IMIP – Exercise

Vesselness Filtering & Bilateral Filtering

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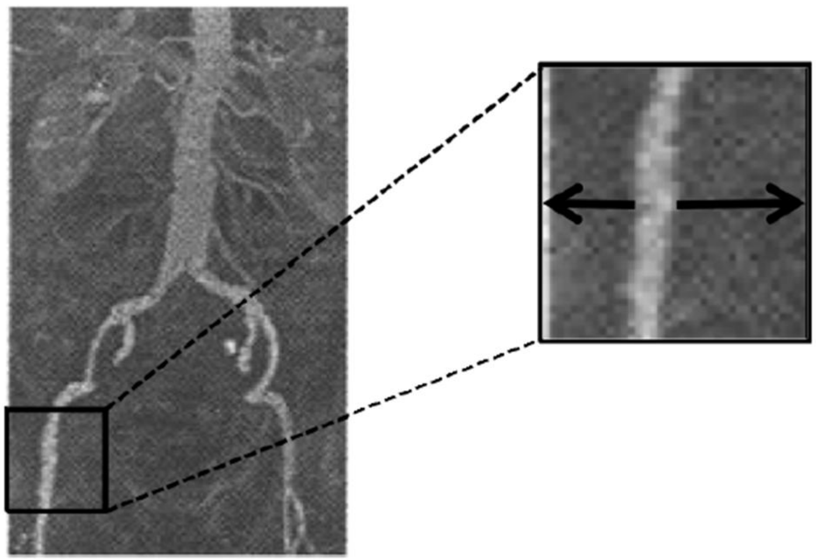
Vessel Segmentation



- *Problem 1*: Vessels appear in different diameters
  → We need to model different scales
- *Problem 2: Edges are only a weak model of vessels*
  → Structure tensor insufficient for vessel modelling

Problem 1: Scale Modelling

- Solution using scale space *s*

$$I(x,s) = I(x) * G(x,s)$$

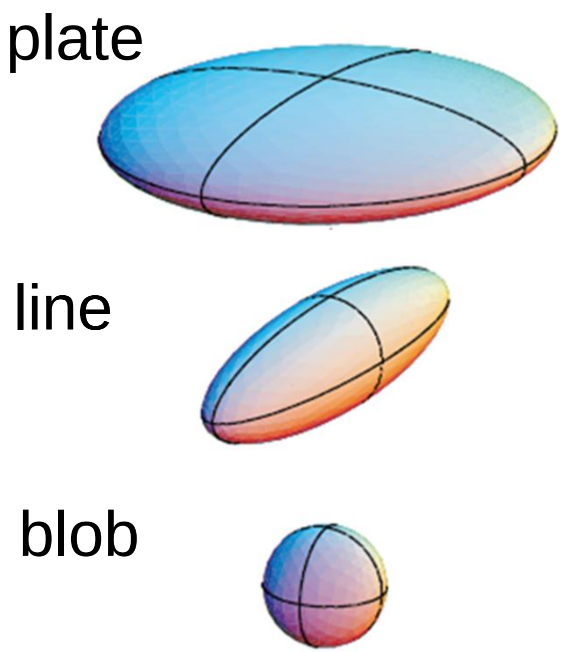
$$G(x,s) = \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{||x||^2}{2s^2}}$$

- Derivative of Gaussians

$$\frac{\delta}{\delta x} I(x,s) = I(x) * \frac{\delta}{\delta x} G(x,s)$$

Problem 2: Vessel Model

- Edges are not a good model of a vessel
  → Compute Hessian (The second order structure is exploited for local shape properties) $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|, \quad \lambda_n \in \mathbb{R}$



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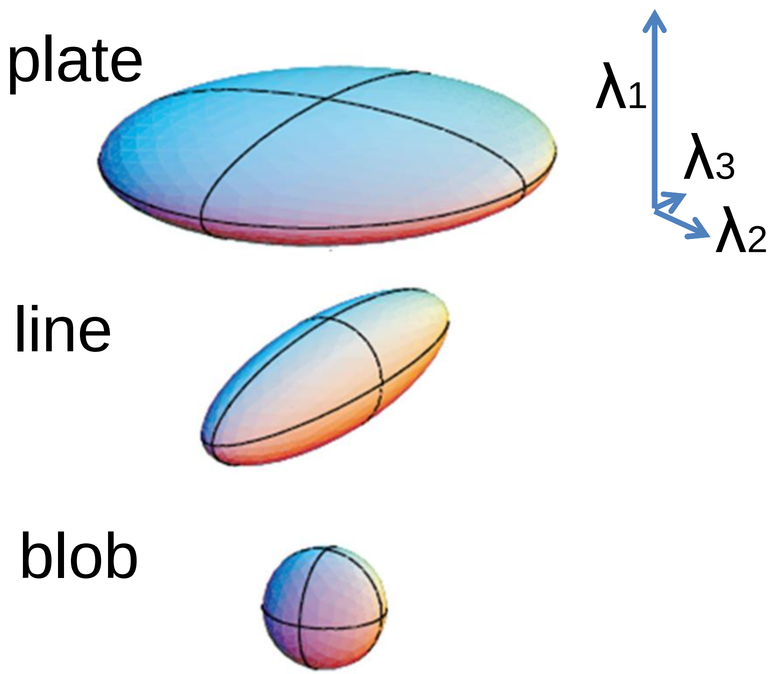

λ_1	λ_2	λ_3
-	0	0
+	0	0

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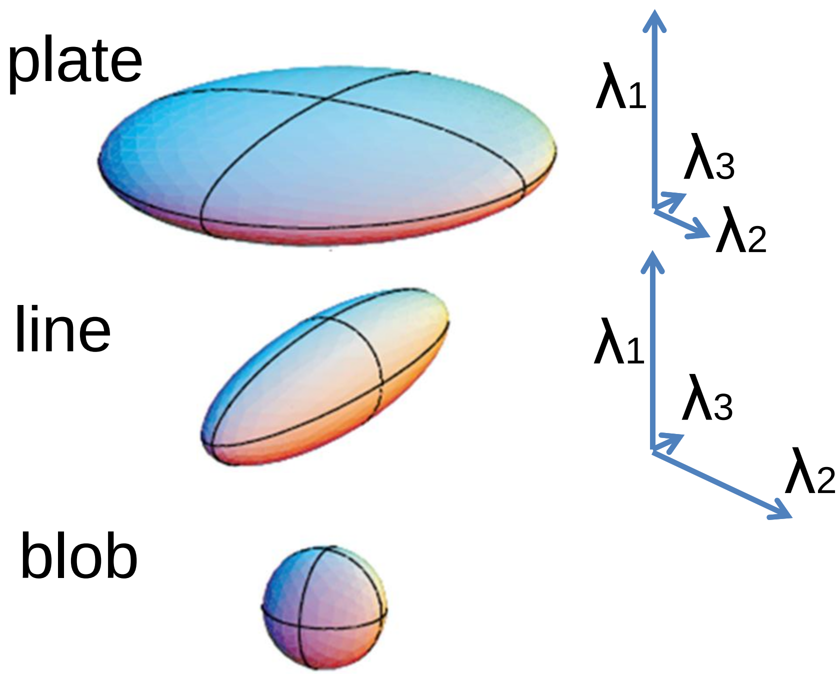

λ_1	λ_2	λ_3
-	0	0
+	0	0
-	-	0
+	+	0

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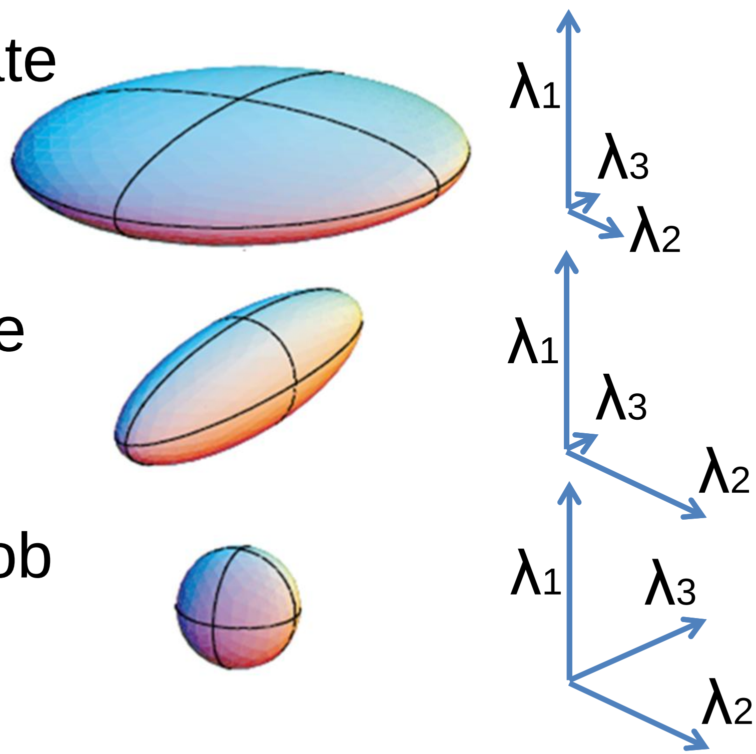
plate

line

blob



λ1	λ2	λ3
-	0	0
+	0	0
-	-	0
+	+	0
-	-	-
+	+	+

Problem 2: Vessel Model

In sum, for an ideal vessel in 3D image:

$$
\begin{aligned}
|\lambda_3| &\approx 0 \\
|\lambda_2| &\gg \lambda_3 \\
\lambda_1 &\approx \lambda_2
\end{aligned}
$$

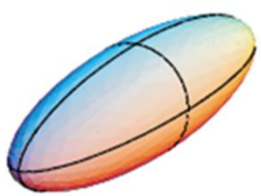
essel

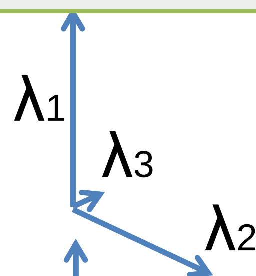
order structure is exploited

$\geq |\lambda_3|, \quad \lambda_n \in \mathbb{R}$

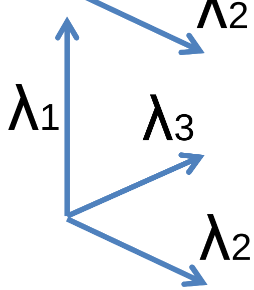
λ_1	λ_2	λ_3
-	0	0
+	0	0
-	-	0
+	+	0
-	-	-
+	+	+

line





blob



Three vesselness measures

Deviation from blob-like structure:

$$\mathcal{R}_B = \frac{|\lambda_3|}{\sqrt{|\lambda_1 \lambda_2|}}, \quad \mathcal{R}_B \in [0,1] \subset \mathbb{R}$$

Similarity to a plate-like structure:

$$\mathcal{R}_A = \frac{|\lambda_2|}{|\lambda_1|}, \quad \mathcal{R}_A \in [0,1] \subset \mathbb{R}$$

Frobenius norm, second-order-like structure:

$$\mathcal{S} = \|\mathbf{H}[L(\mathbf{x})]\|_F = \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}, \quad \mathcal{S} \in \mathbb{R}^{0+}$$

	λ_1	λ_2	λ_3
plate	-	0	0
	+	0	0
line	-	-	0
	+	+	0
blob	-	-	-
	+	+	+

In regions with high contrast, this norm will become larger since at least one of the eigenvalues will be large.

Problem 2: Vessel Model

- In the definition of vesselness the three properties are combined

$$V(\boldsymbol{x}, s) = \begin{cases} 0 \\ \left(1 - \exp\left(-\frac{R_A^2}{2\lambda^2}\right)\right) \exp\left(-\frac{R_B^2}{2\beta^2}\right) \left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) \end{cases}$$

$\lambda, \beta, c \rightarrow$ Parameter to select by user

($\lambda = \beta = 0,5$; c depends on contrast)

The idea behind this expression is to map the features into probability-like estimators of vesselness.

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The idea behind this expression is to map the features into probability-like estimators of vesselness.

The vesselness measure is analyzed at different scales. We integrate them to obtain the final estimate of vesselness:

$$V(\boldsymbol{x}) = \max_s V(\boldsymbol{x}, s)$$

Vessel Model in 2D (for exercise)

- Compute Hessian in 2D

$$H_s = \begin{pmatrix} \frac{\delta^2}{\delta x^2} I(x,s) & \frac{\delta^2}{\delta x \delta y} I(x,s) \\ \frac{\delta^2}{\delta y \delta x} I(x,s) & \frac{\delta^2}{\delta y^2} I(x,s) \end{pmatrix}$$

- Ideal vessel structure in 2D

$$|\lambda_1| < |\lambda_2| \quad \wedge \quad |\lambda_1| \approx 0$$

Note: in exercise we sort eigenvalues in ascending order!

Two vesselness measures

Blobness measure and second-order structure in 2D:

$$R_B = \frac{\lambda_1}{\lambda_2}$$ (Close to 0, if vessel)

$$S = \sqrt{\lambda_1^2 + \lambda_2^2}$$ (High contrast , if vessel)

Two vesselness measures

Blobness measure and second-order structure in 2D:

$$R_B = \frac{\lambda_1}{\lambda_2} \qquad \text{(Close to 0, if vessel)}$$

$$S = \sqrt{\lambda_1^2 + \lambda_2^2} \qquad \text{(High contrast , if vessel)}$$

Combining them into probability estimator:

$$V(\boldsymbol{x}, s) = \begin{cases} 0 & \text{if} \quad \lambda_2 > 0 \\ \exp\left(-\frac{R_B^2}{2\beta^2}\right)\left(1 - \exp\left(-\frac{S^2}{2c^2}\right)\right) \end{cases}$$

β, c: Control parameters ($\beta = 0.5$; c depends on scaling)

$$V(\boldsymbol{x}) = \max_{s_{\min} < s < s_{\max}} V(\boldsymbol{x}, s)$$

Bilateral Filtering

- *Problem*: Conventional filters smooth across edges

- *Idea:* Incorporate edge-stopping functionality based on pixel similarity

General idea

- Every sample is replaced by a weighted average of its neighbours

- These weights reflect two properties
  → How close are the neighbour and the center sample, so that that larger weight to closer samples (*Spatial closeness*)

  → How similar are the neighbour and the center sample – larger weight to similar samples (*Range similarity*)

- All the weights should be normalized to preserve the local mean

In an Equation

$$BF[I]_x = \frac{1}{W_x} \sum_{x' \in \omega_x} G_{\sigma_c}\left(\| x - x' \|\right) G_{\sigma_s}\left(| I(x) - I(x') |\right) I_{x'}$$

In an Equation

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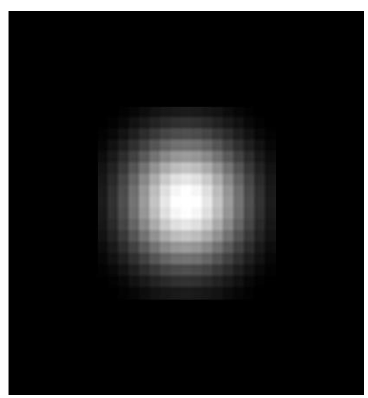
space weight



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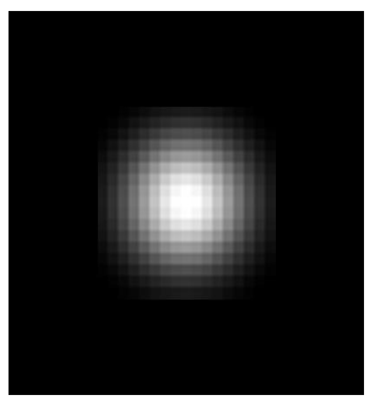
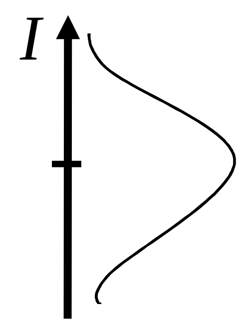


range weight



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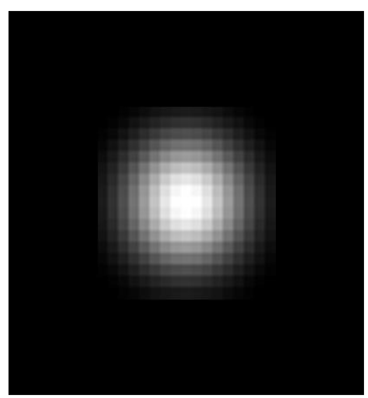
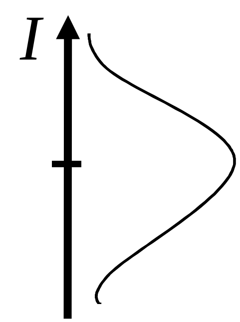
normalization factor

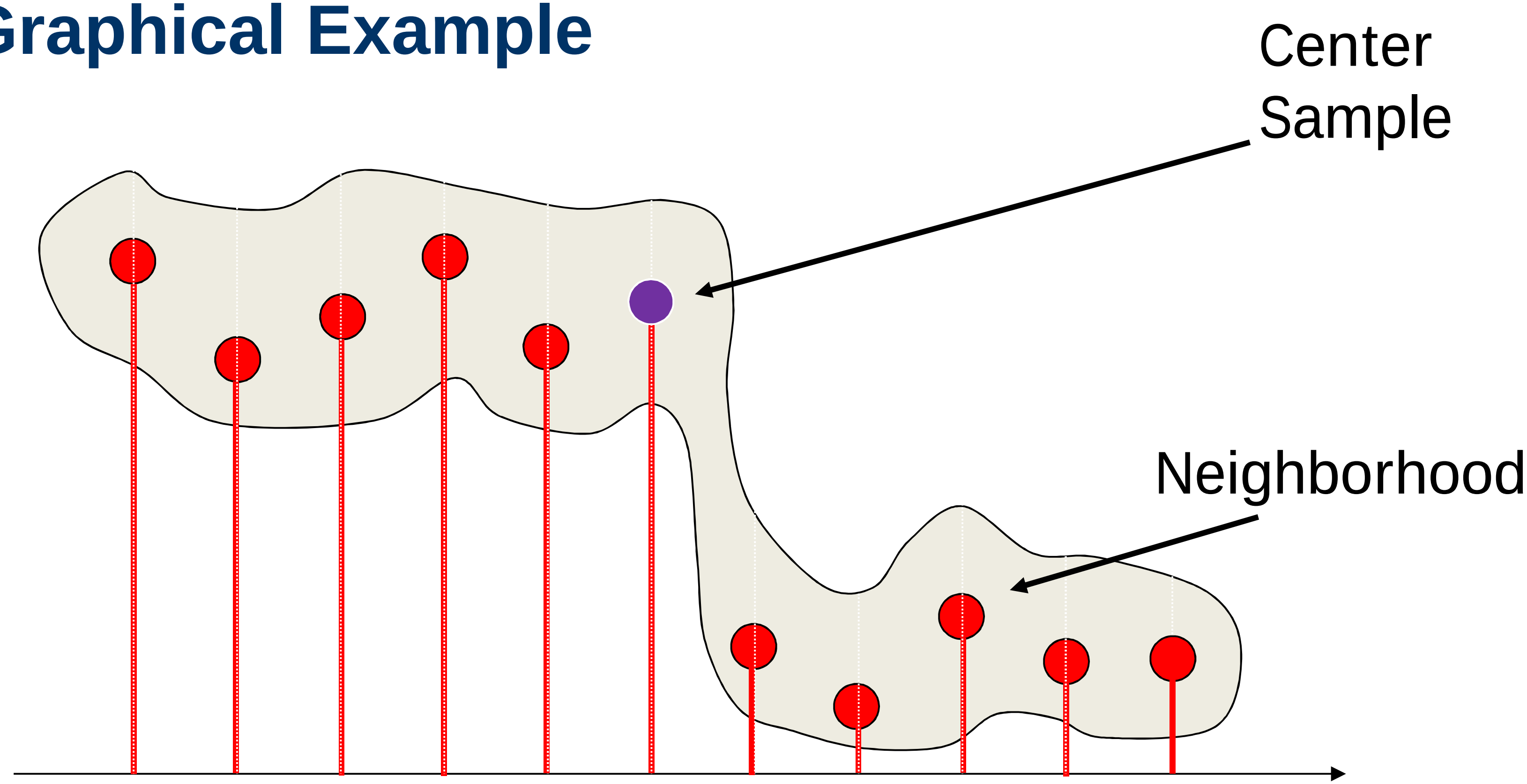
space weight



range weight

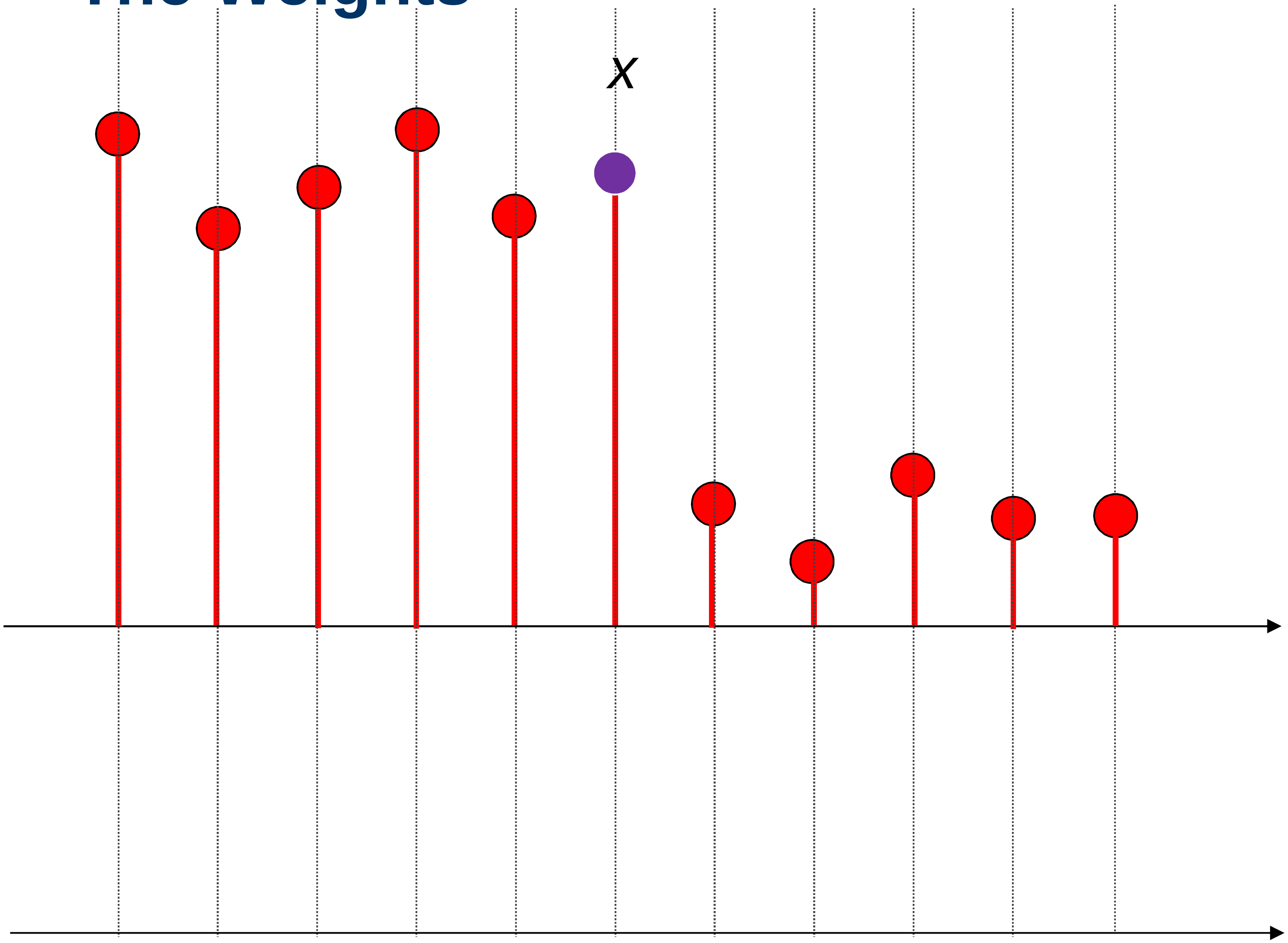


Graphical Example



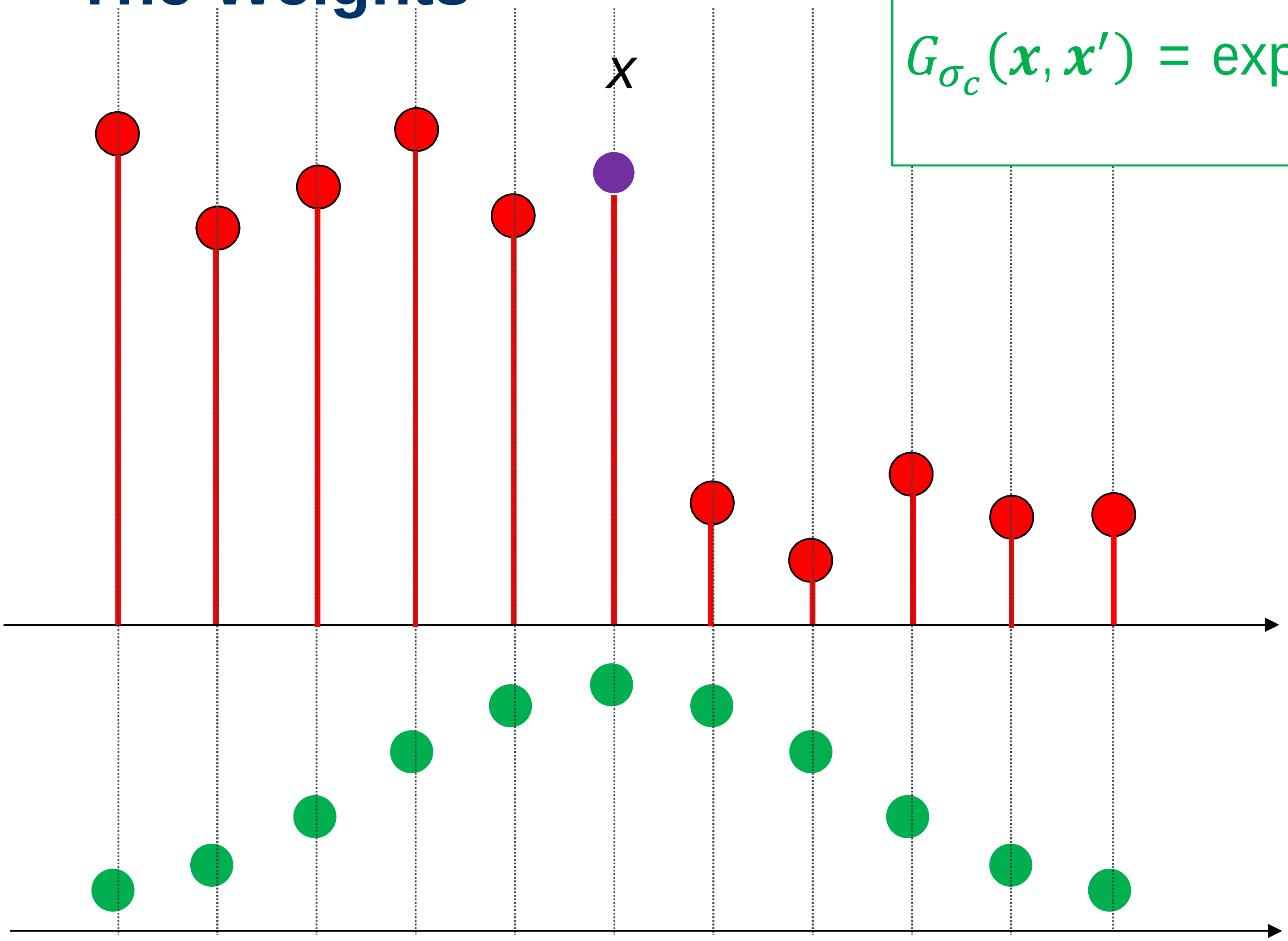
It is clear that in weighting this neighborhood,
we would like to preserve the step

The Weights



The Weights

$$G_{\sigma_c}(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(-\frac{\|\boldsymbol{x} - \boldsymbol{x}'\|^2}{{\sigma_c}^2}\right)$$



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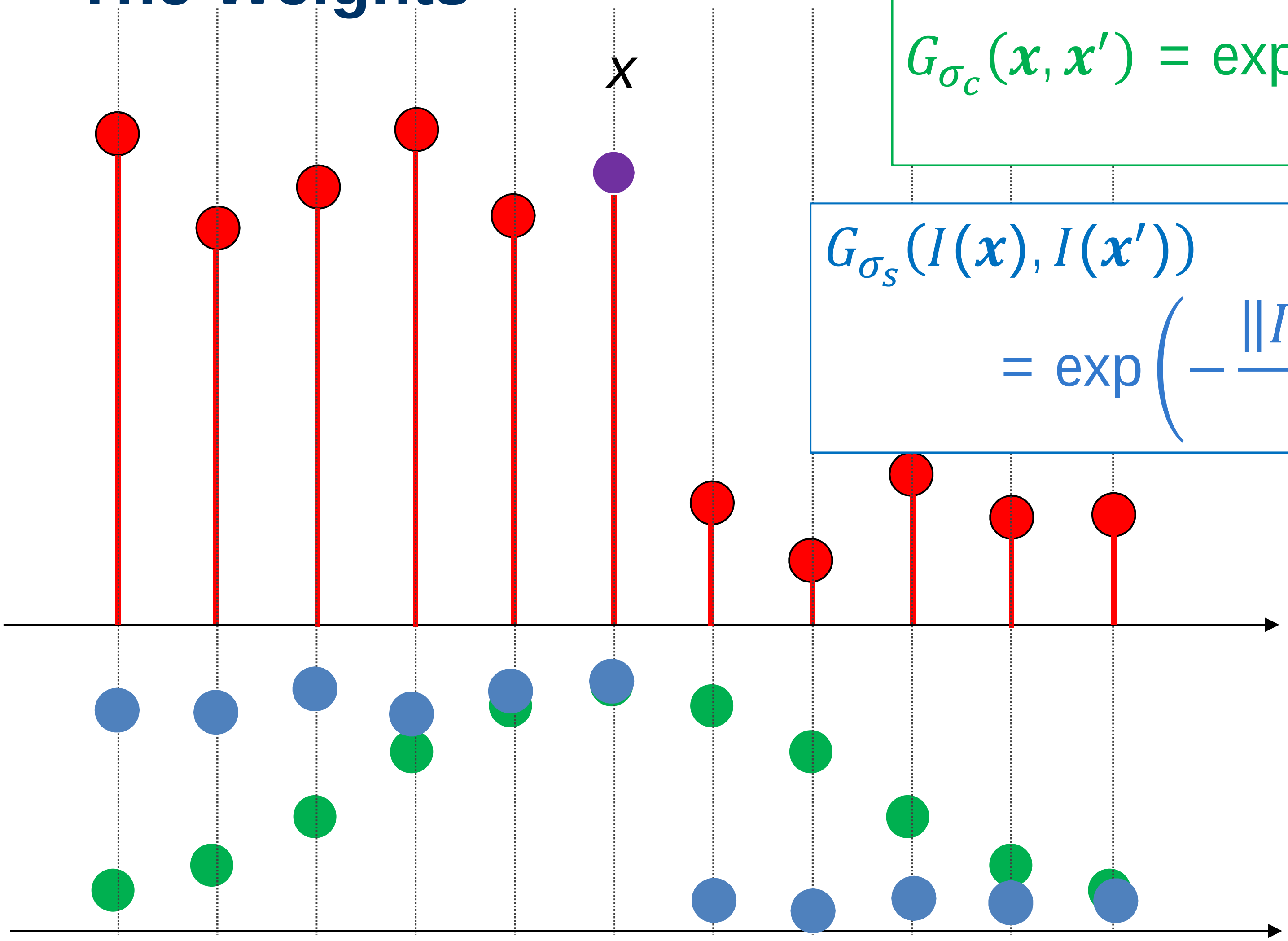

$$G_{\sigma_c}(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(-\frac{\|\boldsymbol{x} - \boldsymbol{x}'\|^2}{{\sigma_c}^2}\right)$$

$$G_{\sigma_s}(I(\boldsymbol{x}), I(\boldsymbol{x}')) = \exp\left(-\frac{\|I(\boldsymbol{x}) - I(\boldsymbol{x}')\|^2}{{\sigma_s}^2}\right)$$

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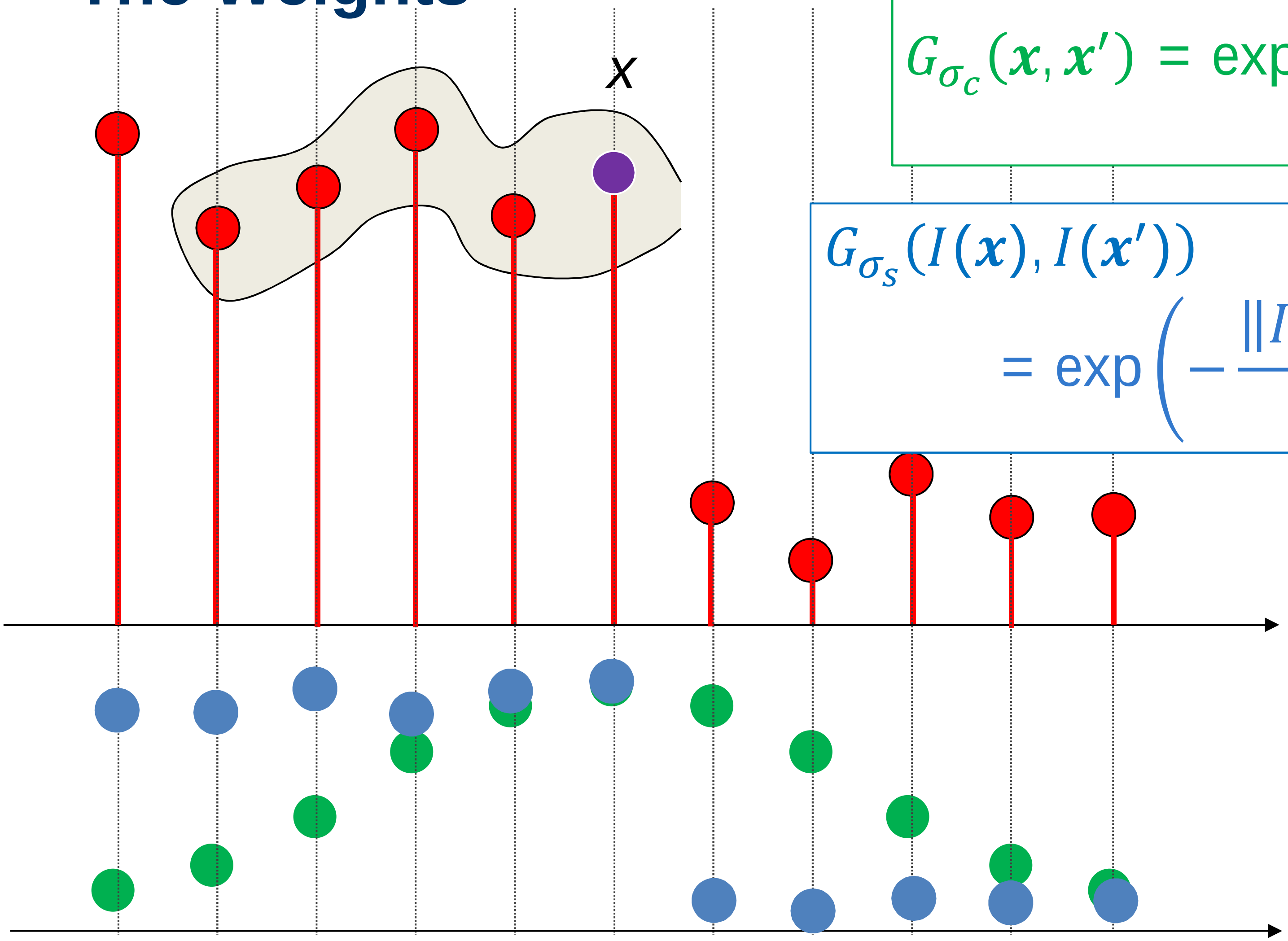

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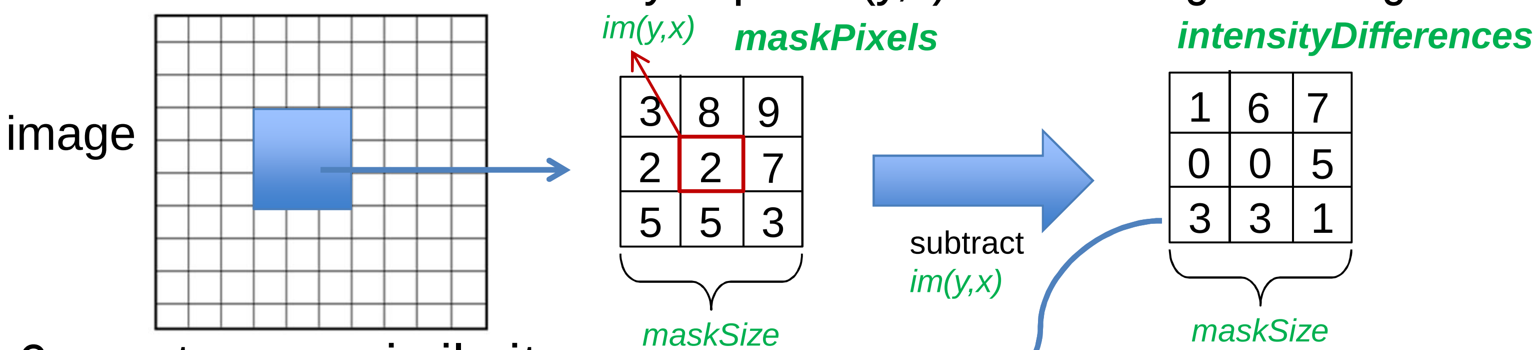
Implementation for Exercise

- Pre-computed spatial closeness in the mask (it's independent to image)

$$\text{closeness} = \frac{1}{\sqrt{2\pi\sigma_c^2}} \exp\left(-\frac{X^2+Y^2}{2\sigma_c^2}\right)$$

- Compute range similarity (depends on image intensity)
  1. Extract a sub-region of the image which is inside the filter mask
  2. Get the difference of intensity in point (y,x) and its neighbouring's



  3. Compute range similarity

$$\text{similarity} = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{\|I(x)-I(x')\|^2}{2\sigma_s^2}\right)$$

- Combine these two weights for each pixel inside the mask
- New intensity is obtained by a weighted average of its neighbours

$$\text{filtered(x)} = \sum_{\text{x'}\in\omega_x} \text{I(x')}W(x,x') / \sum_{\text{x'}\in\omega_x} W(x,x')$$