Material Decomposition for Energy Resolving Detectors using Weighted Levenberg-Marquardt Optimization

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Introduction: Energy resolving X-ray photon counting detectors are able to assign each detected photon to energy bins [1] (Figure 1). This allows the decomposition of an X-ray image into the materials of the acquired object [1, 2], which has potential benefits in angiography. Contrast agent can be extracted without acquiring a mask image, possibly saving dose and avoiding problems with patient motion. We introduce and evaluate physical model fitting approaches, which are solved using a modified Levenberg-Marquardt (LM) algorithm [3].

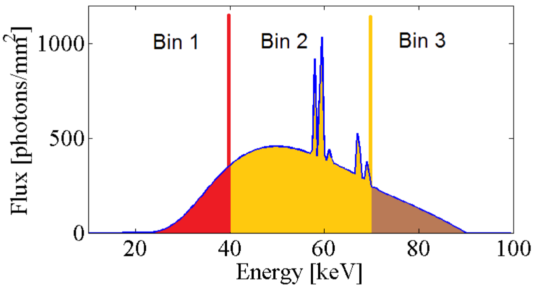

Figure 1 Binned polychromatic X-ray spectrum

Materials and Methods: For each image pixel, we intend to recover the density line integrals $\boldsymbol{l} \in \mathbb{R}^M$ of M materials from B bins, assuming $M \leq B$. The photon count in bin b is modeled by $\hat{I}_b(\boldsymbol{l})$

$$\hat{I}_b(\boldsymbol{l}) = \sum_{i=1}^{N} C_{i,b} \exp\left(-\sum_{m=1}^{M} \mu(m, E_i) l_m\right) \text{ with } \boldsymbol{l} = (l_1, \dots, l_M)^T \in \mathbb{R}^M ,$$

where $C_{i,b}$ denote the un-attenuated photon flux at energy level E_i and $\mu(m, E_i)$ the material and energy dependent attenuation coefficients. We fit $\hat{I}_b(\boldsymbol{l})$ to the measured photon counts I_b; $b = 1 \dots B$ applying three different approaches by solving with LM optimization

$$\textbf{LM}: \quad \operatorname{argmin}_{\boldsymbol{l}} \sum_{b=1}^{B} \left(I_b - \hat{I}_b(\boldsymbol{l})\right)^2, \quad \textbf{LMLog/LMWLog}: \quad \operatorname{argmin}_{\boldsymbol{l}} \sum_{b=1}^{B} w_b \left(\ln I_b - \ln \hat{I}_b(\boldsymbol{l})\right)^2.$$

The LMLog method is applied to the logarithmic image and uses equal weighting ($w_b = 1$; $b = 1 \dots B$). The LMWLog method additionally applies an uncertainty weighting. As the photon measurements correspond to Poisson processes, the SNR of the measurements is proportional to the photon count and we set $w_b = I_b$; $b = 1 \dots B$.

The approaches are evaluated using a numerical projection image created from a digital perfusion brain phantom [4] using the CONRAD framework [5]. The phantom consists of *M=3* materials: bone, brain tissue and Iopromide contrast agent. A polychromatic forward projection was simulated with *B*=3 bins, 90 kV peak voltage and 2.5 mAs time current product (Figure 1).

Results:

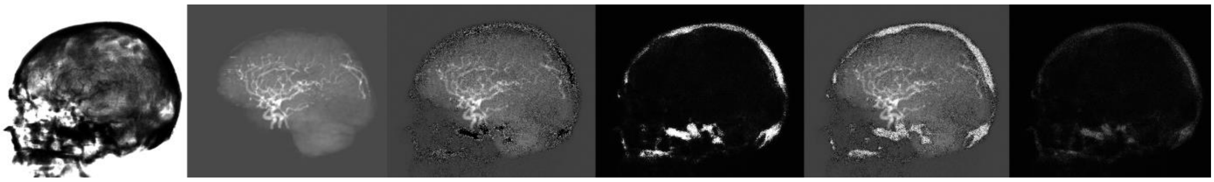

Figure 2 Left to right: bin 1 projection image, reference contrast agent enhancement (CAE), LMLog CAE, LMLog absolute error image (AEI), LMWLog CAE, LMWLog AEI

Input Data	**without noise**		**with noise**		
Method	**LM**	**LMLog**	**LM**	**LMLog**	**LMWLog**
RMSE [mg/mm²]	0.38	0.38	9.7	17.86	1.7
MAD [mg/mm²]	0.82	0.82	70.0	126.56	25.7
Avg. Computation Time [ms]	0.436	0.075	0.560	0.080	0.132
Avg. Function Evaluations	84.4	14.4	105.2	15.1	24.9

Table 1 Root mean square error (RMSE), maximum absolute difference (MAD), average computation time, and average number of evaluations of the quadratic error term

Discussion: Table 1 shows that in the noiseless case the LMLog method achieves same RMSE and MAD as the ML approach, but is almost 6 times faster. In the noisy case, however, the LMLog RMSE and MAD are worse. By applying the uncertainty weighting, the LMWLog achieves the best RMSE and MAD, while being 4 times faster than LM. The AEIs of LMLog and LMWLog in Figure 2 show that the error is mainly reduced by LMWLog in areas with low photon counts in bin 1, i.e., areas with strong attenuation by the skull bone.

The logarithm in the LMLog method improves the optimization speed as it yields an approximately linear relationship between $\boldsymbol{l}$ and $\ln I_b$. However, the intrinsic uncertainty weighting of the LM method is lost, but can be reintroduced by the SNR weighting of the LMWLog method. In future work a comparison to the maximum-likelihood method by Proska and Roessl is desirable [6].

Summary: Material decomposition methods based on the Levenberg-Marquardt algorithm are investigated. We discovered that logarithmic processing can help to reduce computation time and uncertainty weighting to improve image quality.

References:

1. N. Maaß et al., “Dose minimization for material-selective CT with energy-selective detectors,” IEEE NSS/MIC, pp. 4447–4452, 2011.
2. M. Manhart et al.: “Guided Noise Reduction for Spectral CT with Energy-Selective Photon Counting Detectors”, 3rd CT Meeting, Salt Lake City, UT, USA, 2014, pp. 91-94.
3. S. Burton et al., “Argonne National Laboratory MINPACK project”, Technical Report, 1980.
4. A. J. Riordan et al., “Validation of CT brain perfusion methods using a realistic dynamic head phantom”, Medical Physics, Vol. 38, No. 6, pp. 3212–3221, 2011.
5. A. Maier et al., “CONRAD - A software framework for cone-beam imaging in radiology,” Medical Physics, vol. 40, no. 11, pp. 111 914–1–8, 2013.
6. E Roessl and R Proksa: “K-edge imaging in x-ray computed tomography using multi-bin photon counting detectors”, Phys. Med. Biol., vol. 52, pp. 4679–4696, 2007.