction method II—SSFDK (Super Short-FDK)
Reconstruction method II: SSFDK

\[ f(x) = \frac{1}{2\pi} \int_{\Lambda} d\lambda \frac{m(\lambda, x)}{R - x \cdot e_w(\lambda)} g'_F(\lambda, u^*, v^*) \]

where: \[ g_F(\lambda, u, v) = \int_{-u_m}^{u_m} du' \frac{\sqrt{D^2 + v^2}}{\sqrt{u'^2 + v^2 + D^2}} g'(\lambda, \alpha(\lambda, u, v)) h_H(u - u') \]

with: \[ u^* = \frac{D(x - a(\lambda)) \cdot e_u(\lambda)}{R - x \cdot e_w(\lambda)} \]
and \[ v^* = \frac{D(x - a(\lambda)) \cdot e_v(\lambda)}{R - x \cdot e_w(\lambda)} \]

Steps:

1: differentiation
2: filtering
3: weighting
4: back-projection
SSFDK--Derivation

\[ g'(\lambda, \alpha) = \frac{\partial g(\lambda, \alpha)}{\partial \lambda} \text{ for fixed } \alpha \]

1: blended chain-rule (1)
2: split chain-rule(1)
3: scheme from Faridani (2)
4: new scheme from Noo 2007 (1)

\[ g'(\lambda, \alpha) = \frac{g(\lambda + \varepsilon \Delta \lambda, \alpha) - g(\lambda - \varepsilon \Delta \lambda, \alpha)}{2\varepsilon \Delta \lambda} \]


SSFDK--weighting

weighting for the \( \pi \)-line region

Each point in region I and II is covered by one and only one \( \pi \)-line. Therefore, in these two regions, we reconstruct each point from projections within its \( \pi \)–line.
SSFDK--weighting

weighting for the core

\[ \lambda_c = \frac{z_0^+ - z}{z_0^+ - z_i} \times (\lambda_\pi - \lambda_i) + \lambda_i \]

\[ \Delta = \beta \min(\lambda_\pi - \lambda_c, \lambda_c - \lambda_i) \]

with: \( \beta \in (0,0.5) \)

\[ w(\lambda) = \begin{cases} 1 & \lambda_c + \Delta \leq \lambda \leq \lambda_\pi \\ \frac{1}{2} + \frac{\sin^2}{2} \left( \frac{(\lambda + \Delta - \lambda_c)\pi}{4\Delta} \right) & \lambda_c - \Delta \leq \lambda < \lambda_c + \Delta \\ \frac{1}{2} & \lambda_i \leq \lambda < \lambda_c - \Delta \\ 0 & \text{otherwise} \end{cases} \]
SSFDK--weighting

Problem shooting

\[ \lambda_x = \frac{z_0^+ - z}{z_0^+ - z_i} \times (\lambda_e - \lambda_\pi) + \lambda_\pi \]

Parker's weighting for \([\lambda_i, \lambda_x]\)
Example

ROI
π-line region I
Core (no π-line)
π-line region II
kink
Example
Example

ROI

π-line region I

Core (no π-line)

π-line region II

kink
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## Experiments overview

<table>
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<tr>
<th>Experiment</th>
<th>Region</th>
<th>Pitch</th>
<th>Kink</th>
<th>Detector</th>
<th>Fusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP001: HEAD</td>
<td>Head</td>
<td>10cm</td>
<td>-0.44cm</td>
<td>38*38cm</td>
<td>-2.8~2.8cm</td>
</tr>
<tr>
<td>EXP002: HEAD</td>
<td>Head</td>
<td>20cm</td>
<td>-0.88cm</td>
<td>38*38cm</td>
<td>-2.8~2.8cm</td>
</tr>
<tr>
<td>EXP003: HEAD</td>
<td>Head</td>
<td>20cm</td>
<td>2cm</td>
<td>48*48cm</td>
<td>-0.84~4.76cm</td>
</tr>
<tr>
<td>EXP004: THORAX</td>
<td>Thorax</td>
<td>10cm</td>
<td>-0.44cm</td>
<td>89*89cm</td>
<td>-5.4~4.6cm</td>
</tr>
<tr>
<td>EXP005: THORAX</td>
<td>Thorax</td>
<td>20cm</td>
<td>-0.88cm</td>
<td>89*89cm</td>
<td>-5.9~5.1cm</td>
</tr>
</tbody>
</table>
Results: Experiment 001

Angular range: $1.2\pi$;  
Kink: -0.44cm;  
Pitch: 10cm;  
Fusion: $z \in [-2.8\text{cm}, 2.8\text{cm}]$; No truncation: [-5.5cm 4.6cm]
Results: Experiment 001

Angular range: $1.2\pi$;  
Kink: -0.44cm;  
Pitch: 10cm;  
Fusion: $z \in [-2.8\text{cm}, 2.8\text{cm}]$;  
No truncation: $[-5.5\text{cm}, 4.6\text{cm}]$
Results: Experiment 001

Angular range: $1.2\pi$; Kink: -0.44cm; Pitch: 10cm; Fusion: $z \in [-2.8\text{cm}, 2.8\text{cm}]$; No truncation: [-5.5cm, 4.6cm]
Results: Experiment 001

(a) $x = 1.44 \text{ cm} \ z \in [-4.6 \text{ cm}, 3.8 \text{ cm}]$

(b) $y = 0 \ z \in [-4.6 \text{ cm}, 3.8 \text{ cm}]$

Angular range: $1.2\pi$; Kink: $-0.44\text{ cm}$; Pitch: $10\text{ cm}$; Fusion: $z \in [-2.8\text{ cm}, 2.8\text{ cm}]$; No truncation: $[-5.5\text{ cm}, 4.6\text{ cm}]$
Results: Experiment 002

Angular range: $1.2\pi$;  
Kink: $z = -0.88$ cm;  
Pitch: 20 cm;  
Fusion: $z \in [-2.8 \text{ cm}, 2.8 \text{ cm}]$;  
Fusion zone is only visible by one half helix.
Results: Experiment 002

(a) $x = 1.44\text{cm} \ z \in [-9.1\text{cm}, 9.1\text{cm}]$

(b) $y = 0 \ z \in [-9.1\text{cm}, 9.1\text{cm}]$

Angular range: $1.2\pi$;  
Kink: $z = -0.88\text{cm}$;  
Pitch: $20\text{cm}$;  
Fusion: $z \in [-2.8\text{cm}, 2.8\text{cm}]$;  
Fusion zone is only visible by one half helix.
Results: Experiment 003

\[ z = 1.12\text{cm}, \text{ FFDK} \]

\[ z = 1.12\text{cm}, \text{ SSFDK} \]

Angular range : \(1.2\pi\);  
Kink : \(z = 2\text{cm}\);  
Pitch : \(10\text{cm}\);  
Fusion : \(z \in [-0.84\text{cm} 4.76\text{cm}]\);  
No truncation :\([-6\text{cm} 10\text{cm}]\)
Results: Experiment 003

\[ z = 1.68 \text{cm}, \text{FFDK} \]

\[ z = 1.68 \text{cm}, \text{SSFFDK} \]

Angular range: \(1.2\pi\); Kink: \(z = 2\text{cm}\); Pitch: 10cm;
Fusion: \(z \in [-0.84\text{cm} \ 4.76\text{cm}]\); No truncation: \([-6\text{cm} \ 10\text{cm}]\)
Angular range: $1.2\pi$;  
Kink: $z = -0.44\text{cm}$;  
Pitch: 10cm;  
Fusion: $z \in [-5.4\text{cm}, 4.6\text{cm}]$;  
No truncation: $[-15.1\text{cm}, 14.2\text{cm}]$
Angular range: $1.2\pi$;  
Kink: $z = -0.88\text{cm}$;  
Pitch: 20cm;  
Fusion: $z \in [-5.9\text{cm}, 4.1\text{cm}]$;  
No truncation: $[-9.5\text{cm}, 7.7\text{cm}]$
Summary and Outlook

Summary

- two approximate FDK-type reconstruction methods
- five experiments using HEAD and THORAX phantoms
- reconstruction around the kink plane, SSFDK is similar to FFDK
- reconstruction between the kink plane and the pi-surface, different artifacts
- pi-line region: SSFDK has much less cone-beam artifacts

Outlook

- Filtering along a tilted line in the detector
- Reconstruction for the torsion free reverse helix
Thank you for your attention!