Construction of Daubechies filters D4 und D6

The D4 filter pair

Polynomials for low-pass and high-pass filters of length 4

```
d4h[z_] := Sum[h4[k] z^k, {k, 0, 3}]; d4h[z]
```

$h4[0] + z\, h4[1] + z^2\, h4[2] + z^3\, h4[3]$

```
d4hcoeffs = CoefficientList[d4h[z], z]
```

{h4[0], h4[1], h4[2], h4[3]}

```
g4[k_] := (-1)^k h4[3 - k]
```

```
d4g[z_] := Sum[g4[k] z^k, {k, 0, 3}]; d4g[z]
```

$-z^3\, h4[0] + z^2\, h4[1] - z\, h4[2] + h4[3]$

Orthogonality conditions

```
O41 = Sum[h4[k] h4[k], {k, 0, 3}] == 1
```

$h4[0]^2 + h4[1]^2 + h4[2]^2 + h4[3]^2 == 1$

```
O42 = Sum[h4[k] h4[k + 2], {k, 0, 1}] == 0
```

$h4[0]\, h4[2] + h4[1]\, h4[3] == 0$

Low-pass conditions

```
ansatz[ω_] := d4h[Exp[I ω]]; ansatz[ω]
```

$h4[0] + e^{i\omega}\, h4[1] + e^{2 i \omega}\, h4[2] + e^{3 i \omega}\, h4[3]$

```
TP41 = ansatz[Pi] == 0
```

$h4[0] - h4[1] + h4[2] - h4[3] == 0$

```
TP42 = ansatz'[Pi] == 0
```

$-i\, h4[1] + 2\, i\, h4[2] - 3\, i\, h4[3] == 0$

Solving the conditions

```
Solve[{O41, O42, TP41, TP42}, d4hcoeffs]
```

$$\left\{\left\{h4[0] \to \frac{1}{12}\left(2\sqrt{3}\left(2-\sqrt{3}\right)^{3/2}-7\sqrt{3\left(2-\sqrt{3}\right)}\right),\right.\right.$$
$$h4[1] \to \frac{1}{4}\left(\sqrt{3}\left(2-\sqrt{3}\right)^{3/2}-4\sqrt{3\left(2-\sqrt{3}\right)}\right),\ h4[2] \to -\frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)},$$
$$\left.h4[3] \to \frac{1}{12}\left(-\sqrt{3}\left(2-\sqrt{3}\right)^{3/2}+2\sqrt{3\left(2-\sqrt{3}\right)}\right)\right\},$$
$$\left\{h4[0] \to \frac{1}{12}\left(-2\sqrt{3}\left(2-\sqrt{3}\right)^{3/2}+7\sqrt{3\left(2-\sqrt{3}\right)}\right),\right.$$
$$h4[1] \to \frac{1}{4}\left(-\sqrt{3}\left(2-\sqrt{3}\right)^{3/2}+4\sqrt{3\left(2-\sqrt{3}\right)}\right),\ h4[2] \to \frac{1}{4}\sqrt{3\left(2-\sqrt{3}\right)},$$
$$\left.h4[3] \to \frac{1}{12}\left(\sqrt{3}\left(2-\sqrt{3}\right)^{3/2}-2\sqrt{3\left(2-\sqrt{3}\right)}\right)\right\},$$
$$\left\{h4[0] \to \frac{1}{12}\left(2\sqrt{3}\left(2+\sqrt{3}\right)^{3/2}-7\sqrt{3\left(2+\sqrt{3}\right)}\right),\right.$$
$$h4[1] \to \frac{1}{4}\left(\sqrt{3}\left(2+\sqrt{3}\right)^{3/2}-4\sqrt{3\left(2+\sqrt{3}\right)}\right),\ h4[2] \to -\frac{1}{4}\sqrt{3\left(2+\sqrt{3}\right)},$$
$$\left.h4[3] \to \frac{1}{12}\left(-\sqrt{3}\left(2+\sqrt{3}\right)^{3/2}+2\sqrt{3\left(2+\sqrt{3}\right)}\right)\right\},$$
$$\left\{h4[0] \to \frac{1}{12}\left(-2\sqrt{3}\left(2+\sqrt{3}\right)^{3/2}+7\sqrt{3\left(2+\sqrt{3}\right)}\right),\right.$$
$$h4[1] \to \frac{1}{4}\left(-\sqrt{3}\left(2+\sqrt{3}\right)^{3/2}+4\sqrt{3\left(2+\sqrt{3}\right)}\right),\ h4[2] \to \frac{1}{4}\sqrt{3\left(2+\sqrt{3}\right)},$$
$$\left.\left.h4[3] \to \frac{1}{12}\left(\sqrt{3}\left(2+\sqrt{3}\right)^{3/2}-2\sqrt{3\left(2+\sqrt{3}\right)}\right)\right\}\right\}$$

```
Simplify[%]
```

$$\Big\{\Big\{h4[0]\to -\frac{1}{4}\sqrt{2-\sqrt{3}}\,\left(2+\sqrt{3}\right),\ h4[1]\to -\frac{1}{4}\sqrt{2-\sqrt{3}}\,\left(3+2\sqrt{3}\right),\ h4[2]\to -\frac{1}{4}\sqrt{6-3\sqrt{3}},\ h4[3]\to \frac{\sqrt{2-\sqrt{3}}}{4}\Big\},\ \Big\{h4[0]\to \frac{1}{4}\sqrt{2-\sqrt{3}}\,\left(2+\sqrt{3}\right),\ h4[1]\to \frac{1}{4}\sqrt{2-\sqrt{3}}\,\left(3+2\sqrt{3}\right),\ h4[2]\to \frac{1}{4}\sqrt{6-3\sqrt{3}},\ h4[3]\to -\frac{1}{4}\sqrt{2-\sqrt{3}}\Big\},$$

$$\Big\{h4[0]\to -\frac{1}{4}\left(-2+\sqrt{3}\right)\sqrt{2+\sqrt{3}},\ h4[1]\to \frac{1}{4}\left(3-2\sqrt{3}\right)\sqrt{2+\sqrt{3}},\ h4[2]\to -\frac{1}{4}\sqrt{3\left(2+\sqrt{3}\right)},\ h4[3]\to -\frac{1}{4}\sqrt{2+\sqrt{3}}\Big\},$$

$$\Big\{h4[0]\to \frac{1}{4}\left(-2+\sqrt{3}\right)\sqrt{2+\sqrt{3}},\ h4[1]\to \frac{1}{4}\sqrt{2+\sqrt{3}}\,\left(-3+2\sqrt{3}\right),\ h4[2]\to \frac{1}{4}\sqrt{3\left(2+\sqrt{3}\right)},\ h4[3]\to \frac{\sqrt{2+\sqrt{3}}}{4}\Big\}\Big\}$$

```
NSolve[{O41, O42, TP41, TP42}, d4hcoeffs]
```

```
{{h4[0] → -0.482963, h4[1] → -0.836516, h4[2] → -0.224144, h4[3] → 0.12941},
 {h4[0] → 0.482963, h4[1] → 0.836516, h4[2] → 0.224144, h4[3] → -0.12941},
 {h4[0] → 0.12941, h4[1] → -0.224144, h4[2] → -0.836516, h4[3] → -0.482963},
 {h4[0] → -0.12941, h4[1] → 0.224144, h4[2] → 0.836516, h4[3] → 0.482963}}
```

```
daub4 = %[[2]]
```

```
{h4[0] → 0.482963, h4[1] → 0.836516, h4[2] → 0.224144, h4[3] → -0.12941}
```

Polynomials and frequency representation

```
H4[z_] := d4h[z] /. daub4; H4[z]
```

$$0.482963 + 0.836516\,z + 0.224144\,z^2 - 0.12941\,z^3$$

```
fourierH4[ω_] := H4[Exp[I ω]]; fourierH4[ω]
```

$$0.482963 + 0.836516\,e^{i\omega} + 0.224144\,e^{2i\omega} - 0.12941\,e^{3i\omega}$$

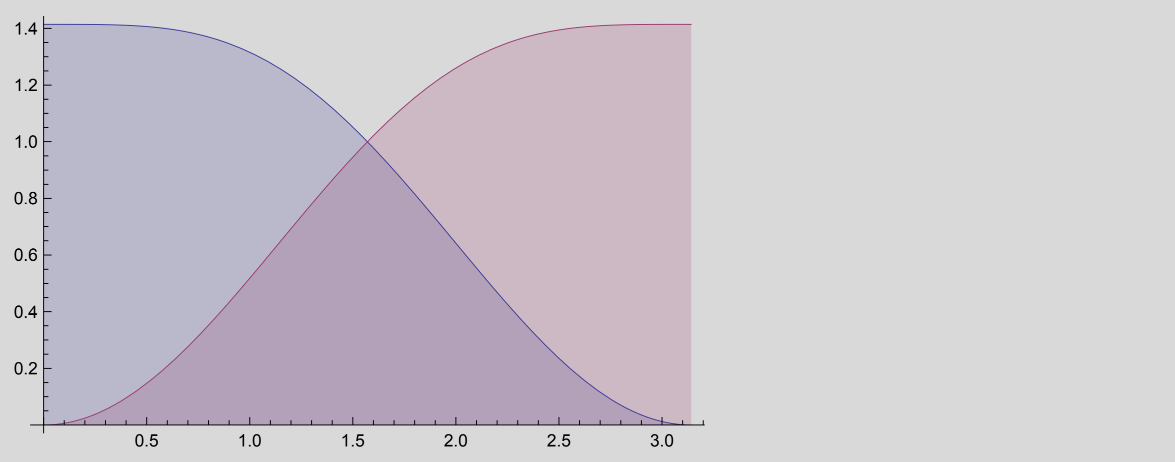
```
G4[z_] := d4g[z] /. daub4; G4[z]
```

$$-0.12941 - 0.224144\,z + 0.836516\,z^2 - 0.482963\,z^3$$

```
fourierG4[ω_] := G4[Exp[I ω]]; fourierG4[ω]
```

$$-0.12941 - 0.224144\,e^{i\omega} + 0.836516\,e^{2i\omega} - 0.482963\,e^{3i\omega}$$

```
Plot[{Abs[fourierH4[ω]], Abs[fourierG4[ω]]}, {ω, 0, Pi}, Filling → Axis]
```



The D6 filter pair

Polynomials for low-pass and high-pass filters of length 6

```
d6h[z_] := Sum[h6[k] z^k, {k, 0, 5}]; d6h[z]
```

$h6[0] + z\, h6[1] + z^2\, h6[2] + z^3\, h6[3] + z^4\, h6[4] + z^5\, h6[5]$

```
d6hcoeffs = CoefficientList[d6h[z], z]
```

{h6[0], h6[1], h6[2], h6[3], h6[4], h6[5]}

```
g6[k_] := (-1)^k h6[5 - k]
```

```
d6g[z_] := Sum[g6[k] z^k, {k, 0, 5}]; d4g[z]
```

$-z^3\, h4[0] + z^2\, h4[1] - z\, h4[2] + h4[3]$

Orthogonality conditions

```
O61 = Sum[h6[k] h6[k], {k, 0, 5}] == 1
```

$h6[0]^2 + h6[1]^2 + h6[2]^2 + h6[3]^2 + h6[4]^2 + h6[5]^2 == 1$

```
O62 = Sum[h6[k] h6[k + 2], {k, 0, 3}] == 0
```

$h6[0]\, h6[2] + h6[1]\, h6[3] + h6[2]\, h6[4] + h6[3]\, h6[5] == 0$

```
O63 = Sum[h6[k] h6[k + 4], {k, 0, 1}] == 0
```

$h6[0]\, h6[4] + h6[1]\, h6[5] == 0$

Low pass conditions

```
ansatz[ω_] := d6h[Exp[I ω]]; ansatz[ω]
```

$$h6[0] + e^{i\omega} h6[1] + e^{2 i \omega} h6[2] + e^{3 i \omega} h6[3] + e^{4 i \omega} h6[4] + e^{5 i \omega} h6[5]$$

```
TP61 = ansatz[Pi] == 0
```

$$h6[0] - h6[1] + h6[2] - h6[3] + h6[4] - h6[5] == 0$$

```
TP62 = ansatz'[Pi] == 0
```

$$-i\, h6[1] + 2 i\, h6[2] - 3 i\, h6[3] + 4 i\, h6[4] - 5 i\, h6[5] == 0$$

```
TP63 = ansatz''[Pi] == 0
```

$$h6[1] - 4\, h6[2] + 9\, h6[3] - 16\, h6[4] + 25\, h6[5] == 0$$

Solving the conditions

```
Solve[{O61, O62, O63, TP61, TP62, TP63}, d6hcoeffs]
```

$$\Bigg\{\Bigg\{h6[0] \to \frac{1}{275\,400}\Bigg(\frac{135\,225}{8}\sqrt{\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)} - \frac{57\,855}{8}\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)^{3/2} + \frac{3085}{8}\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)^{5/2} - \frac{47}{8}\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)^{7/2}\Bigg),$$

$$h6[1] \to \frac{1}{18\,360}\Bigg(-\frac{5895}{8}\sqrt{\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)} - \frac{5265}{8}\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)^{3/2} + \frac{313}{8}\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)^{5/2} - \frac{5}{8}\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)^{7/2}\Bigg),$$

$$h6[2] \to -\frac{1}{8}\sqrt{\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)},$$

$$h6[3] \to \frac{1}{6885}\Bigg(\frac{26\,415}{8}\sqrt{\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)} - \frac{5235}{16}\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)^{3/2} + \frac{67}{8}\left(20 - 4\sqrt{10} - \sqrt{\left(5\left(-5 + 4\sqrt{10}\right)\right)}\right)^{5/2} -$$

$$\frac{1}{16}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\Bigg),$$

$$\text{h6}[4]\to\frac{1}{18\,360}\left(\frac{127\,305}{8}\sqrt{\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}-\right.$$

$$\frac{15\,735}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$

$$\frac{581}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$

$$\left.\frac{7}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[5]\to\frac{1}{91\,800}\left(\frac{267\,075}{8}\sqrt{\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}-\right.$$

$$\frac{36\,735}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$

$$\frac{1475}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$

$$\left.\frac{19}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right)\Big\},$$

$$\Big\{\text{h6}[0]\to\frac{1}{275\,400}\left(-\frac{135\,225}{8}\sqrt{\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\right.$$

$$\frac{57\,855}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$

$$\frac{3085}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\left.\frac{47}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[1]\to\frac{1}{18\,360}\left(\frac{5895}{8}\sqrt{\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\right.$$

$$\frac{5265}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$

$$\frac{313}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\left.\frac{5}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[2]\to\frac{1}{8}\sqrt{\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)},$$

$$\text{h6}[3]\to$$

$$\frac{1}{6885}\left(-\frac{26\,415}{8}\sqrt{\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\right.$$

$$\frac{5235}{16}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$

$$\frac{67}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\left.\frac{1}{16}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[4]\to\frac{1}{18\,360}\left(-\frac{127\,305}{8}\sqrt{\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\right.$$

$$\frac{15735}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$
$$\frac{581}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$
$$\frac{7}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\Bigg),$$

$$\text{h6}[5]\to\frac{1}{91800}\left(-\frac{267075}{8}\sqrt{\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\right.$$
$$\frac{36735}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$
$$\frac{1475}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$
$$\left.\frac{19}{8}\left(20-4\sqrt{10}-\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right)\Big\},$$

$$\Big\{\text{h6}[0]\to\frac{1}{275400}\left(\frac{135225}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}-\right.$$
$$\frac{57855}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$
$$\frac{3085}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$
$$\left.\frac{47}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[1]\to\frac{1}{18360}\left(-\frac{5895}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}-\right.$$
$$\frac{5265}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$
$$\frac{313}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$
$$\left.\frac{5}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[2]\to-\frac{1}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)},$$

$$\text{h6}[3]\to$$
$$\frac{1}{6885}\left(\frac{26415}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}-\right.$$
$$\frac{5235}{16}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$
$$\frac{67}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$
$$\left.\frac{1}{16}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[4]\to\frac{1}{18360}\left(\frac{127305}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}-\right.$$
$$\frac{15735}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$
$$\frac{581}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$

$$\frac{7}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\Big),$$

$$\text{h6}[5]\to\frac{1}{91\,800}\left(\frac{267\,075}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}-\frac{36\,735}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}+\frac{1475}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}-\frac{19}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right)\Big\},$$

$$\Big\{\text{h6}[0]\to\frac{1}{275\,400}\left(-\frac{135\,225}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\frac{57\,855}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-\frac{3085}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+\frac{47}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[1]\to\frac{1}{18\,360}\left(\frac{5895}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\frac{5265}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-\frac{313}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+\frac{5}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[2]\to\frac{1}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)},$$

$$\text{h6}[3]\to\frac{1}{6885}\left(-\frac{26\,415}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\frac{5235}{16}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-\frac{67}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+\frac{1}{16}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[4]\to\frac{1}{18\,360}\left(-\frac{127\,305}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\frac{15\,735}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-\frac{581}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+\frac{7}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[5]\to\frac{1}{91\,800}\left(-\frac{267\,075}{8}\sqrt{\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)}+\right.$$

$$\frac{36735}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$

$$\frac{1475}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\frac{19}{8}\left(20-4\sqrt{10}+\sqrt{\left(5\left(-5+4\sqrt{10}\right)\right)}\right)^{7/2}\Bigg)\Bigg\},$$

$$\Bigg\{\text{h6}[0]\to\frac{1}{275400}\left(\frac{135225}{8}\sqrt{\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-\right.$$

$$\frac{57855}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$

$$\frac{3085}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$

$$\left.\frac{47}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[1]\to\frac{1}{18360}\left(-\frac{5895}{8}\sqrt{\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-\right.$$

$$\frac{5265}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$

$$\frac{313}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$

$$\left.\frac{5}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[2]\to-\frac{1}{8}\sqrt{\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)},$$

$$\text{h6}[3]\to$$

$$\frac{1}{6885}\left(\frac{26415}{8}\sqrt{\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-\right.$$

$$\frac{5235}{16}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$

$$\frac{67}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$

$$\left.\frac{1}{16}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[4]\to\frac{1}{18360}\left(\frac{127305}{8}\sqrt{\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-\right.$$

$$\frac{15735}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$

$$\frac{581}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$

$$\left.\frac{7}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\text{h6}[5]\to\frac{1}{91800}\left(\frac{267075}{8}\sqrt{\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-\right.$$

$$\frac{36735}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$

$$\frac{1475}{8}\left(20+4\sqrt{10}-\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$

$$\frac{19}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\Big)\Big\},$$

$$\Big\{h6[0]\to\frac{1}{275\,400}\Big(-\frac{135\,225}{8}\sqrt{\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+$$

$$\frac{57\,855}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$

$$\frac{3085}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\frac{47}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\Big),$$

$$h6[1]\to\frac{1}{18\,360}\Big(\frac{5895}{8}\sqrt{\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+$$

$$\frac{5265}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$

$$\frac{313}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\frac{5}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\Big),$$

$$h6[2]\to\frac{1}{8}\sqrt{\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)},$$

$$h6[3]\to$$

$$\frac{1}{6885}\Big(-\frac{26\,415}{8}\sqrt{\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+$$

$$\frac{5235}{16}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$

$$\frac{67}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\frac{1}{16}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\Big),$$

$$h6[4]\to\frac{1}{18\,360}\Big(-\frac{127\,305}{8}\sqrt{\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+$$

$$\frac{15\,735}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$

$$\frac{581}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\frac{7}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\Big),$$

$$h6[5]\to\frac{1}{91\,800}\Big(-\frac{267\,075}{8}\sqrt{\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+$$

$$\frac{36\,735}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$

$$\frac{1475}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\frac{19}{8}\left(20+4\sqrt{10}-i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\Big)\Big\},$$

$$\Big\{h6[0]\to\frac{1}{275\,400}\Big(\frac{135\,225}{8}\sqrt{\left(20+4\sqrt{10}+i\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-$$

$$\frac{57855}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$
$$\frac{3085}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$
$$\left.\frac{47}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\mathrm{h6}[1]\to\frac{1}{18360}\left(-\frac{5895}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-\right.$$
$$\frac{5265}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$
$$\frac{313}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$
$$\left.\frac{5}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\mathrm{h6}[2]\to-\frac{1}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)},$$

$$\mathrm{h6}[3]\to$$
$$\frac{1}{6885}\left(\frac{26415}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-\right.$$
$$\frac{5235}{16}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$
$$\frac{67}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$
$$\left.\frac{1}{16}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\mathrm{h6}[4]\to\frac{1}{18360}\left(\frac{127305}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-\right.$$
$$\frac{15735}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$
$$\frac{581}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$
$$\left.\frac{7}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$\mathrm{h6}[5]\to\frac{1}{91800}\left(\frac{267075}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}-\right.$$
$$\frac{36735}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}+$$
$$\frac{1475}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}-$$
$$\left.\frac{19}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right)\Big\},$$

$$\Big\{\mathrm{h6}[0]\to\frac{1}{275400}\left(-\frac{135225}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+\right.$$
$$\frac{57855}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$
$$\frac{3085}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$

$$\frac{47}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\Bigg),$$

$$h6[1]\to\frac{1}{18\,360}\left(\frac{5895}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+\right.$$
$$\frac{5265}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$
$$\frac{313}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$
$$\left.\frac{5}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$h6[2]\to\frac{1}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)},$$

$$h6[3]\to$$
$$\frac{1}{6885}\left(-\frac{26\,415}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+\right.$$
$$\frac{5235}{16}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$
$$\frac{67}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$
$$\left.\frac{1}{16}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$h6[4]\to\frac{1}{18\,360}\left(-\frac{127\,305}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+\right.$$
$$\frac{15\,735}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$
$$\frac{581}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$
$$\left.\frac{7}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right),$$

$$h6[5]\to\frac{1}{91\,800}\left(-\frac{267\,075}{8}\sqrt{\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)}+\right.$$
$$\frac{36\,735}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{3/2}-$$
$$\frac{1475}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{5/2}+$$
$$\left.\frac{19}{8}\left(20+4\sqrt{10}+\mathrm{i}\sqrt{\left(5\left(5+4\sqrt{10}\right)\right)}\right)^{7/2}\right)\}\}$$

```
NSolve[{O61, O62, O63, TP61, TP62, TP63}, d6hcoeffs]
```

```
{{h6[0] → -0.0352263, h6[1] → 0.0854413, h6[2] → 0.135011,
  h6[3] → -0.459878, h6[4] → -0.806892, h6[5] → -0.332671},
 {h6[0] → 0.0352263, h6[1] → -0.0854413, h6[2] → -0.135011,
  h6[3] → 0.459878, h6[4] → 0.806892, h6[5] → 0.332671},
 {h6[0] → -0.0955601 + 0.0508628 i, h6[1] → 0.0812166 + 0.152588 i,
  h6[2] → 0.72145 + 0.101726 i, h6[3] → 0.72145 - 0.101726 i,
  h6[4] → 0.0812166 - 0.152588 i, h6[5] → -0.0955601 - 0.0508628 i},
 {h6[0] → -0.0955601 - 0.0508628 i, h6[1] → 0.0812166 - 0.152588 i,
  h6[2] → 0.72145 - 0.101726 i, h6[3] → 0.72145 + 0.101726 i,
  h6[4] → 0.0812166 + 0.152588 i, h6[5] → -0.0955601 + 0.0508628 i},
 {h6[0] → 0.0955601 + 0.0508628 i, h6[1] → -0.0812166 + 0.152588 i,
  h6[2] → -0.72145 + 0.101726 i, h6[3] → -0.72145 - 0.101726 i,
  h6[4] → -0.0812166 - 0.152588 i, h6[5] → 0.0955601 - 0.0508628 i},
 {h6[0] → 0.0955601 - 0.0508628 i, h6[1] → -0.0812166 - 0.152588 i,
  h6[2] → -0.72145 - 0.101726 i, h6[3] → -0.72145 + 0.101726 i,
  h6[4] → -0.0812166 + 0.152588 i, h6[5] → 0.0955601 + 0.0508628 i},
 {h6[0] → -0.332671, h6[1] → -0.806892, h6[2] → -0.459878,
  h6[3] → 0.135011, h6[4] → 0.0854413, h6[5] → -0.0352263},
 {h6[0] → 0.332671, h6[1] → 0.806892, h6[2] → 0.459878,
  h6[3] → -0.135011, h6[4] → -0.0854413, h6[5] → 0.0352263}}
```

```
daub6 = %[[8]]
```

```
{h6[0] → 0.332671, h6[1] → 0.806892, h6[2] → 0.459878,
 h6[3] → -0.135011, h6[4] → -0.0854413, h6[5] → 0.0352263}
```

Polynomials and frequency representation

```
H6[z_] := d6h[z] /. daub6; H6[z]
```

$$0.332671 + 0.806892\, z + 0.459878\, z^2 - 0.135011\, z^3 - 0.0854413\, z^4 + 0.0352263\, z^5$$

```
fourierH6[ω_] := H6[Exp[I ω]]; fourierH6[ω]
```

$$0.332671 + 0.806892\, e^{i\omega} + 0.459878\, e^{2 i\omega} - 0.135011\, e^{3 i\omega} - 0.0854413\, e^{4 i\omega} + 0.0352263\, e^{5 i\omega}$$

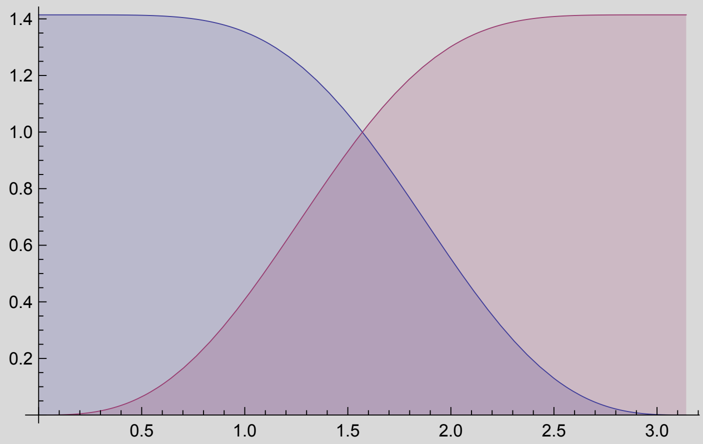
```
G6[z_] := d6g[z] /. daub6; G6[z]
```

$$0.0352263 + 0.0854413\, z - 0.135011\, z^2 - 0.459878\, z^3 + 0.806892\, z^4 - 0.332671\, z^5$$

```
fourierG6[ω_] := G6[Exp[I ω]]; fourierG6[ω]
```

$$0.0352263 + 0.0854413\, e^{i\omega} - 0.135011\, e^{2 i\omega} - 0.459878\, e^{3 i\omega} + 0.806892\, e^{4 i\omega} - 0.332671\, e^{5 i\omega}$$

```
Plot[{Abs[fourierH6[ω]], Abs[fourierG6[ω]]}, {ω, 0, Pi}, Filling → Axis]
```



D2 filter (Haar-filter)

```
H2[z_] = (1 + z) / Sqrt[2]
```

$$\frac{1+z}{\sqrt{2}}$$

```
fourierH2[ω_] := H2[Exp[I ω]]; fourierH2[ω]
```

$$\frac{1+e^{i\,\omega}}{\sqrt{2}}$$

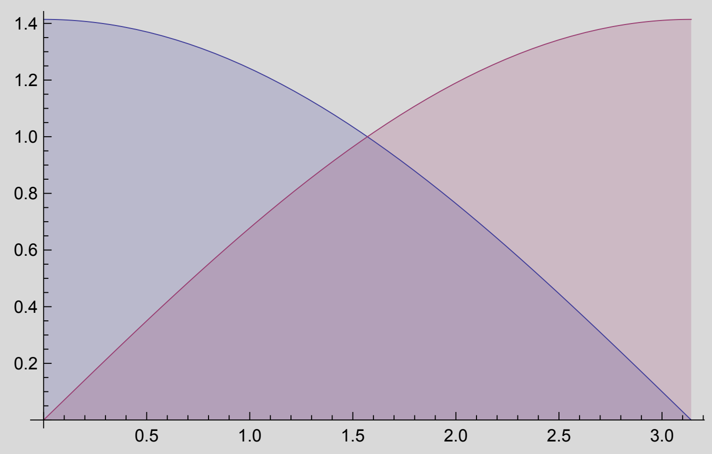
```
G2[z_] = (1 - z) / Sqrt[2]
```

$$\frac{1-z}{\sqrt{2}}$$

```
fourierG2[ω_] := G2[Exp[I ω]]; fourierG2[ω]
```

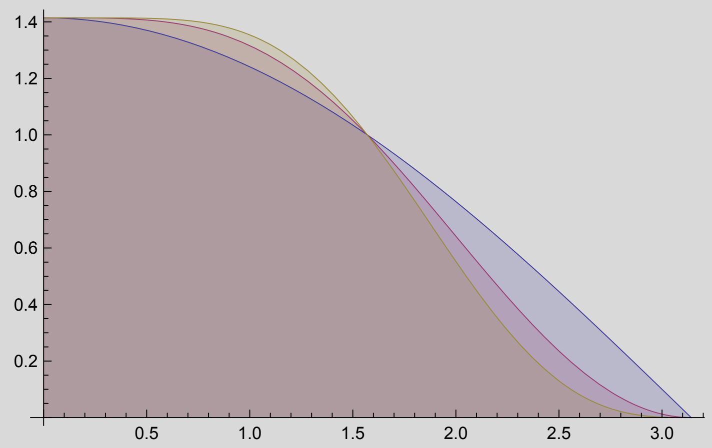
$$\frac{1-e^{i\,\omega}}{\sqrt{2}}$$

```
Plot[{Abs[fourierH2[ω]], Abs[fourierG2[ω]]}, {ω, 0, Pi}, Filling → Axis]
```



Comparing D2, D4, and D6

```
Plot[{Abs[fourierH2[ω]], Abs[fourierH4[ω]], Abs[fourierH6[ω]]},
 {ω, 0, Pi}, Filling → Axis]
```




```
Plot[{Abs[fourierG2[ω]], Abs[fourierG4[ω]], Abs[fourierG6[ω]]},
 {ω, 0, Pi}, Filling → Axis]
```