Signal-to-noise Monte-Carlo analysis of base material decomposed CT projections

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Abstract — The noise transfer properties of the water / bone base-material decomposition method has been analyzed. For typical CT projections of bone attenuators with lengths l_1 = 0 … 4.5 cm embedded into 30 cm of attenuating water length, the SNR loss compared to the native CT data is in the range of a factor 3 to 10. Especially the bone coefficient suffers from inferior SNR performance. This is due to the numerical structure of the inversion of the projection formula to integrated water and bone coefficients.

I. Introduction

The base material decomposition invented by Alvarez and Macovski [1, 2] converts dual-energy CT measurements into two material coefficient images. Simulations on the noise transfer of base material decompositions [3] indicate that depending on the absorber geometry we have to expect an average noise increase of 3 to 5 compared to normal CT imaging. For small decomposition coefficients, the noise increase gets even more significant.

In this paper we present simulation results on the most typical patient absorption geometry with a length l_1 of water and l_2 of bone (Fig. 1) to analyze the SNR behavior of bone / water material composition for dual-energy CT.

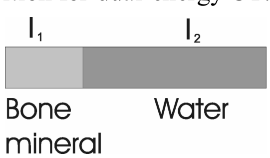

Fig. 1: Model geometry of bone mineral and water attenuator.

II. Theory

The projection $P(n)$ in a CT measurement is given by:

$$P(n)=\frac{I}{I_0}=\int_E w(E)e^{-\int_{L(n)}\kappa(E,\vec{r})d\vec{r}}dE \quad (1)$$

with: I := measured intensity with absorber,
I_0 := measured intensity without absorber,
$\kappa(E, \boldsymbol{r})$:= spectral attenuation coefficient.

The weighting function $w(E)$ is given by

$$w(E)=\frac{S(E)D(E)}{\int_E S(E)D(E)dE} \quad (2)$$

with: $S(E)$:= X-ray tube emission spectrum,
$D(E)$:= detector absorption probability.

Dual-energy measurements generate two sets of projections P_1, P_2 with different spectral weightings w_1, w_2. Fig 1 shows the weighting functions of 80 kV and 140 kV dual-energy measurements. The base material decomposition method [1, 2] separates κ into spatial and spectral coordinates:

$$\kappa(E,\vec{r})=b_1(\vec{r})F_1(E)+b_2(\vec{r})F_2(E) \quad (3)$$

$F_1(E)$, $F_2(E)$ are energy-dependent attenuation functions, e.g. photo absorption and Compton scatter attenuation functions or effective base materials like water and bone mineral. The spatially-dependant coefficients b_1, b_2 describe the effective concentration of the two components or base materials. With dual-energy data we can insert (3) into (1) to obtain a two-step process. First, the projections P_1, P_2 are converted to B_1, B_2 by inverting

$$\begin{pmatrix}P_1\\P_2\end{pmatrix}=\begin{pmatrix}\int_E w_1(E)\left(e^{-F_1(E)}\right)^{B_1}\left(e^{-F_2(E)}\right)^{B_2}dE\\ \int_E w_2(E)\left(e^{-F_1(E)}\right)^{B_1}\left(e^{-F_2(E)}\right)^{B_2}dE\end{pmatrix}. \quad (4)$$

Secondly, the B_1, B_2 are transformed to $b_1(\boldsymbol{r})$, $b_2(\boldsymbol{r})$ by an inverse Radon transformation of

$$B_i=\int_L b_i(\vec{r})d\vec{r}\,. \quad (5)$$

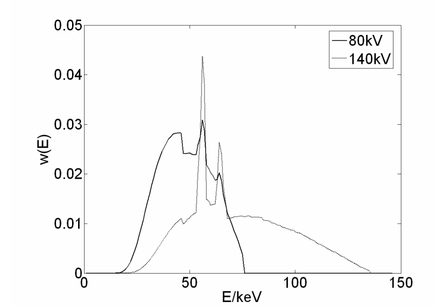

Fig 2. Dual-energy weighting functions w_1 and w_2 for 80 and 140 kV X-ray tube voltage measurements with a GdOS solid state CT detector.

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III. Simulation

The noise transfer properties of the base material decomposition depend critically on (4). The statistics of the dual-energy projections translate into statistics of the coefficients B_1, B_2. We perform a Monte-Carlo (MC) simulation of the noise transfer with the following steps:

1. The dual-energy CT data of the projections P_1, P_2 is calculated according to (1) with the $w_i(E)$ of Fig. 2 and using the attenuation geometry of Fig. 1.
2. For the MC simulation of the probability distributions of B_1, B_2 we repeat the following procedure M times:
   - First, Poisson noise is added to the ideal projections P_1, P_2 of step 1 with a σ-value of

$$\sigma(P_i) = \sigma\left(\frac{N_i}{N_{i,0}}\right) \overset{\sigma(N_{i,0})\,negl.}{=} \sqrt{\frac{P_i}{N_{i,0}}} \quad (6)$$

   The quantum numbers N_i and $N_{i,0}$ are given by the intensities I_i and $I_{i,0}$ of the two measurements $i = 1,2$ multiplied with the sensor area and integration time. Since the $I_{i,0}$ measurements are averaged significantly in practical CT, the noise contribution $\sigma(N_{i,0})$ is negligible in (6).
   - Secondly, the B_1, B_2 are calculated from (4) for each P_1, P_2. We obtain the probability distributions of B_1, B_2 and calculate mean, variance and SNR from this.
3. Finally we compare the SNR of B_1, B_2 to the SNR of the combined projection $P = (N_1 + N_2) / (N_{1,0} + N_{2,0})$:

$$SNR(P) = \sqrt{P_1 N_{1,0} + P_2 N_{2,0}} \quad (7)$$

The results in this paper were obtained with F_1, F_2 = Water and bone mineral (CaHOP) base functions, L = 30 cm, l_1 = 0…4.5 cm of dense bone material, $N_0 = 0.5 \cdot 10^6$, $N_{1,0} = N_{2,0} = N_0 / 2$, w_1, w_2 = 80 kV and 140 kV weighting functions (Fig. 2), M = 2000 MC cycles.

IV. Results and Discussion

Fig. 3 shows the simulated signal-to-noise ratios of B_1, B_2 and P. Fig. 4 depicts the ratios SNR(B_1) / SNR(P) and SNR(B_2) / SNR(P).

The base material decomposition (4) decreases the SNR significantly. Compared to the reference SNR(P), the water coefficient B_1 has about 70% SNR for 2 cm and about 60% SNR for 4 cm of the bone absorber. SNR(B_2) for the bone component obviously approaches 0 for small bone lengths l_1 and has about 20% SNR(P) for the maximum l_1 = 4.5cm

The critical noise transfer is due to the numerical structure of (4). Since exp($-F_2(E)$) << exp($-F_1(E)$), statistical fluctuations in (P_1, P_2) translate into larger variations of B_2, see also [3] for a detailed analysis on the dependence of B_1, B_2 on P_1 and P_2.

As a practical consequence the base material decomposition has to provide a significant increase in contrast or information value to justify the SNR loss. Tissue differentiation by measuring characteristic (B_1, B_2) pairs could potentially yield new information. However, the absolute SNR of the bone coefficient B_2 hardly exceeds 20%. It will be in the range of SNR(B_2) = 2 and below for small lengths of absorbing tissue. This renders tissue characterization at an acceptable dose a very difficult task. If we consider 5, 10 and 15HU as the optimum, good and average 1σ precision of a soft tissue finding in CT, the 1σ precision of the B_2 value will be in the range of 5 x [5,10,15] / 1000 = 2.5 % to 7.5%. This seems to be very difficult hurdle for practical tissue characterization with the bone water decomposition.

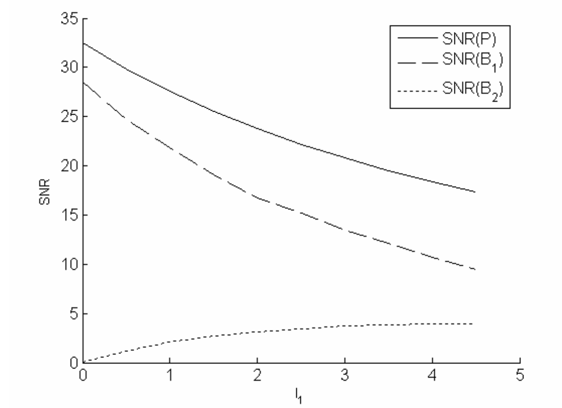

Fig.3 Simulated SNR(B_1), SNR(B_2) as obtained MC-procedure and SNR(P) according to (7).

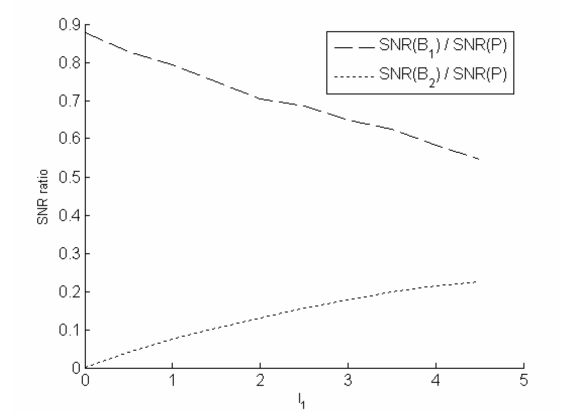

Fig.4 Ratios of SNR(B_1), SNR(B_2) to the reference SNR(P) of the combined dual-energy-scans.

V. Bibliography

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