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A REFINED ICP ALGORITHM FOR ROBUST 3-D CORRESPONDENCE ESTIMATION

Timo Zinßer, Jochen Schmidt, Heinrich Niemann*

Lehrstuhl für Mustererkennung, Universität Erlangen-Nürnberg
Martensstr. 3, 91058 Erlangen, Germany
{zinsser, jschmidt, niemann}@informatik.uni-erlangen.de

ABSTRACT

Robust registration of two 3-D point sets is a common problem in computer vision. The Iterative Closest Point (ICP) algorithm is undoubtedly the most popular algorithm for solving this kind of problem. In this paper, we present the Picky ICP algorithm, which has been created by merging several extensions of the standard ICP algorithm, thus improving its robustness and computation time. Using pure 3-D point sets as input data, we do not consider additional information like point color or neighborhood relations.

In addition to the standard ICP algorithm and the Picky ICP algorithm proposed in this paper, a robust algorithm due to Masuda and Yokoya and the RICP algorithm by Trucco *et al.* are evaluated. We have experimentally determined the basin of convergence, robustness to noise and outliers, and computation time of these four ICP based algorithms.

1. INTRODUCTION

There are many applications for robust registration of two 3-D point sets in computer vision. As acquiring the shape of a 3-D object usually takes several scans from different viewpoints, the resulting point sets have to be registered in order to create a complete 3-D model [1]. It is also possible to compare a scanned 3-D object with an existing CAD model of this object for quality assurance in manufacturing [2]. When computing the registration of two point sets, not only the point correspondences are recovered, but also the motion parameters aligning the two point sets. Therefore, registration algorithms can also be used for motion estimation [3].

The ICP algorithm for registering two point sets was introduced by Chen and Medioni [4] and Besl and McKay [5]. Basically, this algorithm iteratively performs two operations until convergence. The first operation consists of finding the closest point in one point set for each point in the other set. In the second operation, the motion between the two point sets is estimated using only the corresponding point pairs. A comprehensive summary of different extensions to this algorithm can be found in [6].

Like most non-linear minimization algorithms, the ICP algorithm needs a good initialization. Sometimes, this initialization can be obtained by using knowledge about the position of the 3-D sensors or by user input. If this is not possible, more elaborate techniques like principal component analysis or a constrained exhaustive search [7] become necessary. By evaluating the basin of convergence for the examined algorithms, we will determine how accurate the initial motion estimate has to be.

In the next section, we will introduce the newly created Picky ICP algorithm, showing mostly the differences to the standard ICP algorithm. Afterwards, we will shortly present two other robust ICP based algorithms used as additional benchmarks in Section 4. This section contains the experimental evaluation of the four registration algorithms, showing results for the basin of convergence, robustness to noise and outliers, and computation time.

2. THE PICKY ICP ALGORITHM

The solution of the registration problem for the set of 3-D data points A and the set of model points B can be given in two equivalent ways. On the one hand, it can be specified by the set of point pairs

$$C = \{(i,j)\,|\,\boldsymbol{a}_i \in A \text{ and } \boldsymbol{b}_j \in B \text{ are corresponding points}\}.$$

On the other hand, the motion $(\boldsymbol{R}, \boldsymbol{t})$ aligning the two point sets can be used to represent the registration. The motion parameters comprise a rotation matrix $\boldsymbol{R} \in \mathbb{R}^{3\times 3}$ and a translation vector $\boldsymbol{t} \in \mathbb{R}^3$.

As the Picky ICP algorithm is an extension of the ICP algorithm, it can best be described by stating the differences to the latter. For this purpose, the ICP algorithm has been divided into several stages, slightly deviating from the classification used in [6].

The **data representation** has an important impact on the later stages of the registration algorithm. We are focussing on the use of unstructured 3-D point sets without

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additional information. Many derivatives of the original ICP algorithm were designed to take advantage of other representations like range images, line sets, parametric surfaces or triangle meshes. It is also possible to improve the performance of the registration algorithm by utilizing point intensities or colors, if these are available [8].

In order to obtain point pairs, control points have to be chosen from the point sets. The standard ICP algorithm uses a simple strategy for the **selection of control points**: all data points are used as control points. We augmented the Picky ICP algorithm by adding hierarchical point selection. At first, only every 2^h-th data point is employed as a control point, where $h+1$ is the number of hierarchy levels. After convergence of the registration algorithm for one set of control points, the computation is continued on the next hierarchy level. Especially for large point sets, this extension will considerably speed up the computation time.

The first operation in the main loop of the ICP algorithm is the **computation of point pairs**. For each control point from the set of data points, the closest model point is found using nearest neighbor search. As this is the most time consuming operation of the registration algorithm, we use a highly optimized k-D tree nearest neighbor algorithm for maximum performance [9]. Relying on additional information provided by alternative data representations, even faster methods for finding control points have been devised [6, 10].

After a corresponding point has been found for each control point, erroneous point pairs can be rejected. When no additional information is available, only the distance of the two points in a pair can be used to discriminate good pairs from outliers. A versatile method for computing the maximum allowable distance is described in [11]. The main idea is to robustly estimate the standard deviation of the distances, and to reject point pairs with a distance greater than a chosen multiple of this standard deviation.

Another extension implemented in Picky ICP is the ability to prevent that a model point is present in more than one pair. If such a model point is found, all pairs except the one with smallest distance are rejected. This method is particularly useful if the points sets A and B overlap only partially. Rejecting point pairs greatly increases the robustness to noise in the coordinates of the 3-D points and outliers, but it has the disadvantage of slowing the convergence of the algorithm. Furthermore, the proof of convergence presented by Besl and McKay [5] no longer holds and the registration algorithm does not necessarily converge.

The **computation of motion** is the second operation in the main loop of the ICP algorithm. Like Besl and McKay, we use the sum of squared distances of the corresponding point pairs as error measure:

$$(\boldsymbol{R}', \boldsymbol{t}') = \operatorname*{argmin}_{\boldsymbol{R},\boldsymbol{t}} \sum_{(i,j)\,\in C} \left\| \boldsymbol{b}_j - \boldsymbol{R}\boldsymbol{a}_i - \boldsymbol{t} \right\|^2 .$$

Besl and McKay also proposed a motion parameter extrapolation method to speed up convergence of their algorithm. For extrapolation, they use a 7-D vector consisting of a quaternion representing the rotation and a vector representing the 3-D translation. Simon further improved this method by separating the extrapolation of rotation and translation [12]. Finally, the introduction of a dampening factor can help alleviating problems caused by overshoot [6]. All of these extensions are implemented in the Picky ICP algorithm.

The standard ICP algorithm is stopped when the change of the registration error falls below a specified threshold. This criterion cannot be used for Picky ICP, because the registration error might temporarily increase due to the effects of outlier rejection. Instead, the change of motion parameters is monitored and must be above a specified threshold for the algorithm to continue. Additionally, a maximum number of iterations can be given to prevent an infinite loop in the rare case of divergence.

3. ROBUST ICP BASED ALGORITHMS

Another robust ICP based registration algorithm, in this paper called MICP for brevity, was presented by Masuda and Yokoya [13]. The MICP algorithm consists of three main operations: choosing control points by random sampling, motion estimation for these control points with the standard ICP algorithm, and evaluating the computed motion with a robust error measure. These operations are iteratively executed, applying the computed motion only if the corresponding error is the smallest ever found.

We have made several changes to the original MICP algorithm. Firstly, it has been adopted to work with unstructured 3-D point sets as data representation. Masuda and Yokoya use the maximum number of iterations as the stopping criterium. In order to automatically find the point where a good registration has been reached, we set a threshold for the maximum number of iterations during which a new motion update has to be found.

The motion computed in the first iteration of the MICP algorithm will always be applied, even if the control point selection was unfavorable and the registration error is large. This behavior can possibly prevent the algorithm from finding the correct registration. To circumvent this weakness, we start the algorithm by doing several iterations without applying any motion, and then select the best motion found in this warm-up phase.

The RICP algorithm proposed by Trucco *et al.* in [14] is the fourth algorithm evaluated in this paper. Its structure resembles the standard ICP algorithm, but inside the main loop a least median of squares method is used for outlier elimination. The least median of squares method uses three point pairs to compute motion parameters, the number of

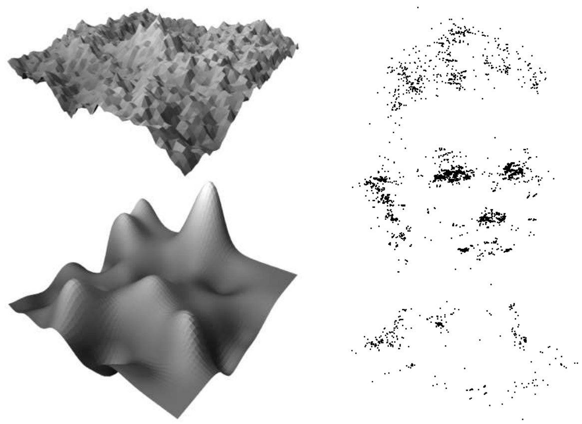

Fig. 1. Illustration of test point sets *Fractal* (top left), *Wave* (bottom left), and *Head* (right).

	Random	*Fractal*	*Wave*	*Room*	*Head*
# points	50	4096	4096	2400	3600
ICP	**0.814**	306	275	232	562
Picky ICP	0.874	**223**	**219**	**213**	**454**
MICP	3.39	426	447	312	484
RICP	57.0	1350	1180	673	1010

Table 1. Computation times in milliseconds on a Pentium IV 2.4 GHz CPU for several test sets (see text for further description). The best time for each set is emphasized.

trials must be chosen manually. Trucco *et al.* do not employ the distance between point pairs as error measure for the least median of squares method, but the absolute difference for each coordinate component of the point pairs.

Our implementation of the RICP algorithm updates the motion parameters only if the registration error decreases. This extension prevents the registration result from becoming worse, which can happen when all control point triplets in the least median of squares method contained at least one outlier. Additionally, we can now employ the new adaptive stopping criterium also used in the MICP algorithm.

4. EXPERIMENTS

The four presented registration algorithms have been evaluated using several different types of point sets, including point sets uniformly distributed in a cube, point sets representing fractal surfaces and point sets of real objects obtained by structure from motion algorithms. The sizes of the point sets range from 50 points to over 16000 points.

Table 1 presents computation times determined using different point sets. Point set *Random* has been artificially created by uniformly distributing 50 points in a unit cube. The point sets *Fractal* and *Wave* are both shown in Fig. 1.

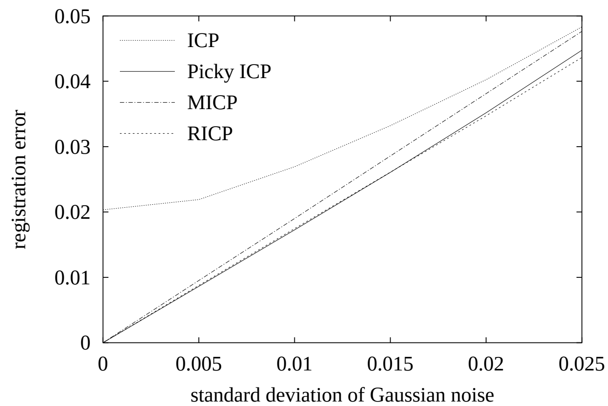

Fig. 2. Error e for different standard deviations of added Gaussian noise computed with point set *Random*. Note that this point set is located within a cube of edge length 1. The results have been averaged over 1000 trials using different initial motions. Outliers in the data point set were simulated by deleting 20% of the model points. The robustness of Picky ICP is comparable to that of RICP, which takes much longer to compute (cf. Table 1).

Room and *Head* (cf. Fig. 1) are point sets created from real image sequences using structure–from–motion techniques.

All computation times have been averaged over thousands of trials with different initial rotations and translations. For very small point sets like *Random*, the outlier rejection of Picky ICP makes it minimally slower than the original ICP algorithm. In comparison, both other robust algorithms need much more computation time. When working with larger point sets, the movement extrapolation of the Picky ICP algorithm comes into effect, making it faster than the ICP algorithm. The MICP algorithm is able to outperform the ICP algorithm when using large point sets, but will not reach the speed of the Picky ICP algorithm. RICP is always the slowest algorithm, needing at least twice as much time as the Picky ICP algorithm.

Another series of experiments was conducted to evaluate the algorithms' robustness against outliers and noise. For this purpose, point sets have been created using Gaussian noise with different standard deviations and different percentages of outliers among data and model points. The registration error for one example configuration has been plotted in Fig. 2. The registration accuracy was measured by

$$e = \sqrt{\frac{1}{|\hat{C}|} \sum_{(i,j)\,\in\,\hat{C}} \|\boldsymbol{b}_j - \boldsymbol{R}\boldsymbol{a}_i - \boldsymbol{t}\|^2},$$

where $\boldsymbol{R}$ and $\boldsymbol{t}$ are the computed motion parameters, and $\hat{C}$ is the set of correct point pairs.

The diagram in Fig. 2 illustrates the high sensitivity of the ICP algorithm to outliers in the data point set. The outlier rejection built into the Picky ICP algorithm clearly improves the obtained results. As the MICP algorithm uses very few control points, it is more sensitive to noise than the other algorithms. The robustness of Picky ICP is comparable to that of RICP for outlier rates of up to 20%, but the former is much faster. The RICP algorithm proves to be more robust for higher outlier rates, but only converges to the correct solution when the initial motion is small.

We also evaluated the basin of convergence of the four algorithms, with respect to the intial rotation and translation between data and model point set. Due to lack of space, the results of the experiments cannot be presented in detail here. They have shown that the basin of convergence depends very much on the topology of the point sets. In general, ICP and Picky ICP have the largest basin of convergence, infrequently outperformed by MICP for large rotations. The RICP algorithm only converged to the correct solution for rather small initial motions.

5. CONCLUSION

In this paper we have presented a refinement of the ICP algorithm for robust registration of 3-D point sets. The new algorithm, called Picky ICP, has been augmented by hierarchical control point selection, robust outlier elimination, refined motion parameter extrapolation and a new stopping criterium. We have shortly described two further algorithms for robust registration and the modifications we implemented to improve them.

We have conducted a thorough experimental evaluation of all four algorithms. The experiments have shown that we succeeded in making the ICP algorithm both faster and more robust to outliers. Compared to the MICP algorithm, the Picky ICP algorithm is more robust to strong Gaussian noise and has a larger basin of convergence considering the initial motion. The RICP algorithm can only beat the robustness of Picky ICP for very high outlier rates in the data point set, but has a much smaller basin of convergence and is by far the slowest of the tested algorithms.

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