Project Flat-Panel CT Reconstruction
Fan Beam Reconstruction

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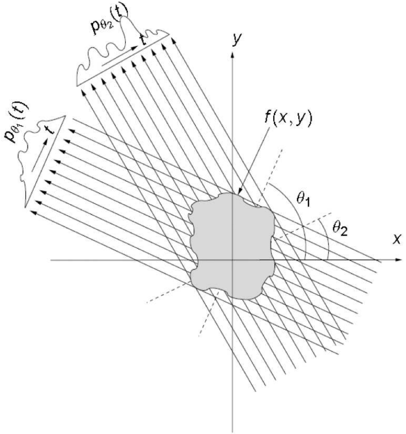
Topics

Fan Beam Geometry

Parallel Beam to Fan Beam Conversion

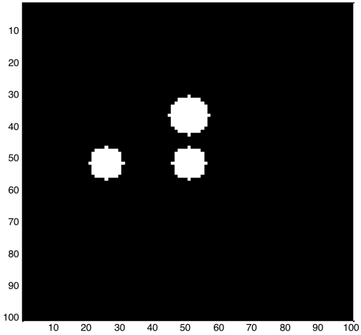
Short Scan

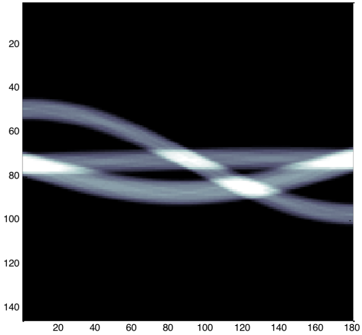
Parallel Beam Geometry



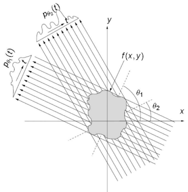
- Earliest Acquisition Geometry
- Principle: Rotate & Translate

Parallel Beam Geometry – Sinogram





Parallel Beam Geometry – Historical Remarks

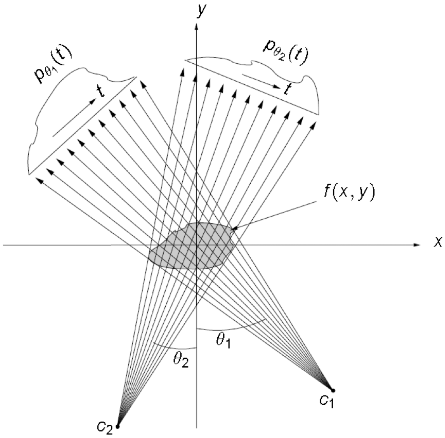


- Acquisition took 5 Minutes
- Reconstruction took 30 Minutes
- Slice resolution was 80 x 80 pixels

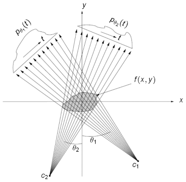
First CT Scanner: EMI (1971)

Image: Wikipedia

Fan Beam Geometry



Fan Beam Geometry – Historical Remarks



- Fan beam Scanners became available in 1975 (20s / slice)
- Fast rotations became possible 1987 with slip rings (300ms / slice)

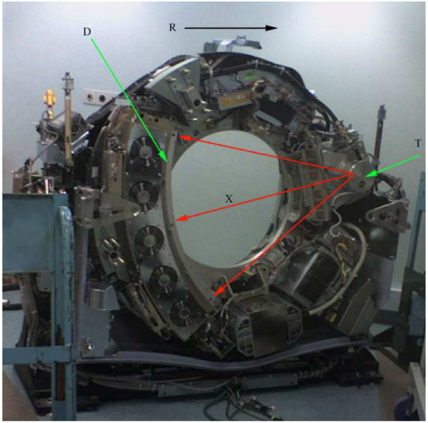


Image: Wikipedia

Fan Beam vs Parallel Beam

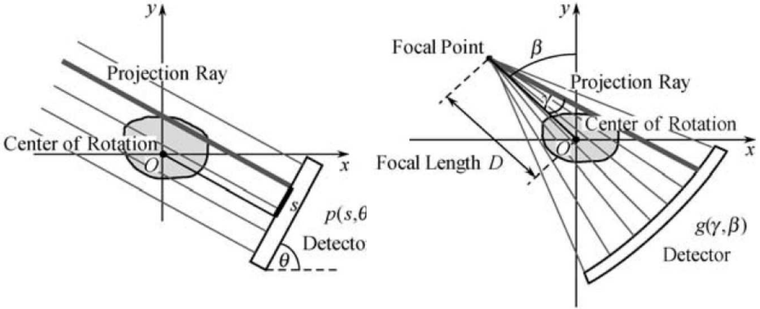


Image: Zeng, 2009

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Parallel Beam to Fan Beam Conversion

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Reconstruction Algorithm for Fan Beam?

- Parallel beam algorithms cannot be applied directly anymore
- We do not have a central slice theorem anymore
- It can be shown that the full circle PSF is equivalent to the parallel beam PSF

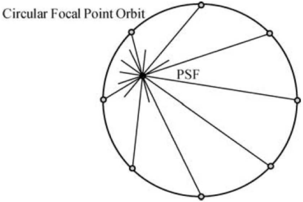
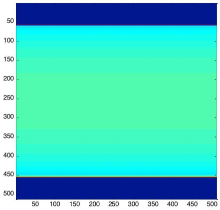
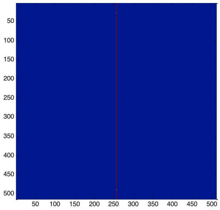


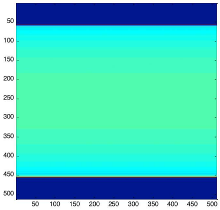
Image: Zeng, 2009

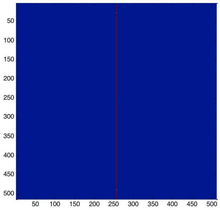
Backprojection and Fourier Slice Theorem

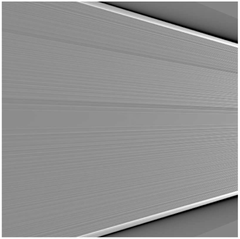


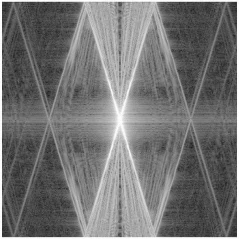


Backprojection and Fourier Slice Theorem









Parallel Beam to Fan Beam Conversion

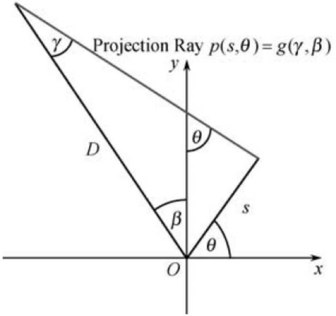


Image: Zeng, 2009

- Idea: Find equal rays in both geometries:

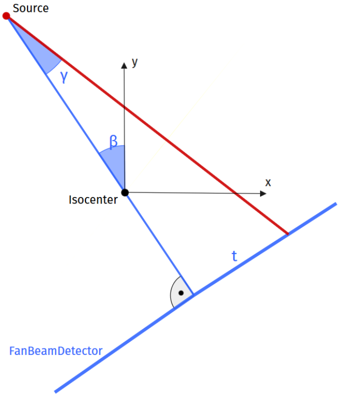
$$\theta = \gamma + \beta$$
$$s = D \sin \gamma$$

- Then set

$$p(s, \theta) = g(\gamma, \beta)$$

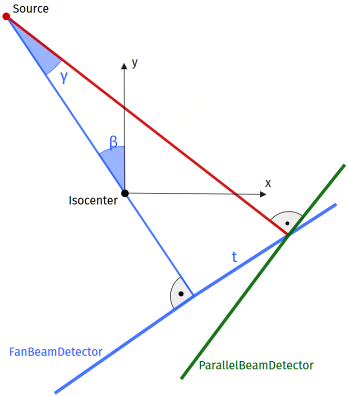
- This process is called "Rebinning"

Parallel Beam to Fan Beam Conversion - Flat-Panel (1)

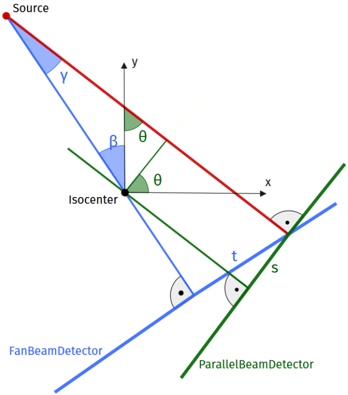


$$\tan \gamma = \frac{t}{D_{sd}}$$

Parallel Beam to Fan Beam Conversion - Flat-Panel (2)



Parallel Beam to Fan Beam Conversion - Flat-Panel (3)



$\sin\gamma = \frac{s}{D_{si}}$
$\theta = \beta + \gamma$

Parallel Beam to Fan Beam Conversion - Practical Aspects

- Rebinning is a feasible solution
- Change of coordinate systems requires interpolation which may introduce inaccuracies
- Hence, rebinning may not be the method of choice

$\Rightarrow$ Derive reconstruction method for fan beam data by conversion of the reconstruction algorithm

Parallel Beam to Fan Beam Conversion - Principle

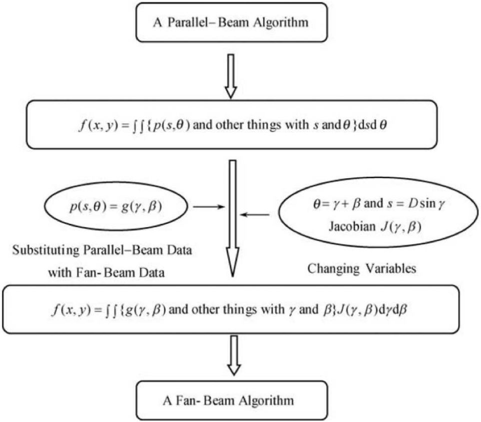


Image: Zeng, 2009

Equally-spaced and Equiangular Detectors

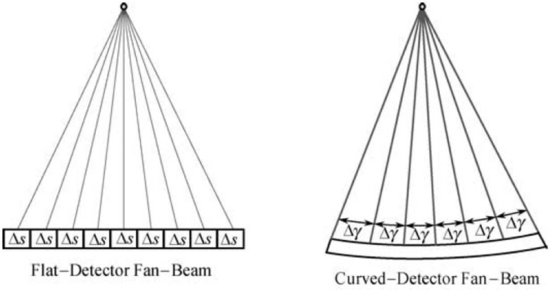


Image: Zeng, 2009

- Sampling is different in both geometries.
- Hence, different reconstruction formulas are obtained.

FBP for the Equiangular Case (1)

1. We start with a parallel beam backprojection:

$$f(x, y) = \frac{1}{2} \int_0^{2\pi} \int_{-\infty}^{\infty} p(s, \theta) h(x \cos \theta + y \sin \theta - s) \mathrm{d}s \mathrm{d}\theta$$

FBP for the Equiangular Case (2)

2. Perform cosine weighting:

$$g_1(\gamma, \beta) = g(\gamma, \beta) \cos \gamma$$

3. Apply fan beam filter:

$$g_2(\gamma, \beta) = g_1(\gamma, \beta) * h_{\text{fan}}(\gamma)$$

$$h_{\text{fan}}(\gamma) = \frac{D}{2} \left(\frac{\gamma}{\sin \gamma} \right)^2 h(\gamma)$$

4. Backproject with distance weight:

$$f(r, \varphi) = \int_0^{2\pi} \frac{1}{D'^2} g_2(\gamma', \beta) \mathrm{d}\beta$$

Example

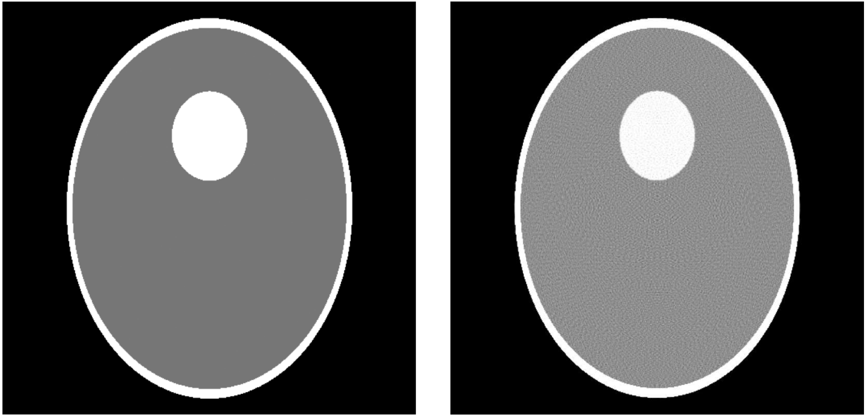


Figure: Reconstruction from fan-beam data.

FBP for the Equally-spaced Case

- Here we start with a parallel beam backprojection using polar coordinates (r, φ),
  - where $x = r\cos\varphi, y = r\sin\varphi$,
  - and $x\cos\theta + y\sin\theta = r\cos(\theta - \varphi)$.
- Derive reconstruction algorithm then from

$$f(r,\varphi) = \frac{1}{2}\int_0^{2\pi}\int_{-\infty}^{\infty} p(s,\theta)h(r\cos(\theta-\varphi) - s)\mathrm{d}s\mathrm{d}\theta$$

Parallel Beam to Fan Beam Conversion - Principle

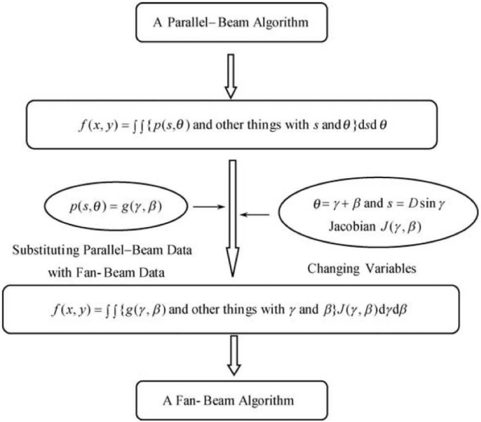


Image: Zeng, 2009

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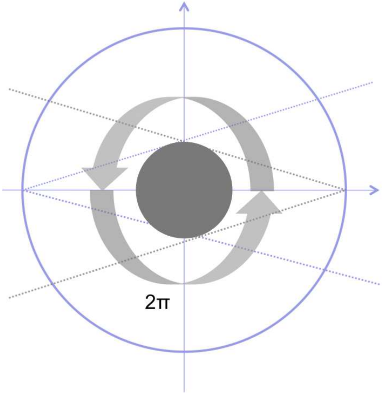
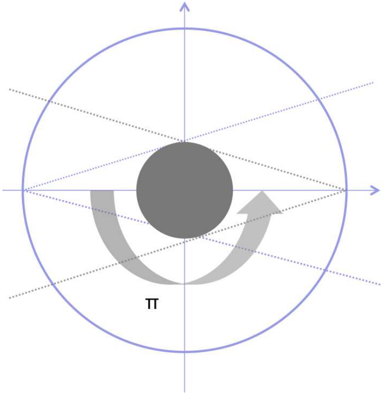
Parallel Beam to Fan Beam Conversion

Short Scan

Full Scan vs Half Scan

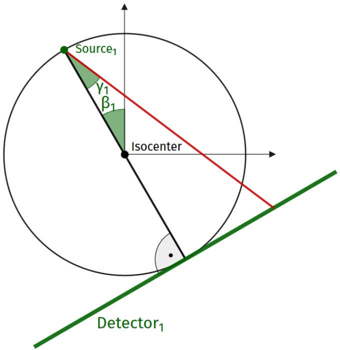


Full Scan

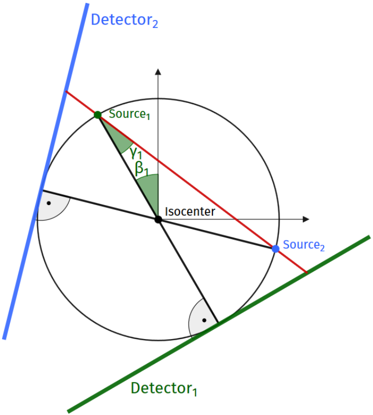


Half Scan

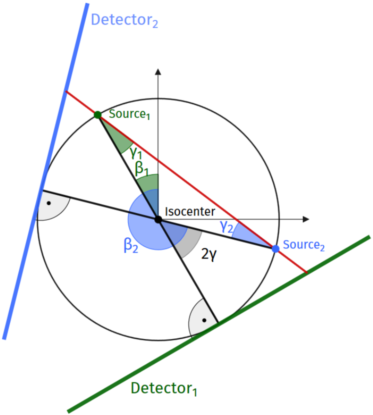
Redundant Areas – Sinogram



Redundant Areas – Sinogram

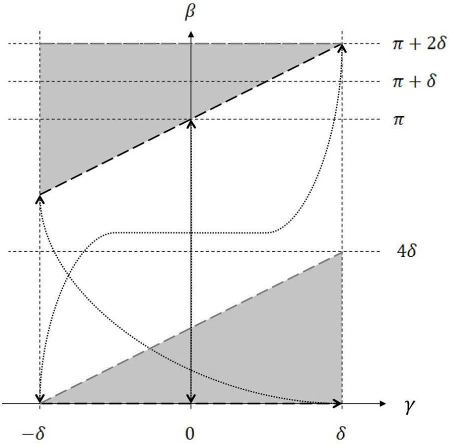


Redundant Areas – Sinogram



$$\gamma_2 = -\gamma_1$$
$$\beta_2 = \beta_1 + 2\gamma_1 + \pi$$

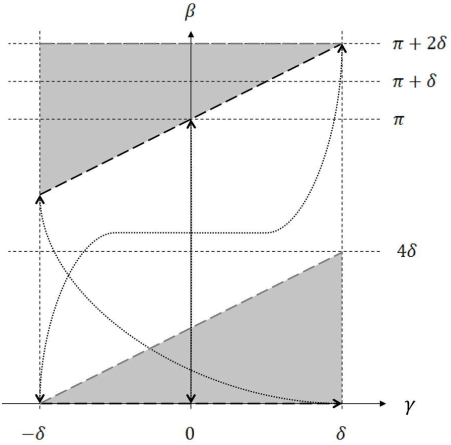
Redundant Areas – Sinogram



Identical rays:

$$\gamma_1 = -\gamma_2$$
$$\beta_2 = \beta_1 + 2\gamma_1 + \pi$$

Redundant Areas – Sinogram



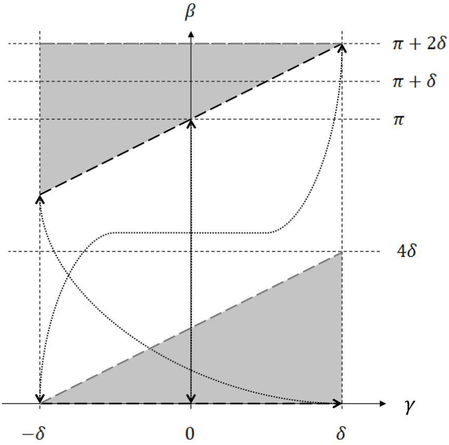
Identical rays:

$$\gamma_1 = -\gamma_2$$
$$\beta_2 = \beta_1 + 2\gamma_1 + \pi$$

Upper triangle:

$$\pi + 2\gamma_1 \leq \beta_1 \leq \pi + 2\delta$$

Redundant Areas – Sinogram



Identical rays:

$$\begin{aligned} \gamma_1 &= -\gamma_2 \\ \beta_2 &= \beta_1 + 2\gamma_1 + \pi \end{aligned}$$

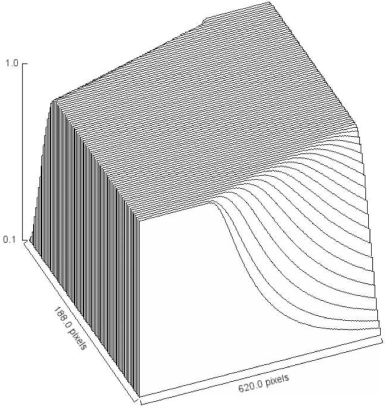
Upper triangle:

$$\pi + 2\gamma_1 \leq \beta_1 \leq \pi + 2\delta$$

Lower triangle:

$$0 \leq \beta_2 \leq 2\gamma_2 + 2\delta$$

Parker Weighting



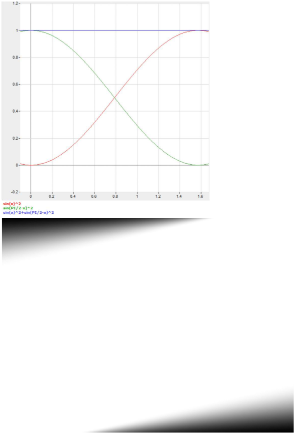


Figure: Parker Weights for a short scan trajectory.

FBP for the Equiangular Case and Parker Weight

- Perform Parker weighting with $w_p(t, \beta)$:

$$g_1(\gamma, \beta) = g(\gamma, \beta) w_p(\gamma, \beta)$$

- Perform cosine weighting:

$$g_2(\gamma, \beta) = g_1(\gamma, \beta) \cos \gamma$$

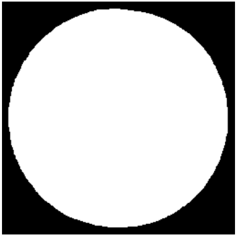
- Apply fan beam filter:

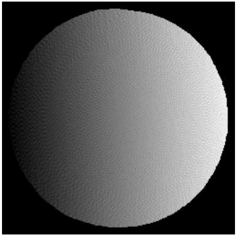
$$g_3(\gamma, \beta) = g_2(\gamma, \beta) * h_{\text{fan}}(\gamma)$$
$$h_{\text{fan}}(\gamma) = \frac{D}{2} \left(\frac{\gamma}{\sin \gamma} \right)^2 h(\gamma)$$

- Backproject with distance weight:

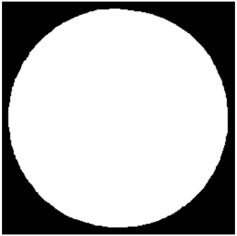
$$f(r, \varphi) = \int_0^{2\pi} \frac{1}{D'^2} g_3(\gamma', \beta) \mathrm{d}\beta$$

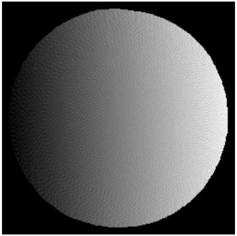
No Redundancy Weights – Example

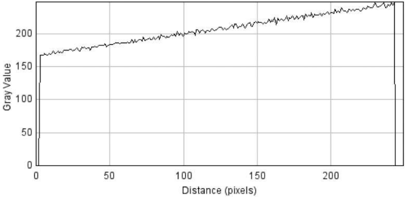




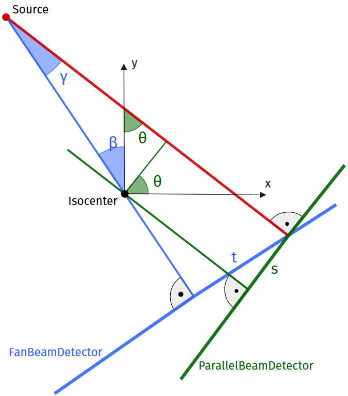
No Redundancy Weights – Example







Reminder - Rebinning Fan Beam with Flat-Panel Detector



$$\tan\gamma = \frac{t}{D_{sd}}$$
$$\sin\gamma = \frac{s}{D_{si}}$$
$$\theta = \beta + \gamma$$

Further Readings

- Gengsheng Lawrence "Larry" Zeng."Medical Image Reconstruction – A Conceptual Tutorial". Springer 2009
- Ronald N. Bracewell."The Fourier Transform and Its Applications". McGraw-Hill Publishing Company. 1999
- Dennis Parker. "Optimal short scan convolution reconstruction for fanbeam CT". Medical Physics. 9(2): 254-257. 1982
- Frederic Noo, Michel Defrise, Rolf Clackdoyle, Hiroyuki Kudo. "Image reconstruction from fan-beam projections on less than a short scan". Physics in Medicine and Biology 47: 2525-2546. 2002

Questions?