Sparse Principal Axes
Statistical Deformation Models for Respiration Analysis and Classification

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Outline

● Motivation
  ● Respiratory Motion
  ● Range Imaging based Surrogates

● Methods
  ● Range Imaging
  ● Principal Component Analysis
  ● Orthomax Rotations

● Results
  ● PCA vs. Varimax Rotations
  ● Applications

● Conclusion and Outlook
Motivation

- Respiratory Motion
- Range Imaging based Surrogates
Motivation

- Internal movement due to respiratory motion
  - Tomographic reconstruction
  - Fractionated radiotherapy

- Gating and beam adjustment based on 1-D respiration surrogates
  - Dedicated sensors attached to the body surface
  - Non-intrusive range imaging (RI)

- Current RI-based 1-D surrogates
  - Manually selected points and regions
  - Heuristic surface partitioning
  - Global statistical models

*Figure:* Gated radiotherapy using RI techniques.
Motivation

- Drawbacks of conventional statistical models
  - Fail to describe local deformations (e.g. thoracic and abdominal breathing)
  - Hinder an intuitive interpretation of the model's inherent variations
  - Produce invalid shapes w.r.t. the human respiration system

- Sparse axes models to produce relief
  - Relate local deformations to sparsity of the model's principal axes
  - Orthomax criterion to derive sparse principal modes of variation [1]

[1] Stegmann M.B, Sjöstrand K, Larsen R.  
Sparse Modeling of Landmark and Texture Variability using the Orthomax Criterion.  
Methods

- Range Imaging
- Principal Component Analysis
- Orthomax Rotations
Range Imaging

- Sensor domain $\Omega : \mathbb{N} \times \mathbb{M} \mapsto \mathbb{R}$
- Range image $f(x), x \in \Omega$
- Linearized range image
  $$f \equiv g = (f(x_1), \ldots, f(x_{N\cdotM}))^T, \ g \in \mathbb{R}^{N\cdotM}$$
- Training data acquired at $K$ different respiration states $i$
  $$\mathcal{G} = \{g_i\}_{i=1}^K, \ g_i \equiv f_i$$

Figure: Body surface captured by the Microsoft Kinect RI device.
Principal Component Analysis (PCA)

- Configuration matrix
  \[ L = [g_1 - \bar{g}, g_2 - \bar{g}, \ldots, g_K - \bar{g}], \quad L \in \mathbb{R}^{N \cdot M \times K}, \quad \bar{g} = \sum_{i=1}^{K} g_i \]

- Eigendecomposition and principal component basis \( \Phi \)
  \[ (L^T L) \tilde{e}_i = \lambda_i \tilde{e} \]
  \[ \Phi = [L\tilde{e}_1, L\tilde{e}_2, \ldots, L\tilde{e}_P], \quad \Phi \in \mathbb{R}^{N \cdot M \times P} \]

- Linear span of the model
  \[ g^* = \bar{g} + \Phi b, \quad b \in \mathbb{R}^P \]
  \[ b = \Phi^T (g^* - \bar{g}) \]

- Note that PCA maximizes the variance of the input data along the basis vectors \( L\tilde{e}_i \).

Thus, global modes are obtained!
Orthomax Rotations

- Optimization problem

\[
R^*_O = \arg\max_R \sum_{j=1}^{P} \sum_{i=1}^{N \cdot M} (\Phi R)_{ij}^4 - \frac{\gamma}{N \cdot M} \sum_{j=1}^{P} \sum_{i=1}^{N \cdot M} (\Phi R)_{ij}^2
\]

- Varimax rotation (\(\gamma = 1\))

\[
R^*_O = \arg\max_R \sum_{j=1}^{P} \left( \frac{1}{N \cdot M} \sum_{i=1}^{N \cdot M} \delta_{ij}^2 - \left( \frac{1}{N \cdot M} \sum_{i=1}^{N \cdot M} \delta_{ij} \right)^2 \right), \quad \delta_{i,j} = (\Phi R)_{i,j}^2
\]

- Varimax rotations transform the model basis according to \(\Phi_O = \Phi R_O\), maximizing the squared variable loadings by bringing several loadings close to zero.

This favors sparse modes!
Results

- PCA vs. Varimax Rotations
- Applications
Results

- Dimension reduction
  - Abdominal and thoracic breathing
  - Two separate and one joint model

<table>
<thead>
<tr>
<th>Mode of variation</th>
<th>Abdominal</th>
<th>Thoracic</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>98.4 % (98.4%)</td>
<td>92.7 % (92.7%)</td>
<td>75.3 % (75.3%)</td>
</tr>
<tr>
<td>#2</td>
<td>~1.0 % (99.4%)</td>
<td>~ 5.9 % (98.6%)</td>
<td>23.2 % (98.5%)</td>
</tr>
<tr>
<td>#3</td>
<td>&lt; 1.0 % (99.8%)</td>
<td>&lt; 1.0 % (99.4%)</td>
<td>~ 1.0 % (99.4%)</td>
</tr>
<tr>
<td>#4</td>
<td>&lt; 1.0 % (99.9%)</td>
<td>&lt; 1.0 % (99.6%)</td>
<td>&lt; 1.0 % (99.6%)</td>
</tr>
<tr>
<td>Total</td>
<td>99.9%</td>
<td>99.6%</td>
<td>99.6%</td>
</tr>
</tbody>
</table>

*Table:* Variance covered by the first four modes of variation.
Results

- PCA vs. varimax rotations

![Figure: Respiratory motion patterns from statistical analysis. PCA (P) and Varimax rotations (V). Magnitude of variation is color coded from blue (low) to red (high).]
Results

- Simulation of respiration states for algorithm benchmarking
Results

- Patient specific respiration analysis and classification

PCC > 0.98

PCC > 0.99
Conclusion and Outlook
Conclusions and Outlook

- Sparse principal axes for respiration analysis
  - Varimax rotations to generate sparse modes
  - Local surface deformations
  - Differentiation between thoracic and abdominal breathing

- Future work
  - Extension to 3-D point clouds
  - Non-linear techniques for model generation

Thank you for your attention!
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