Background and Purpose

In C-arm CT, severe artifacts may show in the reconstructed image due to a limited rotation angle and view projections during acquisition. Iterative methods using Compressed Sensing [3] are designed to compensate those artifacts by iteratively alternating between backprojecting data into the reconstructed image and projecting intermediate reconstruction images back into the raw data domain.

The purpose of our work is to present insights about the data dependence of the combined method’s parameters set by the user, introducing a high dimensional optimization space.

Key Ideas

Analyze the impact of changes to the TV constraint iterative reconstruction method’s parameter set
- Define and validate suitable error metrics
- Select one optimal parameter set for given data
- Investigate the quality of the reconstruction with the parameter set on different data

Method: TV Constraint Iterative Reconstruction

Compressed Sensing deals with the problem of incomplete data by finding solutions to underdetermined linear systems.
- Takes advantage of the signal’s sparseness or compressibility in some domain using sparsifying operator $Ψ$ [3]
- Sparseness can be incorporated into a constraint. [5]

During reconstruction the one solutions which transformed coefficient sequence also minimizes the $ℓ 1$ norm, penalizing artifact creation.

$$
\min ||ψ_1(f)||, \text{ subject to } ||R_f(r) - p|| < \epsilon
$$

Alternating optimization: iTV reconstruction approach. [4]
- SART [2] to minimize $||R_f(r) - p||^2$ 
- Gradient Descent to increase the sparsity of $ψ_1(f) := \nabla f(r)$
- Linear combination of intermediate volumes $f_{n+1} = (1 - \lambda)f_{n+1}^{SART} + \lambda f_{n+1}^{TV}$ after each iteration

An optimal parameter value $\lambda \in [0 : 1]$ is determined in the raw data domain by solving (3), since $\epsilon_{n+1}$ is known and $\omega$ constant.

$$
\epsilon_{n+1} = (1 - \omega) \cdot ||f_{n+1}^{SART}(r) - p||^2 + \omega \cdot \epsilon_n, \quad \omega \in [0 : 1]
$$

$$
||R_f(r) - p||^2 + \lambda ||f_{n+1}^{TV}(r)||^2 = \epsilon_{n+1}
$$

Variables: Select one optimal parameter set

Star-shaped search for the optimal parameters using iTV

| Type | $\omega$ | $\lambda_{max}$ iterations $N_{init}$ $\alpha_{opt}$ GD-Iterations |
|------|---------|------------------------|------------------------------|
| default changes | $\in [0, 1]$, $\in [0, 1]$ | $8$ | $10^4$ | $3$ | $25$ |

The default parameter set for the TV reconstruction and it’s variations. The best result for the FORBILD head phantom comes from the default set plus relaxation parameter $\beta = 0.8$. [1]

Metrics: Measure reconstruction quality

Measures used for 3D image comparison
- Root-Mean-Square Error
- Peak Signal-to-Noise Ratio
- Mean Structural Similarity
- Pearson Correlation
- Total Variation
- Eyeball Measure

Results

Reconstruction results with limited angle relative to ground truth or FDK in percent.

Simultaneous Algebraic Reconstruction Technique

Reconstruction results for the SART method, $\beta = 0.8$, $N = 20$.

<table>
<thead>
<tr>
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<th>FORBILD head</th>
<th>Human head phantom</th>
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<tbody>
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<td>50</td>
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<tr>
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<td>100</td>
</tr>
<tr>
<td>TV</td>
<td>156</td>
<td>172</td>
</tr>
</tbody>
</table>

Improved TV Regularized Reconstruction

Reconstruction results for the iTV method $\beta = 0.8$, $\omega = 0.8$, $\lambda_{max} = 1.2$, $N = 20$, $\text{regul} = 10^7$, $\alpha_{opt} = 0.3$, GD-Iterations $= 25$.

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</thead>
<tbody>
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<td>132</td>
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<tr>
<td>TV</td>
<td>149</td>
<td>138</td>
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</tbody>
</table>

Figure 1: FORBILD head phantom and human head phantom reconstructions from limited angle ($\Theta_{max} = 155^\circ$) and few projections ($\Theta_A = 2.25^\circ$). WC: [0, 1000]

Conclusion

The fixed set of parameters optimized for a limited angle acquisition of the FORBILD head phantom was used during reconstruction of various scenarios.

The inhomogeneous regions resulting from the X-ray photon noise induced and streak artifacts are less prominent in iTV improving the perception of low contrast elements. However, the porous bone structure of the human head phantom got blurred significantly using the same set of parameters for this reconstruction.

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References


