Evaluation of Spectrum Mismatching using Spectrum Binning Approach for Statistical Polychromatic Reconstruction in CT

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Abstract. In CT, the nonlinear attenuation characteristics of polychromatic X-rays cause beam hardening artifacts in the reconstructed images. Statistical algorithms can effectively correct beam hardening artifacts while providing the benefit of noise reduction. In practice, a big challenge for CT is the difficulty at acquiring accurate energy spectrum information, which hinders the efficiency of beam hardening correction approaches that require the spectrum as prior knowledge such as the statistical methods. In this paper, we used proposed energy spectrum binning approach for reducing prior knowledge from full spectrum to three energy bins to compare the results when applying parameters optimized for one spectrum to data measured using a different spectrum.

1 Introduction

In CT, beam hardening artifacts are caused by the polychromaticity of the X-ray source and the energy dependent attenuation coefficients of materials. Many beam hardening correction (BHC) algorithms have been developed for X-ray CT, both analytically [1] and iteratively [2,3,4]. De-Man et al. proposed a maximum likelihood based reconstruction algorithm [3] (IMPACT), in which the energy dependent attenuation coefficients of materials were decomposed into linear combinations of photoelectric and Compton scattering components. Another approach was proposed by Elbakri et al., which reconstructs a density map from pre-segmented base substance images [2]. Statistical algorithms can effectively correct beam hardening artifacts while providing the benefit of noise reduction from iterative reconstruction, and are very flexible with respect to various geometries, prior knowledge, noise statistics, etc. However, it is often the case that those approaches are very computationally intensive with respect to optimization, number of materials, and processing of the full X-ray spectrum.

Wu et al. have proposed a modified optimization problem for polychromatic statistical reconstruction algorithms, and simplified the algorithms with a spectrum binning method to reduce the full spectrum. From previous studies, it is shown that a generalized spectrum binning algorithm using three energy bins has an average to absolute error of logarithmic signal of less than 0.003. [5]

In practice, a big challenge for CT is the difficulty at acquiring accurate energy spectrum information. This always hinders the efficiency of BHC approaches that require the spectrum as prior knowledge such as the statistical methods. In this study, various spectra with different tube voltages or pre-filtering are simulated, and evaluations of the spectrum binning methods with mismatched spectra are carried out.

2 Materials and Methods

2.1 Spectrum Binning

In CT, emitted X-ray photons have varying energies and the detector response is also energy dependent. According to Beer-Lambert's law, the measured detector signal of a polychromatic X-ray beam is the sum of the monochromatic contributions from small energy bins:

$$\bar{Y}_i = \int_{\varepsilon} b_i(\varepsilon) \exp\left(-\int_{L_i} \mu(\varepsilon) \mathrm{d}l\right) \mathrm{d}\varepsilon, \tag{1}$$

where $b_i(\varepsilon)$ is the unattenuated scan signal of detector pixel i at energy ε, and $\mu(\varepsilon)$ is the energy dependent attenuation coefficient of the object. Based on the assumption of the Poisson measurement model and ignoring the electrical noise, images can be reconstructed through minimizing the log-likelihood objected function [2,3] as

$$\Psi = -\sum_{i=1}^{I} (Y_i \ln(\bar{Y}_i) - \bar{Y}_i) + \beta \mathbf{R}(\mu), \tag{2}$$

where $\mathbf{R}$ is the penalty function of image roughness.

Statistical polychromatic reconstruction assumes that for certain base substances k, the energy dependent attenuation coefficient can be approximated as a linear combination of photoelectric and Compton scattering components [3] (at low X-ray energies), such as

$$m_k(\varepsilon) = \phi_k \Phi(\varepsilon) + \theta_k \Theta(\varepsilon), \tag{3}$$

where $\Phi(\varepsilon)$ and $\Theta(\varepsilon)$ denote base functions of photoelectric and Compton scattering components, and ϕ_k and θ_k are the amount of two components for substances k. Define ρ_j as the material density at spatial location j, and f_j^k is the fraction of the kth material, then the measured signal from the detector from Eg.(1) can be decomposed into two components:

$$\mu_j(\varepsilon) = \sum_{k=1}^{K} m_k(\varepsilon) \rho_j f_j^k = \left(\Phi(\varepsilon) \sum_{k=1}^{K} \phi_k f_j^k + \Theta(\varepsilon) \sum_{k=1}^{K} \theta_k f_j^k\right) \rho_j = (\phi_j \Phi(\varepsilon) + \theta_j \Theta(\varepsilon)) \rho_j. \tag{4}$$

Generally, the statistical reconstruction methods are computationally very expensive since they use prior knowledge about materials and energy spectrum. Wu et al. developed an optimal spectrum binning strategy, which uses the reduced number of energy bins instead of full spectrum information. A generalized spectrum binning approach that has freedoms in both the bin sizes and the values of Φ_s and Θ_s is proposed. The energy bin sizes only need to satisfy the following constrains:

$$\sum_{s=1}^{S} B_s = \int_{\varepsilon_{min}}^{\varepsilon_{max}} b(\varepsilon)\mathrm{d}\varepsilon, \quad B_s > 0 \quad \text{for} \quad s = 1, 2, 3, ...S. \tag{5}$$

Therefore, the sum of all bins is identical to the integral over the spectrum. For two components (ϕ_t, θ_t), the true and approximated expected signals are

$$\bar{Y}^S(\phi_t, \theta_t; B_s, \Phi_s, \Theta_s) = \sum_{s=1}^{S} B_s \exp\left(-\Phi_s \phi_t - \Theta_s \theta_t\right). \tag{6}$$

Φ_s and Θ_s are the corresponding values of each bin. The optimal bin sizes, Φ_s and Θ_s are determined by optimizing

$$\underset{B_s, \Phi_s, \Theta_s}{\operatorname{argmin}} \| \log(\bar{Y}^{\text{full}}(\phi_t, \theta_t)) - \log(\bar{Y}^S(\phi_t, \theta_t, B_s, \Phi_s, \Theta_s)) \|_1, \tag{7}$$

where $\bar{Y}^S$ is computed by equation (6). The empirical result shows that the L_1 distance provides relatively good BHC results.

In a previous study, we compared our spectrum binning method with other reconstruction algorithms with respect to the effectiveness of beam hardening correction. In this paper, we examine the performance of optimized parameters applied to datasets with mismatch between assumed and actual spectra.

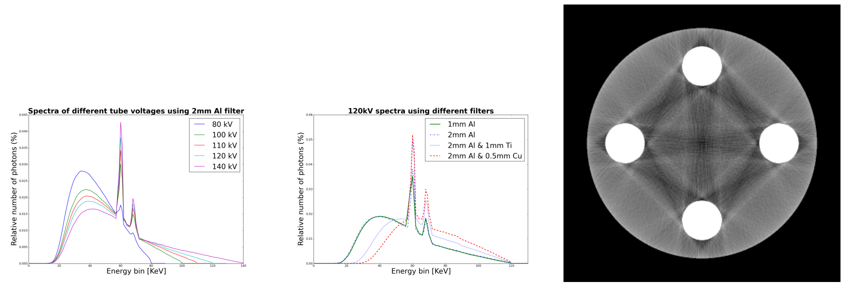
2.2 Experiments

In order to evaluate the stability of the spectrum binning method with respect to mismatched spectra, a digital phantom consisting of soft tissue and bone is used in the simulation. Each dataset consists of 640 projection images over an angular range of 360°, with a size of 512×512 pixels at an isotropic resolution of 0.8×0.8mm^2. The image reconstruction was performed on a 320×320 voxel grid with spacing of 0.8×0.8mm^2.

To perform the comparative analysis, datasets with different tube voltages and pre-filtration are simulated. We chose 120kVp with 2mm Al filter spectrum as reference, and applied the obtained optimized spectral binning parameters to other datasets for examination of influences on mismatched spectra.

3 Results

Fig. 1 shows spectra of different tube voltages Fig. 1(a) and spectra of different filters using the same voltage Fig. 19 (b). A filtered back-projection reconstruction of the phantom is illustrated, where the cupping and streak artifacts can be observed.



(a) (b) (c)

Fig. 1. Simulated spectra and reconstruction of the phantom using filtered back-projection

Denote $C_t = f(P_t)$ as the correct reconstruction for tube voltage t, where P are parameters obtained from spectrum t. $M_t = f(P_g)$ is the reconstruction of datasets with tube voltage t using mismatched parameters obtained from voltage g. We used the 120kVp including 2mm Al filter spectrum g as reference. Then, the optimal parameters B_g, Φ_g and Θ_g from the reference spectrum are applied to datasets simulated with other spectra.

Fig. 2 shows the resulting reconstructions from various spectrum setups in three aspects: reconstructed slices using mismatched parameters M_t, difference images between C_t and M_t, and corresponding center line profiles. Two sets of comparisons are carried out: with different tube voltages, and with various filters applied at identical voltage. From the reconstructed images, it can be seen that beam hardening artifacts are sufficiently suppressed even though the applied parameters are from unmatched spectra. The difference images show that an increase in distance between real and reference spectra results in larger differences, e.g. 80kVp vs. 100kVp, and 1mm Al&1mm Ti vs. 1mm Al&0.5mm Cu. We plot line profiles from the uncorrected reconstructions and from the reconstruction corrected by using matched and mismatched parameters for binning with two and three energy bins, respectively. As discussed previously, three bins generally achieve better results than an optimization using 2 bins.

In Table 1, quantitative comparisons of the mismatched spectra with correct spectra parameters are listed. The binning parameters obtained from the reference spectrum (120kVp & 2mmAl) are applied to other spectra datasets. Root mean square error (RMSE) is used for illustration. As observed from the difference images in Fig. 2, the more the spectra deviate from the reference spectrum, the larger the RMSE value.

4 Discussion

In this work, energy spectrum binning approach is used for reducing prior knowledge from full spectrum to three energy bins in statistical polychromatic reconstruction. A simulation study was carried out that compared the results when

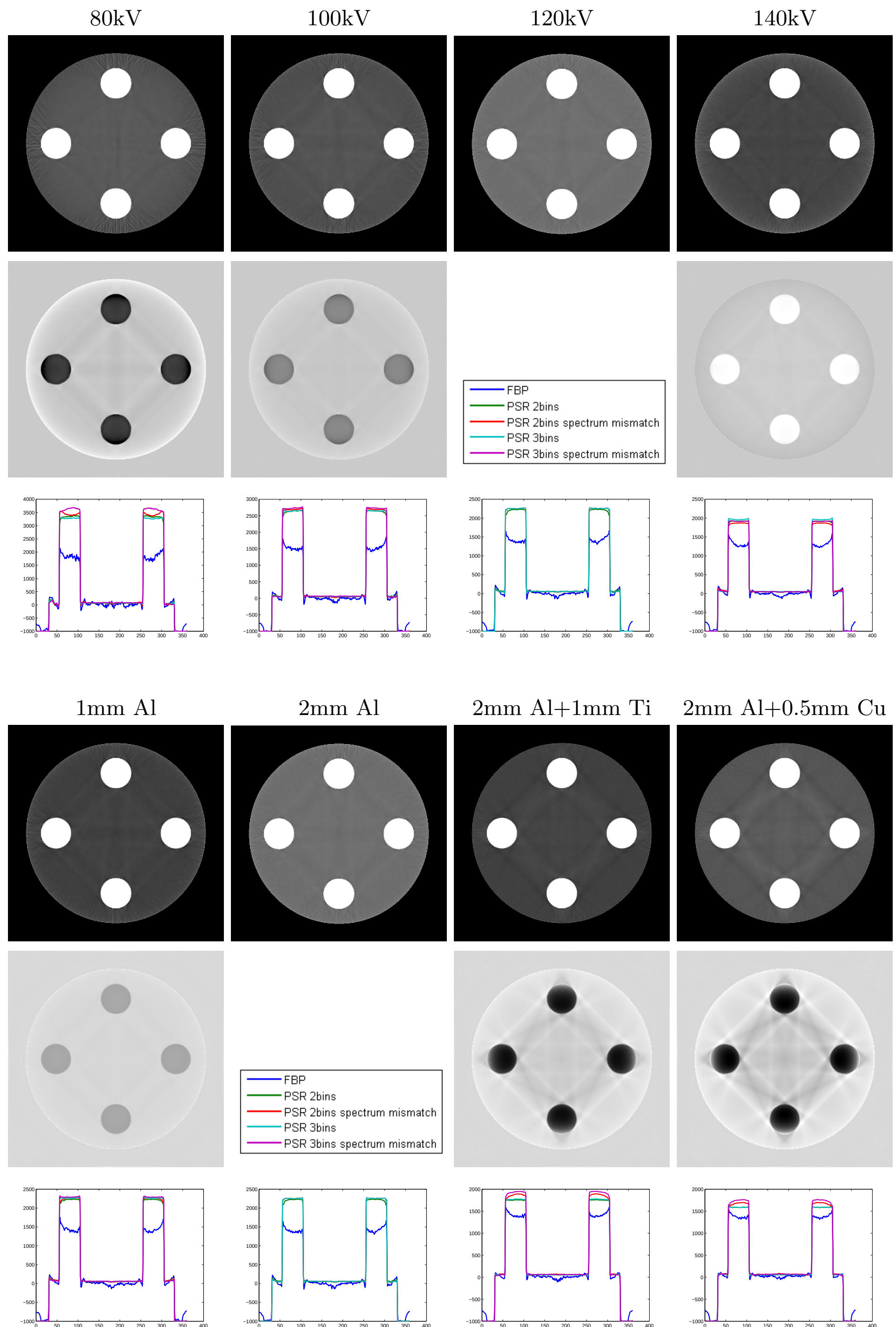


Fig. 2. Results of reconstructed images from mismatched spectral binning parameters and corresponding line profiles. Reconstructed images are displayed with window [-200, 400] HU.

Spectrum	**120kVp** ***2mm Al***	80kVp *2mm Al*	100kVp *2mm Al*	140kVp *2mm Al*	120kVp *1mm Al*	120kVp *2mm Al* *1mm Ti*	120kVp *2mm Al* *0.5mm Cu*
B_s (normalized)	**0.40** **0.45** **0.15**	0.65 0.22 0.13	0.56 0.29 0.15	0.46 0.36 0.18	0.50 0.34 0.16	0.37 0.41 0.22	0.35 0.35 0.30
Φ_s	**3.5754** **0.7658** **0.2021**	1.8080 0.5013 0.2719	2.2787 0.5640 0.2490	3.3056 0.7895 0.2155	2.6732 0.6676 0.2302	2.6816 0.9424 0.3351	2.3862 1.1076 0.4128
Θ_s	**1.0601** **0.9886** **0.9008**	1.0245 0.9673 0.9345	1.0369 0.9705 0.9229	1.0595 0.9865 0.9033	1.0468 0.9780 0.9126	1.0528 0.9965 0.9303	1.0546 1.0120 0.9359
RMSE (HU)	0	**76.814**	**29.205**	**17.415**	**11.430**	**33.429**	**31.688**

Table 1. Spectrum binning results for reference and test spectra, and RMSE of reconstructions between matched and mismatched spectrum binning parameters.

applying parameters optimized for one spectrum to data measured using a different spectrum. The quantitative results indicate, using RMSE, that the spectrum mismatch does not significantly degrade the correction results, with a maximum additional RMSE of 76.814 HU when compared to the corrected image generated using the spectrum for which the optimum parameters were calculated.

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References

1. Joseph P, Spital R. A method for correction bone induced artifacts in computed tomography scanners. Journal of computer assisted tomography. 1978;.
2. Elbakri IA, Fessler JA. Statistical image reconstruction for polyenergetic X-ray computed tomography. IEEE Trans Med Imaging. 2002 Feb;21(2):89–99.
3. De Man B, Nuyts J, Dupont P, Marchal G, Suetens P. An iterative maximum-likelihood polychromatic algorithm for CT. IEEE transactions on medical imaging. 2001 Oct;20(10):999–1008.
4. Van Gompel G, Van Slambrouck K, Defrise M, Batenburg KJ, de Mey J, Sijbers J, et al. Iterative correction of beam hardening artifacts in CT. Medical physics. 2011 Jul;38(7):S36.
5. Wu M, Yang Q, Maier A, Fahrig R. A Practical Statistical Polychromatic Image Reconstruction for Computed Tomography Using Spectrum Binning. In: To be published in SPIE Medical Imaging; 2014. .