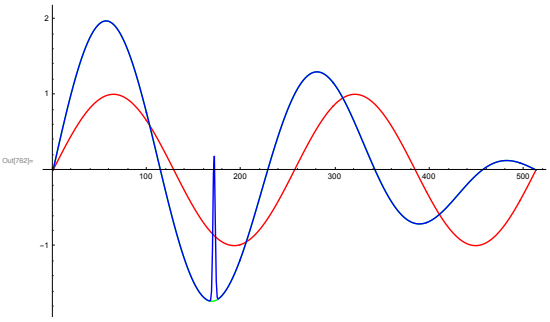
Approximation and vanishing moments

Example (1)

In[758]:=
```
data0 =
  Table[Sin[4 * Pi * t] , {t, 0, 1, 1/2^9}];
data1 =
  data0 + Table[ Sin[5 * Pi * t], {t, 0, 1, 1/2^9}];
data2 =
  data1 + Table[ 2 Exp[-10^5 (1/3 - t)^2], {t, 0, 1, 1/2^9}];
```

In[762]:=
```
ListLinePlot[{data0, data1, data2},
 PlotStyle → {Red, Green, Blue}, ImageSize → Large]
```

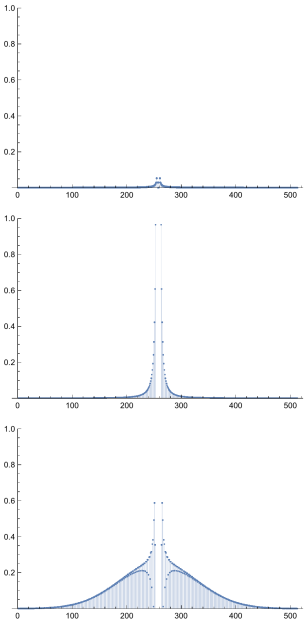
Out[762]=



In[754]:=
```
f0 = Fourier[data0];
f1 = Fourier[data1];
f2 = Fourier[data2];
```

In[786]:=
```
GraphicsColumn[Map[ListPlot[RotateLeft[Abs[#], 256],
   PlotRange → {0, 1}, Filling → Axis, ImageSize → Medium] &, {f0, f1, f2}]]
```

Out[786]=

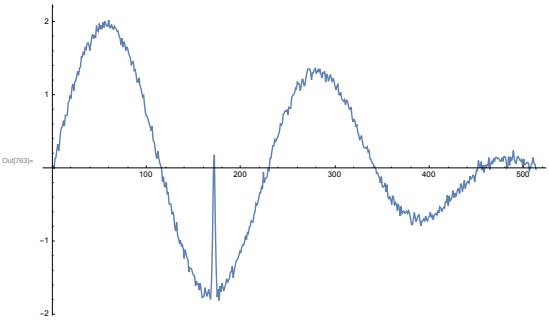


Example (2)

In[702]:=
```
noised1 = data2 + RandomVariate[NormalDistribution[0, 0.05], 2^9 + 1];
```

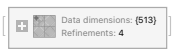

In[763]:=
```
ListLinePlot[noised1, ImageSize → Large]
```

Out[763]=



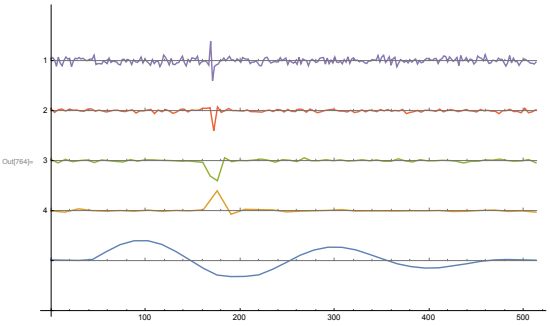
In[704]:=
```
dwt1 = DiscreteWaveletTransform[noised1, DaubechiesWavelet[3], 4]
```

Out[704]=



In[764]:=
```
WaveletListPlot[dwt1, {{1}, {0, 1}, {0, 0, 1}, {0, 0, 0, 1}, {0, 0, 0, 0}},
 PlotLayout → "CommonXAxis", Method → {"Inverse" → True}, ImageSize → Large]
```

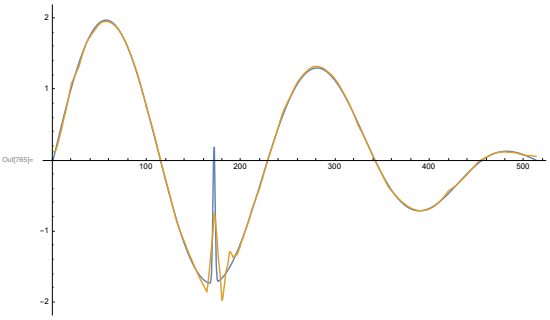
Out[764]=



In[706]:=
```
th1 = InverseWaveletTransform[WaveletThreshold[dwt1, {"SURELevel"}], Automatic];
```


In[765]:=
```
ListLinePlot[{data2, th1}, PlotRange → All, ImageSize → Large]
```

Out[765]=

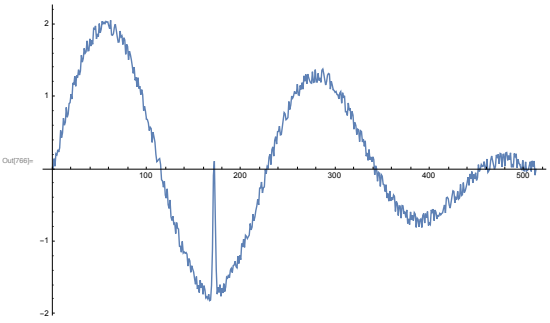


Example (3)

In[708]:=
```
noised2 = data2 + 0.1 * Table[ Sin[t^2], {t, 0, 2^9}];
```

In[766]:=
```
ListLinePlot[noised2, ImageSize → Large]
```

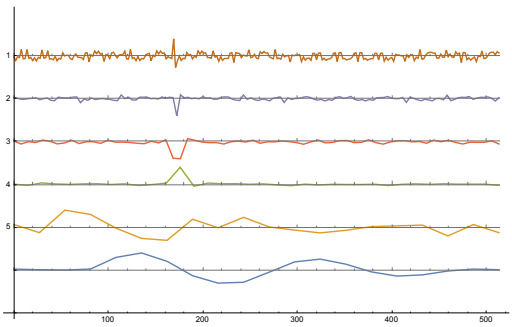
Out[766]=



```
In[710]:= dwt2 = DiscreteWaveletTransform[noised2, DaubechiesWavelet[3], 5]
Out[710]= DiscreteWaveletData[ Data dimensions: {513}  Refinements: 5 ]
```

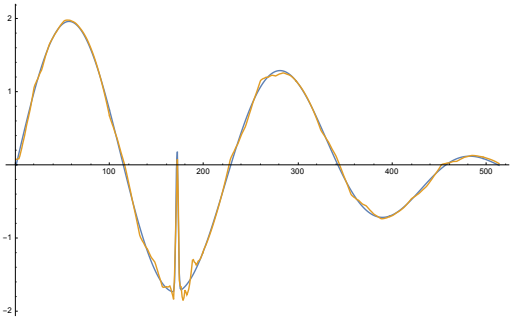
```
In[767]:= WaveletListPlot[dwt2,
 {{1}, {0, 1}, {0, 0, 1}, {0, 0, 0, 1}, {0, 0, 0, 0, 1}, {0, 0, 0, 0, 0}},
 PlotLayout → "CommonXAxis", Method → {"Inverse" → True}, ImageSize → Large]
```

Out[767]=




```
In[712]:= th2 = InverseWaveletTransform[WaveletThreshold[dwt2, {"SURE"}], Automatic];
In[768]:= ListLinePlot[{data2, th2}, PlotRange → All, ImageSize → Large]
```

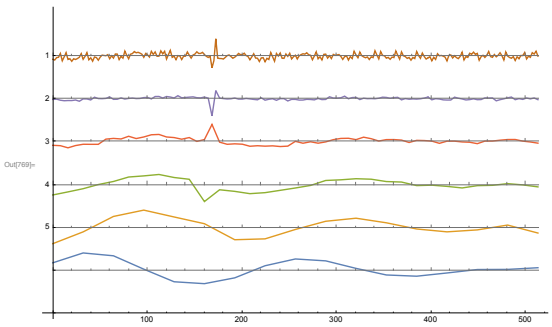
Out[768]=




```
In[714]:= dwt3 = DiscreteWaveletTransform[noised2, HaarWavelet[], 5]
Out[714]= DiscreteWaveletData[ Data dimensions: {513}  Refinements: 5 ]
```

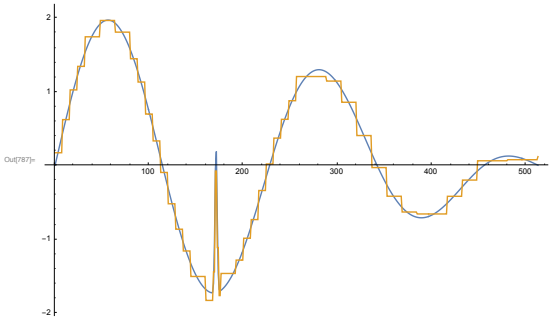
```
In[769]:= WaveletListPlot[dwt3,
 {{1}, {0, 1}, {0, 0, 1}, {0, 0, 0, 1}, {0, 0, 0, 0, 1}, {0, 0, 0, 0, 0}},
 PlotLayout → "CommonXAxis", Method → {"Inverse" → True}, ImageSize → Large]
```

Out[769]=




```
In[716]:= th3 = InverseWaveletTransform[WaveletThreshold[dwt3, {"SURE"}], Automatic];
In[787]:= ListLinePlot[{data2, th3}, PlotRange → All, ImageSize → Large]
```

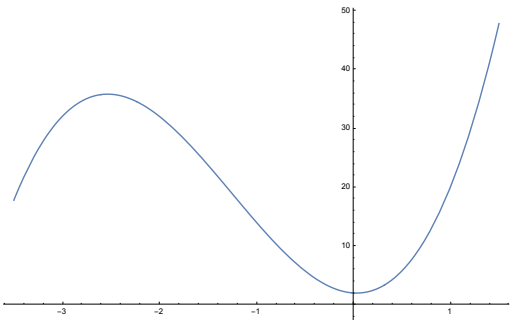
Out[787]=



Example (4)

```
In[427]:= pol[t_] = 4 t^3 + 15 t^2 - t + 2
Out[427]= 2 - t + 15 t^2 + 4 t^3
In[770]:= Plot[pol[t], {t, -3.5, 1.5}, ImageSize → Large]
```

Out[770]=




```
In[497]:= ϕN_[t_] := WaveletPhi[DaubechiesWavelet[N], t]
In[498]:= ψN_[t_] := WaveletPsi[DaubechiesWavelet[N], t]
In[499]:= aN_[k_] := NIntegrate[pol[t + k] * ϕN[t], {t, 0, 2 N - 1}, WorkingPrecision → 20]
In[500]:= dN_[k_] := NIntegrate[pol[t + k] * ψN[t], {t, -N + 1, N}, WorkingPrecision → 20]
In[469]:= outN_[t_] := Sum[aN[k] * ϕN[t - k], {k, -8, 6}]
In[471]:= o1[t_] = out1[t]
```

Out[471]=

$$1731.4\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][-6+t]\quad 0\le -6+t\le 1\right\}_{0}^{\text{True}}+$$

$$1119.77\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][-5+t]\quad 0\le -5+t\le 1\right\}_{0}^{\text{True}}+$$

$$669.798\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][-4+t]\quad 0\le -4+t\le 1\right\}_{0}^{\text{True}}+$$

$$357.536\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][-3+t]\quad 0\le -3+t\le 1\right\}_{0}^{\text{True}}+$$

$$159.028\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][-2+t]\quad 0\le -2+t\le 1\right\}_{0}^{\text{True}}+$$

$$50.3206\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][-1+t]\quad 0\le -1+t\le 1\right\}_{0}^{\text{True}}+$$

$$7.46104\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][t]\quad 0\le t\le 1\right\}_{0}^{\text{True}}+$$

$$6.49609\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][1+t]\quad 0\le 1+t\le 1\right\}_{0}^{\text{True}}+$$

$$23.4726\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][2+t]\quad 0\le 2+t\le 1\right\}_{0}^{\text{True}}+$$

$$34.4375\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][3+t]\quad 0\le 3+t\le 1\right\}_{0}^{\text{True}}+$$

$$15.4375\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][4+t]\quad 0\le 4+t\le 1\right\}_{0}^{\text{True}}-$$

$$57.4803\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][5+t]\quad 0\le 5+t\le 1\right\}_{0}^{\text{True}}-$$

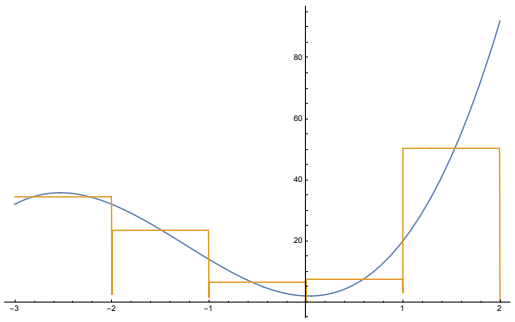
$$208.269\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][6+t]\quad 0\le 6+t\le 1\right\}_{0}^{\text{True}}-$$

$$460.882\left\{\text{InterpolatingFunction}\left[\text{Domain: }\{\{0.,1.\}\}\ \text{Output: scalar}\right][7+t]\quad 0\le 7+t\le 1\right\}_{0}^{\text{True}}-$$

839.272 { InterpolatingFunction[Domain: {{0., 1.}} Output: scalar][8 + t] 0 ≤ 8 + t ≤ 1
0 True

In[771]:= **Plot[{pol[t], o1[t]}, {t, -3, 2}, Exclusions → None, ImageSize → Large]**

Out[771]=



In[473]:= **o2[t_] = out$_2$[t]**

Out[473]= 1822.85 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][-6 + t] 0 ≤ -6 + t ≤ 3
0 True +

1187.32 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][-5 + t] 0 ≤ -5 + t ≤ 3
0 True +

717.011 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][-4 + t] 0 ≤ -4 + t ≤ 3
0 True +

387.915 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][-3 + t] 0 ≤ -3 + t ≤ 3
0 True +

176.034 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][-2 + t] 0 ≤ -2 + t ≤ 3
0 True +

57.3691 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][-1 + t] 0 ≤ -1 + t ≤ 3
0 True +

7.91913 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][t] 0 ≤ t ≤ 3
0 True +

3.68457 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][1 + t] 0 ≤ 1 + t ≤ 3
0 True +

20.6654 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][2 + t] 0 ≤ 2 + t ≤ 3
0 True +

34.8616 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][3 + t] 0 ≤ 3 + t ≤ 3
0 True +

22.2732 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][4 + t] 0 ≤ 4 + t ≤ 3
0 True -

41.0998 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][5 + t] 0 ≤ 5 + t ≤ 3
0 True -

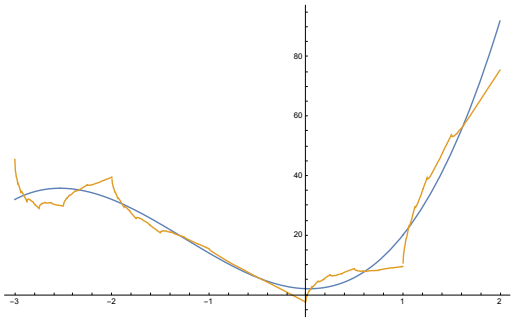
179.257 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][6 + t] 0 ≤ 6 + t ≤ 3
0 True -

416.2 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][7 + t] 0 ≤ 7 + t ≤ 3
0 True -

775.926 { InterpolatingFunction[Domain: {{0., 3.}} Output: scalar][8 + t] 0 ≤ 8 + t ≤ 3
0 True

In[772]:= **Plot[{pol[t], o2[t]}, {t, -3, 2}, ImageSize → Large]**

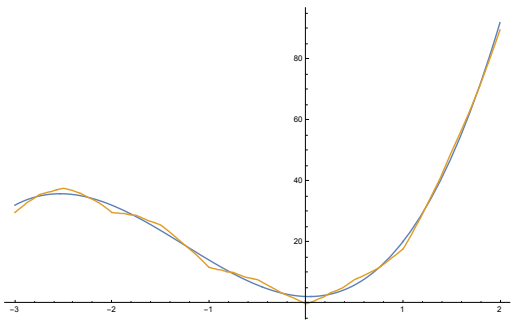
Out[772]=



o3[t_] = out$_3$[t];

In[788]:= **Plot[{pol[t], o3[t]}, {t, -3, 2}, ImageSize → Large]**

Out[788]=



In[477]:= **o4[t_] = out$_4$[t]**

Out[477]= 2105.87 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][-6 + t] 0 ≤ -6 + t ≤ 7
0 True +

1402.86 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][-5 + t] 0 ≤ -5 + t ≤ 7
0 True +

873.989 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][-4 + t] 0 ≤ -4 + t ≤ 7
0 True +

495.244 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][-3 + t] 0 ≤ -3 + t ≤ 7
0 True +

242.628 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][-2 + t] 0 ≤ -2 + t ≤ 7
0 True +

92.1427 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][-1 + t] 0 ≤ -1 + t ≤ 7
0 True +

19.7865 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][t] 0 ≤ t ≤ 7
0 True +

1.55949 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][1 + t] 0 ≤ 1 + t ≤ 7
0 True +

13.462 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][2 + t] 0 ≤ 2 + t ≤ 7
0 True +

31.494 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][3 + t] 0 ≤ 3 + t ≤ 7
0 True +

31.6554 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][4 + t] 0 ≤ 4 + t ≤ 7
0 True -

10.0537 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][5 + t] 0 ≤ 5 + t ≤ 7
0 True -

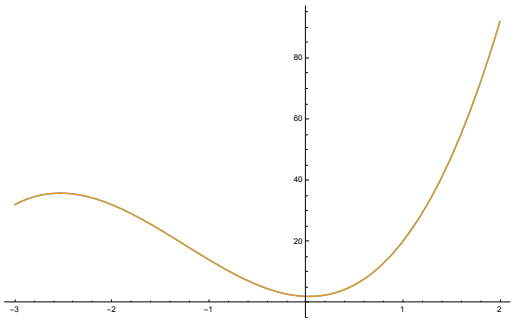
117.633 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][6 + t] 0 ≤ 6 + t ≤ 7
0 True

315.083 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][7 + t] 0 ≤ 7 + t ≤ 7
0 True

626.404 { InterpolatingFunction[Domain: {{0., 7.}} Output: scalar][8 + t] 0 ≤ 8 + t ≤ 7
0 True

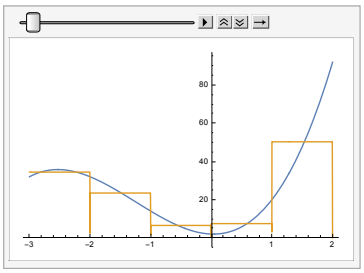
In[789]:=
```
Plot[{pol[t], o4[t]}, {t, -3, 2}, ImageSize → Large]
```

Out[789]=



In[564]:=
```
ListAnimate[Map[Plot[#, {t, -3, 2}] &, {{pol[t], o1[t]},
  {pol[t], o2[t]},
  {pol[t], o3[t]},
  {pol[t], o4[t]}}]]
```

Out[564]=

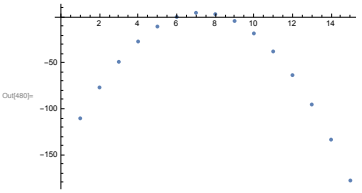


In[479]:=
```
Table[d1[k], {k, -8, 6}]
```

Out[479]= {−110.3414481, −76.70901709, −48.80699384, −26.68225330, −10.38167047, 0.04787964130, 4.559522038, 3.106381719, −4.358416314, −17.88174706, −37.51048552, −63.29150670, −95.27168560, −133.4978972, −178.0170165}

In[480]:=
```
ListPlot[%]
```

Out[480]=

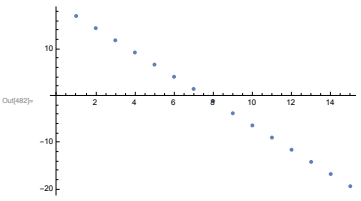


In[481]:=
```
Table[d2[k], {k, -8, 6}]
```

Out[481]= {16.98797632, 14.38990011, 11.79182390, 9.193747687, 6.595671476, 3.997595264, 1.399519053, −1.198557159, −3.796633370, −6.394709581, −8.992785793, −11.59086200, −14.18893822, −16.78701443, −19.38509064}

In[482]:=
```
ListPlot[%]
```

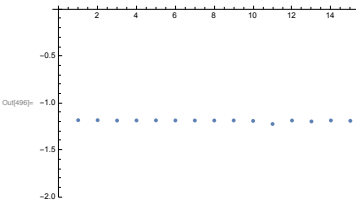
Out[482]=



In[495]:=
```
Table[d3[k], {k, -8, 6}];
```

In[496]:=
```
ListPlot[%, PlotRange → {-2, 0}]
```

Out[496]=



In[491]:=
```
Table[d4[k], {k, -8, 6}]
```

Out[491]= {0.007452550678, 0.002468165166, −0.002446635634, −0.002084425694, 0.0003445420316, −0.0002924807627, 0.0001288785905, −0.0006939128934, 0.0001509561142, −0.0002364899007, 0.003426931729, 0.001742872372, 0.03190869207, 0.003246796893, 0.007239255776}

In[492]:=
```
ListPlot[%, PlotRange → {-1, 1}]
```

Out[492]=