A Variational Framework for Joint Image Registration, Denoising and Edge Detection

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Abstract. In this paper we propose a new symmetrical framework that solves image denoising, edge detection and non–rigid image registration simultaneously. This framework is based on the Ambrosio–Tortorelli approximation of the Mumford–Shah model. The optimization of a global functional leads to decomposing the image into a piecewise–smooth representative, which is the denoised intensity function, and a phase field, which is the approximation of the edge-set. At the same time, the method seeks to register two images based on the segmentation results. The key idea is that the edge set of one image should be transformed to match the edge set of the other. The symmetric non–rigid transformations are estimated simultaneously in two directions. One consistency functional is designed to constrain one transformation to be the inverse of the other. The optimization process is guided by a generalized gradient flow to guarantee smooth viscous flow of the transformations. A multi–scale implementation scheme is applied to ensure the efficiency of algorithm. We have performed preliminary medical evaluation on T1 and T2 MRI data, where the experiments show encouraging results.

1 Introduction

Image registration, image denoising and edge detection are three important and challenging image processing problems in the field of medical image analysis. Traditionally, solutions are developed for each of these three problems mutually independent from each others. However, in the various applications, the solutions of these problems are interdependent. Indeed, tackling each task would benefit significantly from prior knowledge of the solution of the other tasks. Hence, one handles these different image processing problems in an uniform mathematically sound approach.

There already have been some attempts in the literature to develop methods aligning the images and detecting the features simultaneously [1,2,3,4,5]. Due to our knowledge, most of the existing approaches are restricted to lower dimensional parametric transformations for image registration. Recently, the pioneering work of Droske shows the possibility to solve the non–rigid registration problem by edge alignment. The key idea of this work is to modify the Ambrosio–Tortorelli approximation of Mumford–Shah model, which is traditionally used for image segmentation, so that the new functional can also estimate the spatial transformation between images, but in contrast to the method proposed in [6] our method is fully "symmetric".

2 Method

Assume we are given two gray images R and T, whose intensity value are described by the function u_R^0 and u_T^0 respectively. The goal of the joint framework is to find piecewise smooth representatives u_R and u_T (denoising), phase field edge functions v_R and v_T (edge detection) and symmetric non–rigid spatial transformations h and g such that $u_R \circ g$ matches u_T and $u_T \circ h$ matches u_R (registration). For simplification of presentation, we denote all the unknowns with $\Phi = [u_R, u_T, v_R, v_T, h, g]$. The associated functional is defined as

$$E_G[\Phi] = E_{AT}^{u_R^0}[u_R, v_R] + E_{AT}^{u_T^0}[u_T, v_T] + E_{REG}[\Phi] \rightarrow \min, \tag{1}$$

In the following part of this section, we show the definitions and variational formulations of the functionals.

2.1 Denoising and Edge Detection

The $E_{\mathrm{AT}}^{u_0}[u, v]$ denotes the Ambrosio–Tortorelli (AT) approximation functional proposed in [7,8]. This functional is originally designed to approximate the Mumford–Shah model [9] for image segmentation. The functional is defined as

$$\begin{aligned} E_{\mathrm{AT}}^{u_0,\epsilon}[u, v] = & \underbrace{\frac{\alpha}{2} \int_{\Omega} (u - u_0)^2 \, dx}_{E_1} + \underbrace{\frac{\beta}{2} \int_{\Omega} v^2 \|\nabla u\|^2 dx}_{E_2} \\ & + \frac{\nu}{2} \Big(\underbrace{\epsilon \int_{\Omega} \|\nabla v\|^2 \, dx}_{E_3} + \underbrace{\frac{1}{4\epsilon} \int_{\Omega} (v - 1)^2 dx}_{E_4} \Big), \end{aligned} \tag{2}$$

with parameters $\alpha, \beta, \nu \geq 0$. In the Ambrosio–Tortorelli approximation, the edge set is depicted by a phase field function v such that

$$v(x) \approx \begin{cases} 0 & \text{if } x \text{ is an edge point} \\ 1 & \text{otherwise.} \end{cases}$$

The term E_1 favors u to be as similar to u^0 as possible. The term E_2 allows u to be singular (large $\|\nabla u\|^2$) where $v \approx 0$ and favors u to be smooth (small $\|\nabla u\|^2$) where $v \approx 1$. The term E_3 constrains v to be smooth. The last term E_4 prevents the degeneration of v, i.e. without E_4 the functional would be minimized by $v \equiv 0, u \equiv u_0$. For the details of the Ambrosio–Tortorelli approximation, we refer to [7].

2.2 Edge Alignment

The main goal of the registration functional E_{REG} is to find the transformations that match the edge sets of image R and image T to each other. In order to explicitly enforce the bijectivity and invertibility of spatial mapping, we estimate the two transformations in two directions simultaneously: $h : \Omega \to \Omega$ is the transformation from image T to R and $g : \Omega \to \Omega$ is the one from R to T. The functional E_{REG} is a linear combination of an external functional E_{ext}, an internal functional E_{int} and a consistent functional E_{con}:

$$E_{\text{REG}}[\Phi] = \mu E_{\text{ext}}[\Phi] + \lambda E_{\text{int}}[h, g] + \kappa E_{\text{con}}[h, g] \tag{3}$$

where μ, λ and κ are just scaling parameters. The three functional terms are defined as follows:

$$E_{\text{ext}}[\Phi] = \frac{1}{2}\int_\Omega (v_T \circ h)^2 \left\|\nabla u_R\right\|^2 \; dx + \frac{1}{2}\int_\Omega (v_R \circ g)^2 \left\|\nabla u_T\right\|^2 \; dx, \tag{4}$$

$$E_{\text{int}}[h, g] = \frac{1}{2}\int_\Omega \left\|\nabla h - \mathbb{1}\right\|^2 \; dx + \frac{1}{2}\int_\Omega \left\|\nabla g - \mathbb{1}\right\|^2 \; dx, \tag{5}$$

$$E_{\text{con}}[h, g] = \frac{1}{2}\int_\Omega \|h \circ g(x) - x\|^2 \, dx + \frac{1}{2}\int_\Omega \|g \circ h(x) - x\|^2 \, dx. \tag{6}$$

The external functional E_{ext} favors transformations that align zero–regions of phase field of one image to regions of high gradient in the other image. The internal functional E_{int} imposes a common smoothness prior on the transformations. The consistency functional E_{con} constrains the transformations to be inverse to each other, since it is minimized when $h = g^{-1}$ and $g = h^{-1}$.

2.3 Variational Formulation

Since the definition of the global functional $E_G[\Phi]$ is mathematically symmetrical respect with the two groups of unknown $[u_R, v_R, h]$ and $[u_T, v_T, g]$, we present only the variational formulation of $[u_R, v_R, h]$, the other formulation can be deduced in a complementary way.

1. The variation with respect to u_R for $\vartheta \in C_0^\infty(\Omega)$:

$$\begin{aligned}\langle \partial_{u_R} E_G, \vartheta \rangle = \alpha \int_\Omega (u_R - u_R^0)\vartheta \, dx \\ + \beta \int_\Omega v_R^2 \nabla u_R \cdot \nabla\vartheta \, dx + \mu \int_\Omega (v_T \circ h)^2 \nabla u_R \cdot \nabla\vartheta \, dx.\end{aligned} \tag{7}$$

2. The variation with respect to v_R for $\vartheta \in C_0^\infty(\Omega)$:

$$\begin{aligned}\langle \partial_{v_R} E, \vartheta \rangle = \beta \int_\Omega \left\|\nabla u_R\right\|^2 v_R \vartheta \, dx + \frac{\nu}{4\epsilon}\int_\Omega (v_R - 1)\vartheta \, dx \\ + \nu\epsilon \int_\Omega \nabla v_R \cdot \nabla\vartheta \, dx + \mu \int_\Omega \|\nabla u_T \circ g^{-1}\|^2 v_R \vartheta \left|\det Dg\right|^{-1} \; dx.\end{aligned} \tag{8}$$

3. The variation with respect to h for $\psi \in C_0^\infty(\Omega, \mathbb{R}^d)$:

$$
\begin{aligned}
\langle \partial_h E_{\mathrm{G}}, \psi \rangle = \mu \int_\Omega \|\nabla u_R\|^2 (v_T \circ h) \nabla (v_T \circ h) \cdot \psi \, dx + \lambda \int_\Omega Dh : D\psi \, dx \\
+ \kappa \int_\Omega ([h \circ g](x) - x) \cdot [\psi \circ g](x) \, dx \\
+ \kappa \int_\Omega ([g \circ h](x) - x) Dg(h(x)) \cdot \psi(x) \, dx
\end{aligned}
\tag{9}
$$

2.4 Minimizing the Energy

We minimize the functional by finding a zero crossing of the variation. Because of the high dimensionality of the minimization problem (six unknown functions, two of them vector valued), we employ an EM type algorithm, i.e. we iteratively solve for zero crossings of the variations given in section 2.3. Since the variations with respect to the images and the phase fields are linear in the given variable, we can solve these equations directly with Finite-Element methods. The non-linear equations for the transformation are solved with a regularized gradient flow, which combined with the time discretization is closely related to iterative Tikhonov regularization, see [10].

3 Results

The first experiment was performed on a pair of T1/T2 MRI slices (See Fig.1**a**,**b**), which have the same resolution (257×257) and come from the same patient. The experiment results in Fig.1 show that the proposed method successfully removes the noise (**c**,**d**) and detects the edge features (**e**,**f**) of T1/T2 slices. Moreover, the method computes the transformations such that the two transformed slices (**g**,**h**) optimally align to the original images according to the edge features, see (**i**,**j**). The second experiment was designed to demonstrate the effect of the proposed method in 3D. We deformed one MRI volume ($129 \times 129 \times 129$) with Gaussian radial basis function (GRBF) and seek to recover the artificially introduced transformation via symmetric registration method. See the registration results in Fig.2.

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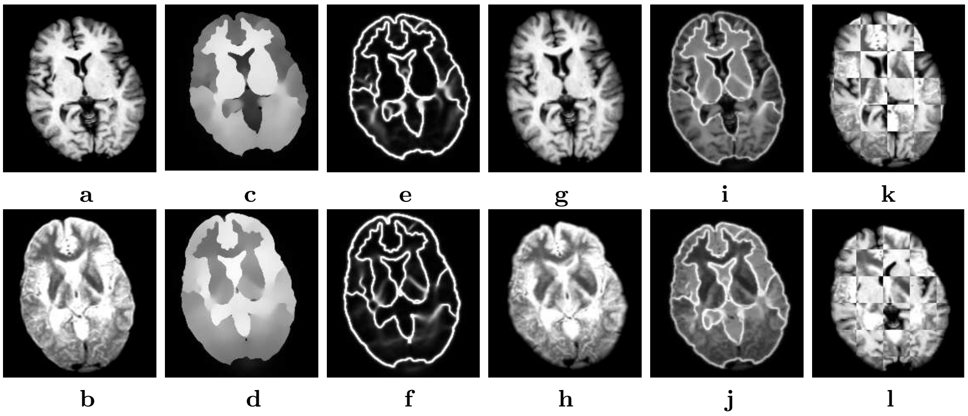

Fig. 1. Results of registration of T1/T2 slices with parameters: $\alpha = 2550, \beta = 1, \nu = 1, \mu = 0.1, \lambda = 20, \kappa = 1, \epsilon = 0.5h$. (**a**, **b**): The original images u^0_{T1} and u^0_{T2}. (**c**, **d**): Piecewise smooth functions u_{T1} and u_{T2}. (**e**, **f**): Phase field functions v_{T1} and u_{T2}. (**g**, **h**): The registered T1 and T2 slices. (**i**): Blending of transformed T1 slice and phase field function of T2 slice. (**j**): Blending of transformed T2 slice and phase field function of T1 slice. (**k**): Checkbox of (**g**) and (**b**). (**l**): Checkbox of (**a**) and (**h**).

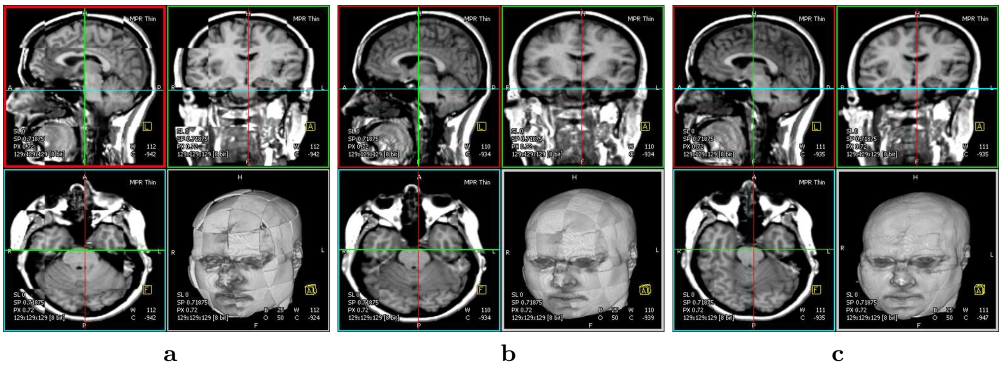

Fig. 2. Results of 3D registration. We denote the original MRI volume as R and the artificially deformed volume as T. After symmetric registration, the resampled volume are denoted as $R^{'}$ and $T^{'}$ respectively. (**a**) The checkbox of volume R and T. (**b**) The checkbox of volume R and $T^{'}$.(**c**) The checkbox of volume T and $R^{'}$. The parameter setting: $\alpha = 2550, \beta = 1, \nu = 1, \mu = 0.1, \lambda = 20, \kappa = 1, \epsilon = 0.5h$.

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