Sparsity Level Constrained Compressed Sensing (SLCCS) for CT Reconstruction

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Abstract. It is a very hot topic to reconstruct images from as few projections as possible in the field of CT reconstruction. Due to the lack of measurements, the reconstruction problem is ill-posed. Thus streaking artifacts are unavoidable in images reconstructed by filtered backprojection algorithm. Recently, compressed sensing [1] takes sparsity as prior knowledge and reconstructs the images with high quality using only few projections. Based on this idea, we propose to further use the sparsity level as a constraint. In the experiments, we reconstructed Shepp-Logan phantom with only 30 views by our method, TVR [2] and stand ART [3] respectively. We also calculated the Euclidean norm of the reconstruction image and the ground truth for each method. The results show that reconstruction results of our method are more accurate than the results of total variation regularization (TVR) [2] and stand ART [3] method.

1 Introduction

In the field of CT reconstruction, it draws a lot of attention to reconstruct the images from few samples (often under the Nyquist sampling rate) to reduce the radiation dose. And in certain applications, it is not possible to sample at Nyquist sampling rate because of motion, cardiac imaging [4]. In these cases, the reconstruction problem is ill-posed. Thus the images reconstructed by FBP (Filtered Backprojection) [2], which is widely used in CT product, contain many streaking artifacts. Recently, compressive sampling (CS) shows that a high quality signal or image can be reconstructed with far fewer measurements than the Nyquist sampling rate. The main idea of CS is to take sparsity as a prior. Based on that, Pan's group developed TVR method [3] and reconstructs high quality images using only few projections. However, the method reduces the contrast of the image. [5] In this paper, we propose to further use the number of the nonzero coefficient (sparsity level) as prior knowledge. This prior knowledge is formulated as an additional constraint in the CS based reconstruction frame work. In practice, the accurate sparsity level is hard to estimate. We suggest to approximate it using the image reconstructed by FBP. From the experiments, even if the sparsity level is 1.4 times as the actual value, the method can still improve the reconstruction accuracy. In the experiments, we used the Shepp-Logan phantom and reconstructed it with our method, TVR [2] and stand ART [3]. The results show that our method reconstruct the images with best accuracy.

2 Materials and methods

A discrete version of the CT scanning process can be described as

$$A\boldsymbol{x} = \boldsymbol{b} \tag{1}$$

Here $\boldsymbol{A} = (a_{ij})$ is the system matrix representing the projection operator, $\boldsymbol{x} = (x_1,..., x_n)$ represents the object and $\boldsymbol{b} = (b_1, ..., b_m)$ is the corresponding projection data. So to reconstruct the object $\boldsymbol{x}$ is to solve the linear system. In our case, the linear system is underdetermined due to undersampling. There exist infinite solutions. As mentioned above, CS takes sparsity as prior knowledge, which formulates the reconstruction problem as

$$\min_{\boldsymbol{x}} ||\Phi\boldsymbol{x}||_{L1} \ s.t. \ ||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_2^2 < \alpha \tag{2}$$

Here, α stands for the variance of the noise. Φ is the sparsifying transform, wavelet transform. The inequality constraint enforces the data fidelity and the L1 norm term promotes the sparsity. For details, we refer to the work []. It is well known that the constrained optimization problem (Eq. (2)) can be transformed to an easier unconstrained optimization problem [].

$$\min_{\boldsymbol{x}} ||\Phi\boldsymbol{x}||_{L1} + \beta||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_2^2 \tag{3}$$

The cost function in the optimization problem is convex. Although there exists a global minimum for a convex function, the minimizer could be non unique (Fig. 1). The function in the picture is convex, but the number of the minimizer is infinite. In this case, the initial guess determines the final solution for the iterative reconstruction. The exact reconstruction of CS is guaranteed when there exists only one minimizer [1]. Otherwise the error bound is described by the 'size' of the solution set. Sometimes, the 'size' of the solution set could be very large. Due to the sampling mechanism, exact reconstruction condition

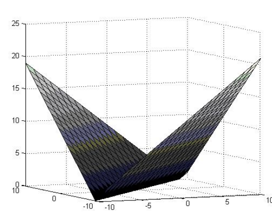

Fig. 1. Convex function with non unique minimizers

of CS can not be satisfied in the context of CT reconstruction. Is there any way to further select the solution? We propose to use the sparsity level. Then the reconstruction problem can be formulated as

$$\min_{\boldsymbol{x}} ||\Phi_1 \boldsymbol{x}||_{L1} + \beta||\boldsymbol{Ax} - \boldsymbol{b}||_2^2 s.t. ||\Phi_2 \boldsymbol{x}||_{L0} < \chi \tag{4}$$

where $||\bullet||_{L0}$ is L0 norm, which counts the number of nonzero entries. $||\Phi_2 \boldsymbol{x}||_{L0} < \chi$ is the sparsity level constraint. χ is a scalar. Φ_1 and Φ_2 both are the sparsifying transforms. We use total variation [2] as Φ_1 and Haar wavelet transform as Φ_2 in our experiments. These two sparsifying transforms are heavily used in the CS based reconstruction, as the medical images can be expressed sparsely with these two transforms. Some other sparsifying transform can also be used. Eq. (3) is a non convex optimization problem. However, all the local minimum should be in the optimal solution set of Eq. (3). The sparsity level constraint selects the solutions which satisfies the constraint within the optimal solution set for Eq. (3). So this formulation can increase the accuracy of the reconstruction compared to Eq. (3). In our experiments, we find that the sparsity level constraint does not have to be very accurate. The most accurate reconstruction is achieved when χ is a little bit bigger than the actual sparsity level. In practice, we can first use FBP to reconstruct the image, then apply the sparsifying transform Φ_2 to estimate the sparsity level. The optimization problem is hard to solve due to the high dimensions. A splitting method is used to solve Eq. (4) [3]. Inspired by this idea, we developed our method based on the splitting method. The algorithm can be summarized as below:

- Step 1) Apply the standard ART update [2].
- Step 2) Solve the optimization problem $\boldsymbol{x'} = \min ||\boldsymbol{x} - \boldsymbol{v}||_2^2 + \beta||\Phi_1 \boldsymbol{x}||_1$ ($\boldsymbol{v}$ is calculated from step 1 which is the volume estimation from step 1)
- Step 3) Apply the sparsifying transform Φ_2 on $\boldsymbol{x'}$ ($\boldsymbol{x'}$ is calculated from step 2)
- Step 4) Keep the χ largest coefficients and set others to zero
- Step 5) Apply the inverse sparsifying transform Φ_2^{-1}
- Step 6) Repeat step 1 to step 5 until $||\boldsymbol{x}^{(t)} - \boldsymbol{x}^{(t+1)}||_2^2$ is less than a certain value or the maximum iteration number is reached.

Step 1 and step 2 solve the problem (Eq. (3)) without sparsity level constraint using a splitting method. Step 3 to step 4 force the current estimate to fulfill the sparsity level constraint. We keep the largest coefficients and set others to zero by the assumption that the energy of natural image concentrate on a few basis of an sparsifying transform.

3 Results

In the experiments, we use the Shepp-Logan phantom. We consider image reconstruction of a 128 × 128 image from projection data containing 30 views and 128 bin on the detector. The projections are equally angular-spaced over 180 degree. We reconstructed the images with standard ART, TVR and our method.

The reconstruction results is listed in Fig. 2. In the picture, we can see that the reconstruction from the standard ART contains many streak artifacts. The reconstruction from TVR is better. But the contrast is reduced. The streaks are nearly can not be seen in the reconstruction from our method. Furthermore the contrast is better than the reconstruction of TVR.

The profile of the reconstruction can be found in Fig.3. The contrast of the reconstruction from our method outperforms the one from TVR.

We also calculated the relative error (RE) over iterations for quantitative evaluation

$$RE = \frac{||\boldsymbol{x} - \boldsymbol{x}^*||_2}{||\boldsymbol{x}^*||_2} \tag{5}$$

where x is the reconstructed image and x^* is the ground truth. It can be seen from Fig. 4 that the convergence speed of our method is faster than the other two. And the accuracy of our method is the best. In Fig. 5, we set χ to different values. The best reconstruction is achieved when χ is a little bit bigger than the actual sparsity level. Our method still gives more accurate results than the ones from TVR and stand ART, even if we set χ to 1.4 times as the actual sparsity level.(see Fig. Fig. 4 and Fig. 5). Thus the estimation of sparsity level need not to be very accurate.

4 Discussion

Although CS claims it can reconstruct the image exactly under Nyquist sampling rate, the exact reconstruction condition is hard to be satisfied due to the sampling mechanism of CT. To further improve the reconstruction quality, we propose to use the sparsity level as prior knowledge as well. The sparsity level constraint shrinks the 'size' of the optimal solution set thus increases the reconstruction quality. In the experiment, by taking the sparsity as a prior, TVR which is CS based reconstruction method increase the reconstruction accuracy a lot compared to the classical method (ART). However, the contrast of the TVR reconstruction is reduced. By using the sparsity level constraint, the contrast

Fig. 2. Reconstruction results. Only 30 projections are used to do the reconstruction. The window level are all [-0.2 0.3]. The reconstruction from the standard ART contains many streak artifacts. The reconstruction from TVR is better. But the contrast is reduced. The streaks are nearly can not be seen in the reconstruction from SLCCS, also the contrast is better than the reconstruction of TVR

of our reconstruction is better and relative error of our method is the smallest among the three reconstruction method. We will test the algorithm with vivo data in the future.

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References

1. Candes E, Romberg J, Tao T. Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. IEEE Trans Inf Theory. 2006;52:489–509.
2. Zeng GL. Medical Image Reconstruction. China: High Education Press; 2009.
3. Sidky EY, Pan X. Image reconstruction in circular cone-beam computed tomography by constrained, total-variation minimization. Phys Med Bio. 2008;53:4777–4807.
4. Hansis E, Schaefer D, Doessel O, Grass M. Evaluation of iterative sparse object reconstruction from few projections for 3-D rotational coronary angiography. IEEE Trans Med Imag. 2008;27(11):1548 – 55.
5. Jia X, Lou Y, Li R, Song W, Jiang SB. GPU-based Fast Cone Beam CT Reconstruction from Undersampled and Noisy Projection Data via Total Variation. Med Phy. 2010;37(11):1757 – 1761.

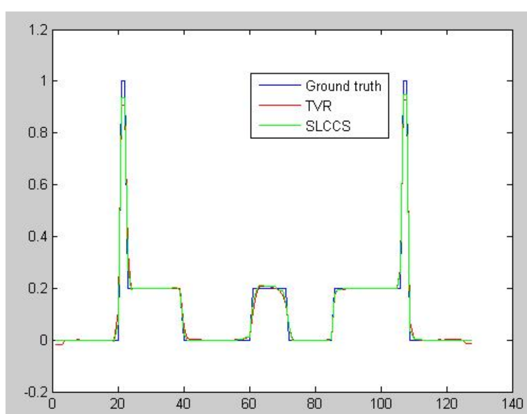

Fig. 3. Profile. The images shows the profile of the center line from the reconstruction of TVR, SLCCS and the ground truth. It can be easily seen that the reconstruction of SLCCS reserves the contrast.

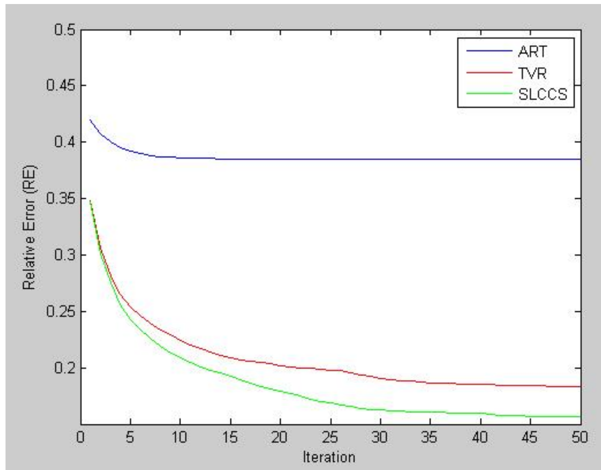

Fig. 4. Converge map. SLCCS converges faster than the other two method and gives best reconstruction quality

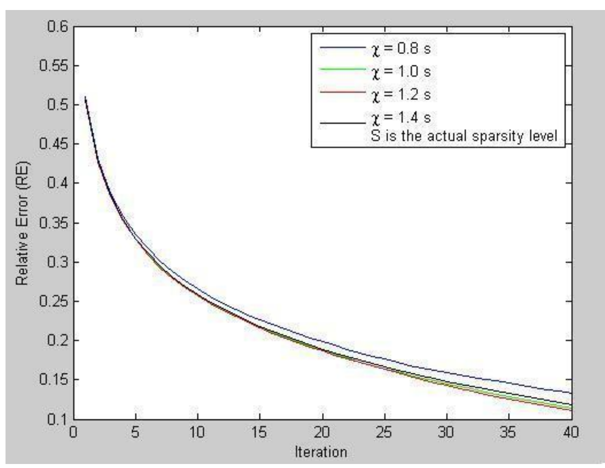

Fig. 5. Test of χ. We set χ to different values. The most accurate reconstruction is achieved when χ is a little bit bigger than the actual sparsity level. Thus the estimation of sparsity level need not to be very accurate but it should not be less than the actual sparsity level