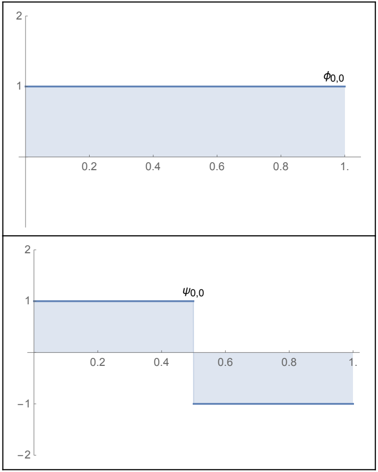
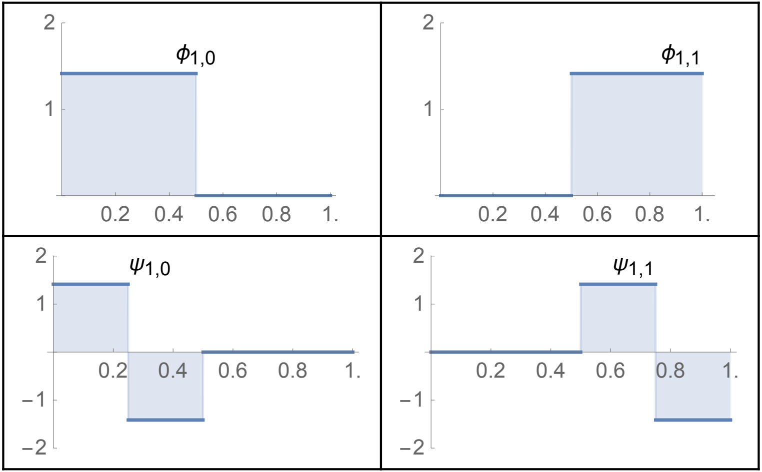
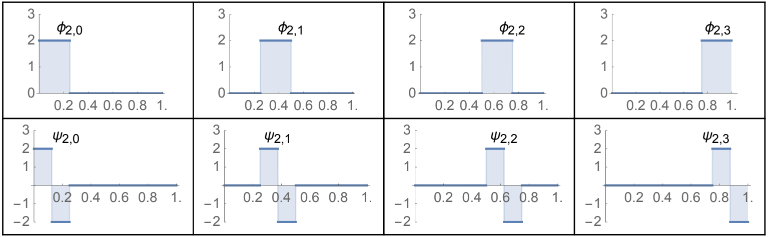
Haar-Basen der Stufe $j = 0$



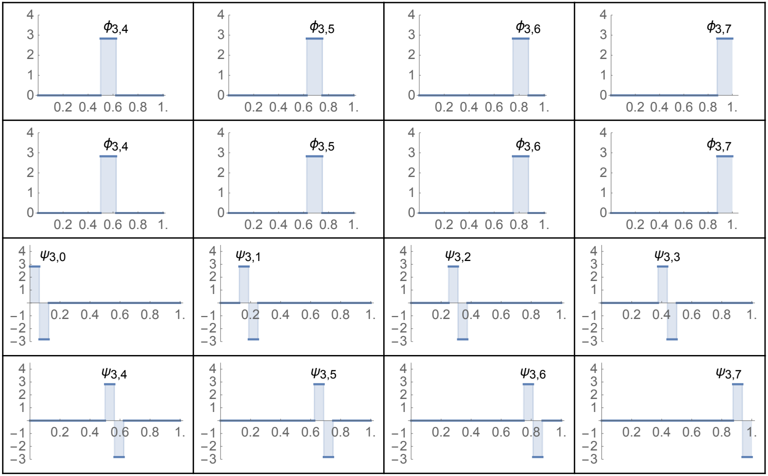
Haar-Basen der Stufe $j = 1$



Haar-Basen der Stufe $j = 2$



Haar-Basen der Stufe $j = 3$



Projektionen

- *Approximation*
  $(P_j f)(t) = \sum_k a_{j,k}\phi_{j,k}(t)$ mit $a_{j,k} = \langle f \mid \phi_{j,k}\rangle$:
  Projektion von $f(t)$ auf den Unterraum $\mathcal{V}_j$
- *Detail*
  $(Q_j f)(t) = \sum_k d_{j,k}\psi_{j,k}(t)$ mit $d_{j,k} = \langle f \mid \psi_{j,k}\rangle$:
  Projektion von $f(t)$ auf den Unterraum $\mathcal{W}_j$
- $P_j f + Q_j f = P_{j+1} f$

Verifikation der Projektionsgleichung

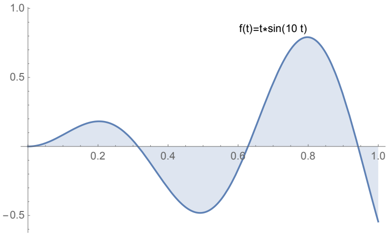
$$\text{Aus} \quad \begin{bmatrix} a_{j,k} \\ d_{j,k} \end{bmatrix} = H \begin{bmatrix} a_{j+1,2k} \\ a_{j+1,2k+1} \end{bmatrix} \quad \text{und} \quad \begin{bmatrix} \phi_{j,k} \\ \psi_{j,k} \end{bmatrix} = H \begin{bmatrix} \phi_{j+1,2k} \\ \phi_{j+1,2k+1} \end{bmatrix}$$

$$\text{folgt} \quad a_{j,k}\phi_{j,k} + d_{j,k}\psi_{j,k} = \begin{bmatrix} a_{j,k} & d_{j,k} \end{bmatrix} \begin{bmatrix} \phi_{j,k} \\ \psi_{j,k} \end{bmatrix} =$$

$$= \begin{bmatrix} a_{j+1,2k} & a_{j+1,2k+1} \end{bmatrix} H^t H \begin{bmatrix} \phi_{j+1,2k} \\ \phi_{j+1,2k+1} \end{bmatrix} = \begin{bmatrix} a_{j+1,2k} & a_{j+1,2k+1} \end{bmatrix} \begin{bmatrix} \phi_{j+1,2k} \\ \phi_{j+1,2k+1} \end{bmatrix}$$

$$= a_{j+1,2k}\phi_{j+1,2k} + a_{j+1,2k+1}\phi_{j+1,2k+1}$$

Beispiel: Haar-Projektionen der Funktion $f(t) := t\sin(10t)$ im Intervall $[0, 1]$



$j = 0$

$$a_{0,0} = \langle f \mid \phi_{0,0} \rangle = 0.0784669$$
$$d_{0,0} = \langle f \mid \psi_{0,0} \rangle = -0.126012$$

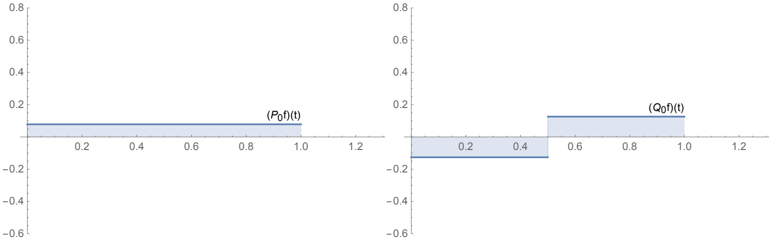


Figure: $(P_0 f)(t) = a_{0,0}\phi_{0,0}(t) \quad (Q_0 f)(t) = d_{0,0}\psi_{0,0}(t)$

$j = 1$

$$a_{1,0} = \langle f \mid \phi_{1,0} \rangle = -0.0336192$$
$$a_{1,1} = \langle f \mid \phi_{1,1} \rangle = 0.144588$$
$$d_{1,0} = \langle f \mid \psi_{1,0} \rangle = 0.107196$$
$$d_{1,1} = \langle f \mid \psi_{1,1} \rangle = -0.124352$$

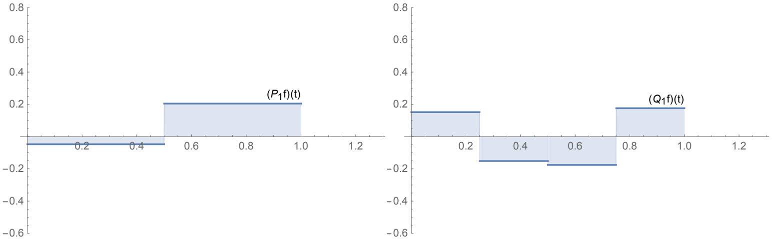


Figure: $(P_1 f)(t) = \sum_{k=0}^{1} a_{1,k}\phi_{1,k}(t) \quad (Q_1 f)(t) = \sum_{k=0}^{1} d_{1,k}\psi_{1,k}(t)$

$j = 2$

$$
\begin{aligned}
(a_{2,k})_{k=0..3} &= (\langle f \mid \phi_{2,k} \rangle)_{k=0..3} \\
&= (0.0520266, -0.0995713, 0.0143094, 0.190169) \\
(d_{2,k})_{k=0..3} &= (\langle f \mid \psi_{2,k} \rangle)_{k=0..3} \\
&= (-0.0298334, 0.0957395, -0.17041, 0.174586)
\end{aligned}
$$

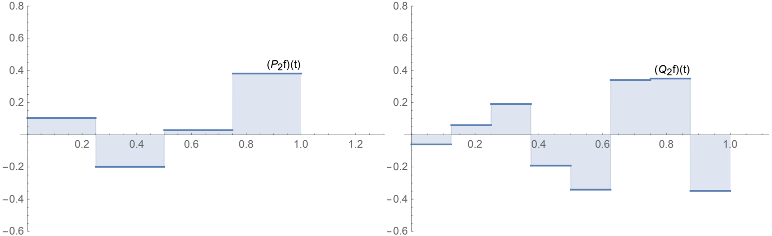


Figure: $(P_2 f)(t) = \sum_{k=0}^{3} a_{2,k}\phi_{2,k}(t) \quad (Q_2 f)(t) = \sum_{k=0}^{3} d_{2,k}\psi_{2,k}(t)$

$j = 3$

$$
\begin{aligned}
(a_{3,k})_{k=0..7} &= (\langle f \mid \phi_{3,k} \rangle)_{k=0..7} \\
&= (0.015693, 0.0578837, -0.00270949, -0.138106, \\
&\quad -0.110379, 0.130616, \ 0.257921, 0.0110187) \\
(d_{3,k})_{k=0..7} &= (\langle f \mid \psi_{3,k} \rangle)_{k=0..7} \\
&= (-0.0112668, -0.0035282, 0.0332469, 0.0249161, \\
&\quad -0.0414734, \ -0.0665895, 0.0136322, 0.0996306)
\end{aligned}
$$

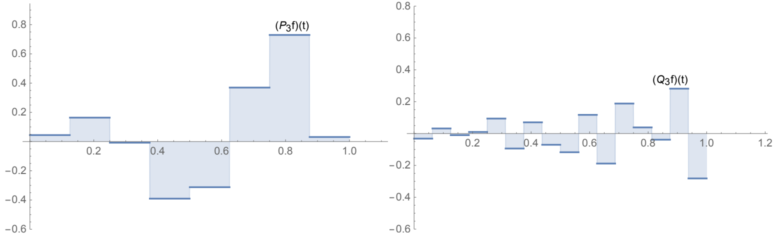


Figure: $(P_3 f)(t) = \sum_{k=0}^{7} a_{3,k}\phi_{3,k}(t) \quad (Q_3 f)(t) = \sum_{k=0}^{3} d_{3,k}\psi_{3,k}(t)$

$j = 4$

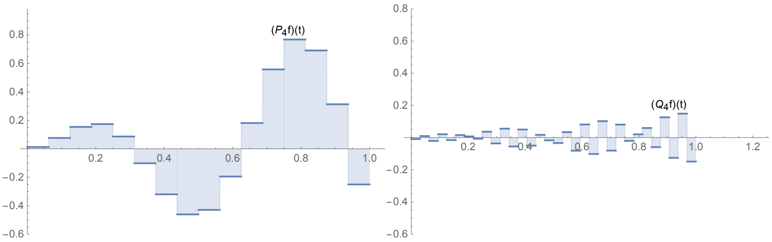


Figure: $(P_4 f)(t) = \sum_{k=0}^{15} a_{4,k}\phi_{4,k}(t) \quad (Q_4 f)(t) = \sum_{k=0}^{15} d_{4,k}\psi_{4,k}(t)$

$j = 5$

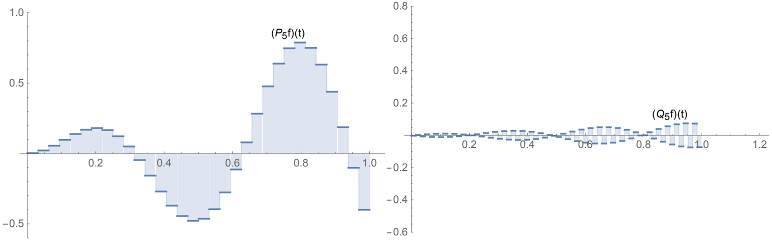


Figure: $(P_5 f)(t) = \sum_{k=0}^{31} a_{5,k}\phi_{5,k}(t) \quad (Q_5 f)(t) = \sum_{k=0}^{31} d_{5,k}\psi_{5,k}(t)$