Exercises for
Pattern Analysis
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Assignment 7, 04.06.2014

General Information:

Lecture (3 SWS):	Mon 08.15 – 09:45 (H16) and Tue 08.15 – 09.45 (H16)
Exercises (1 SWS):	Wed 12.15 – 13.15 (00.151-113) and Thu 12.30 – 13.30 (00.151-113)
Certificate:	Oral exam at the end of the semester
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Mean Shift Algorithm

Exercise 1 The mean shift algorithm can be used to determine a local maximum (or a saddle point) of a probability density function $p(\boldsymbol{x})$. In particular, it is feasible to determine the *mode* of the density. For a mathematical formulation of the mean shift algorithm, the Parzen window approach is used to model $p(\boldsymbol{x})$. The mean shift iterations are equivalent to a gradient ascent for $p(\boldsymbol{x})$.

(a) Outline the main steps of the mean shift algorithm.

(b) Derive the mean shift vector for the following kernels:

- Epanechnikov kernel
- Gaussian kernel

Exercise 2 Let $\mathcal{S} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n\}$ be a set of $N = 8$ samples defined as:

$$\mathcal{S} = \left\{ \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix}, \begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix}, \begin{pmatrix} 0.2 \\ 0.25 \end{pmatrix}, \begin{pmatrix} 0.3 \\ 0.2 \end{pmatrix}, \begin{pmatrix} 0.5 \\ 0.7 \end{pmatrix}, \begin{pmatrix} 0.7 \\ 0.8 \end{pmatrix}, \begin{pmatrix} 0.8 \\ 0.9 \end{pmatrix}, \begin{pmatrix} 0.9 \\ 0.8 \end{pmatrix} \right\}$$

(a) Draw the samples in the 2-dimensional feature space.

(b) Perform one mean shift iteration and draw the corresponding mean shift vectors using the following starting points:

- $\boldsymbol{x}^0 = \begin{pmatrix} 0 & 0 \end{pmatrix}^\top$
- $\boldsymbol{x}^0 = \begin{pmatrix} 1 & 1 \end{pmatrix}^\top$

Use the Epanechnikov kernel with kernel width $\lambda = 0.25$.

(c) Sketch the mean shift vectors if the mean shift iterations are performed until convergence.

(d) Explain how the mean shift algorithm can be used for an automatic clustering. How do you determine the number of clusters? Compare mean shift clustering to hard- and soft-clustering.

Exercise 3 **Matlab exercise:** In terms of image processing, the mean shift algorithm can be employed for edge-preserving smoothing. This filtering technique can be used to denoise images. The key idea of mean shift filtering is to represent each pixel of an image by a feature vector $\boldsymbol{x}$ and to define a joint probability density function $p(\boldsymbol{x})$ for the image. Mean shift iterations are performed to find a local

Figure 1: Noisy (left) and denoised image (right) using mean shift filtering.

maximum of $p(\boldsymbol{x})$ next to a given pixel. For the sake of simplicity, we consider 2-dimensional, intensity (gray value) images. For details of mean shift for edge-preserving smoothing please refer to

Comaniciu, D. and Meer, P. *Mean shift: a robust approach toward feature space analysis*. IEEE Transactions on Pattern Analysis and Machine Intelligence (2002), Volume 24, Issue: 5, pp. 603 - 619

(a) Define a feature vector $\boldsymbol{x}_i$ to model the i-th pixel for a given input image. Explain how the feature vector can be extended to handle color images represented in the RGB color space.

(b) Explain how the mean shift algorithm can be employed to denoise $\boldsymbol{x}_i$. In particular, describe which parameters are required and explain the influence of the parameters to the outcome of mean shift.

(c) Implement the edge-preserving smoothing using the mean shift algorithm. Without loss of generalization, we use the Epanechnikov kernel for the mean shift iterations.

(d) Test your algorithm using synthetic image data:

- Load the example *Cameraman* image.
- Apply your mean shift algorithm to smooth the noisy image.
- The width of the Epanechnikov kernel can be selected empirically by visual inspection of the denoised image.
- Compare the input and the denoised image qualitatively.