Diagnostic Medical Image Processing (DMIP)
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Exercise 6: Fan-beam Reconstruction and Short Scan

The parallel-beam geometry was once used in practical scanner, while the diverging-geometry, e.g., fan-beam geometry, is employed exclusively today. In a fan-beam geometry, short scan that acquires data only over a π plus the fan angle is commonly used, aiming to reduce scan time and X-ray dose. The goal for this exercise is to reconstruct short scan data using the fan-beam FBP algorithm with an appropriate redundancy weighting function, i.e., Parker weights.

1 Theory

1.1 Fan-beam FBP Algorithm

In a fan-beam geometry, a single focal point is the X-ray source and beams are not parallel but characterized by a fan shape with a opening angle δ. A straightforward substitution of the integral variables from the parallel-beam data $p(s,\theta)$ to the fan-beam data $g(\gamma,\beta)$ leads to the corresponding fan-beam geometry algorithm (see Fig. 1a for the notation):

- *Step 1:* Cosine weighting of projection data to obtain $g_1(s,\beta)$:

$$g_1(s,\beta) = \frac{D}{\sqrt{D^2+s^2}} g(s,\beta) \ , \tag{1}$$

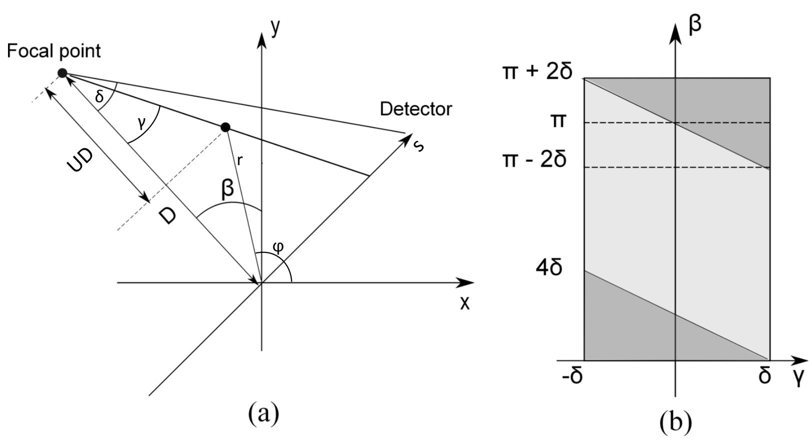

Figure 1: (a) Fan-beam imaging geometry, (b) fan-beam sinogram and data redundancy.

- *Step 2*: Perform fan-beam filtering :

$$g_F(s,\beta) = \int_{-\infty}^{\infty} h_R(s-s')\, g_1(s',\beta)\, ds' , \tag{2}$$

where $h_R(s)$ is the filter kernel.

- *Step 3:* Backprojection with a weighting function of object-focal point distance U:

$$f(r,\varphi) = \int_{0}^{2\pi} \frac{1}{U^2} g_F(s,\beta)\, d\beta \tag{3}$$

$$U = \frac{D + r\sin(\beta - \varphi)}{D} \tag{4}$$

1.2 Short Scan

Short scan measures some redundant rays at the beginning and at the end of data acquisition (see the two dark triangles in Fig. 1b). Corresponding redundant rays can be determined by the relation

$$g(\gamma,\beta) = g(-\gamma, \beta + \pi + 2\gamma) \; . \tag{5}$$

The commonly used weighting function for redundancies is the Parker weighting function, in which the projection rays measured twice are normalized to unity while guaranteeing smooth transitions between non-redundant and redundant data. The weighting can be defined as:

$$\omega(\gamma,\beta) = \begin{cases} \sin^2\left(\frac{\pi}{4}\frac{\beta}{\delta-\gamma}\right), & 0 \le \beta \le 2\delta - 2\gamma \\ 1, & 2\delta - 2\gamma \le \beta \le \pi - 2\gamma \\ \sin^2\left(\frac{\pi}{4}\frac{\pi+2\delta-\beta}{\delta+\gamma}\right), & \pi - 2\gamma \le \beta \le \pi + 2\delta \end{cases}$$

2 Implementation tasks

We will implement a fan-beam based FBP algorithm to reconstruct the different provided sinograms.
Complete the gaps in the Matlab frame that are marked with "**TASK**"

1. Compute the fan angle out of the provided geometry data (line 30)
2. Use the fan angle to compute the short scan range (line 33)
3. Determine the distances from detector boundary to all detector cells (line 44)
4. Compute the intersection points of all ray lines with the detector line (line 222)
5. Compute the distance weights for all pixels, needed during backprojection (line244)
6. Complete the cosine-weights computation (line 280)
7. Complete the parker-weights implementation (line 309/313)