A 3-D Scattering Model for Orientation-dependent X-ray Dark-field Imaging

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Introduction

- X-ray dark-field imaging (XDI) visualizes the microstructure of specimen [1, 2].
- XDI is a promising technique for recovering bone structures or for early detection of breast cancer [3]
- Full 3-D XDI reconstruction is still an open problem.
- Particularly, the relationship between 3-D fiber orientation and the XDI signal is not yet fully described
- **Here, we propose a 3-D scattering projection model**
- This model can be used for developing advanced 3-D XDI reconstruction algorithms

3-D Scattering Model

- Scattering is modeled as a Gaussian distribution with the primal axis along the 3-D orientation
- In a Talbot-Lau interferometer (*Figure 1*), only scattering projected to *x*-axis is measured, integrating the Gaussian distribution along both *y*-axis and *z*-axis results in a scattering function
- 3-D orientation denoted by azimuth angle Θ_a and elevation angle Θ_e
- The observed intensity is calculated as the convolution of the original intensity with the scattering function
- The negative logarithm of dark-field signals (log-dark-field) of an elongated structure is:

$$d(\boldsymbol{p}_d, \omega) = (b_1 + b_2\cos^2(\Theta_e)) + (b_3 - b_2\cos^2(\Theta_e))\cos^2(\Theta_a - \omega)$$

$\boldsymbol{p}_d$: pixel position on the detector
b_1, b_2, b_3: material-dependent constants
ω: rotation angle of the imaging system

Validation and Evaluation

- Dark-field images of carbon fiber bunch at elevation angles 30°, 45°, 60° was examined (*Figure 2*)
- Log-dark-field was averaged over a region of interest
- 40 projections were taken over 720°
- Line fitting was applied to calculate coefficients depending on different elevation angles
- Material-dependent constants were calculated from the data set at angle 30° and angle 60°
- Simulated log-dark-field of carbon fiber bunch at elevation angle 45° was compared to experimental data (*Figure 3*)
- Results are evaluated by normalized square root difference (NSRD):

$$r_d = \frac{\sqrt{\sum_{i=1}^{N_p}\left(d^e_{log}(i) - d^s_{log}(i)\right)^2}}{N_p\left(\max_{i=1..N_p} d^e_{log}(i) - \min_{i=1...N_p} d^e_{log}(i)\right)}$$

N_p=40: number of projections
$d^e_{log}(i)$: averaged log-dark-field from experiment at *i*th projection.
$d^s_{log}(i)$: averaged log-dark-field from simulation at *i*th projection.

- Simulation results match experiments nicely, with 1.1% NSRD

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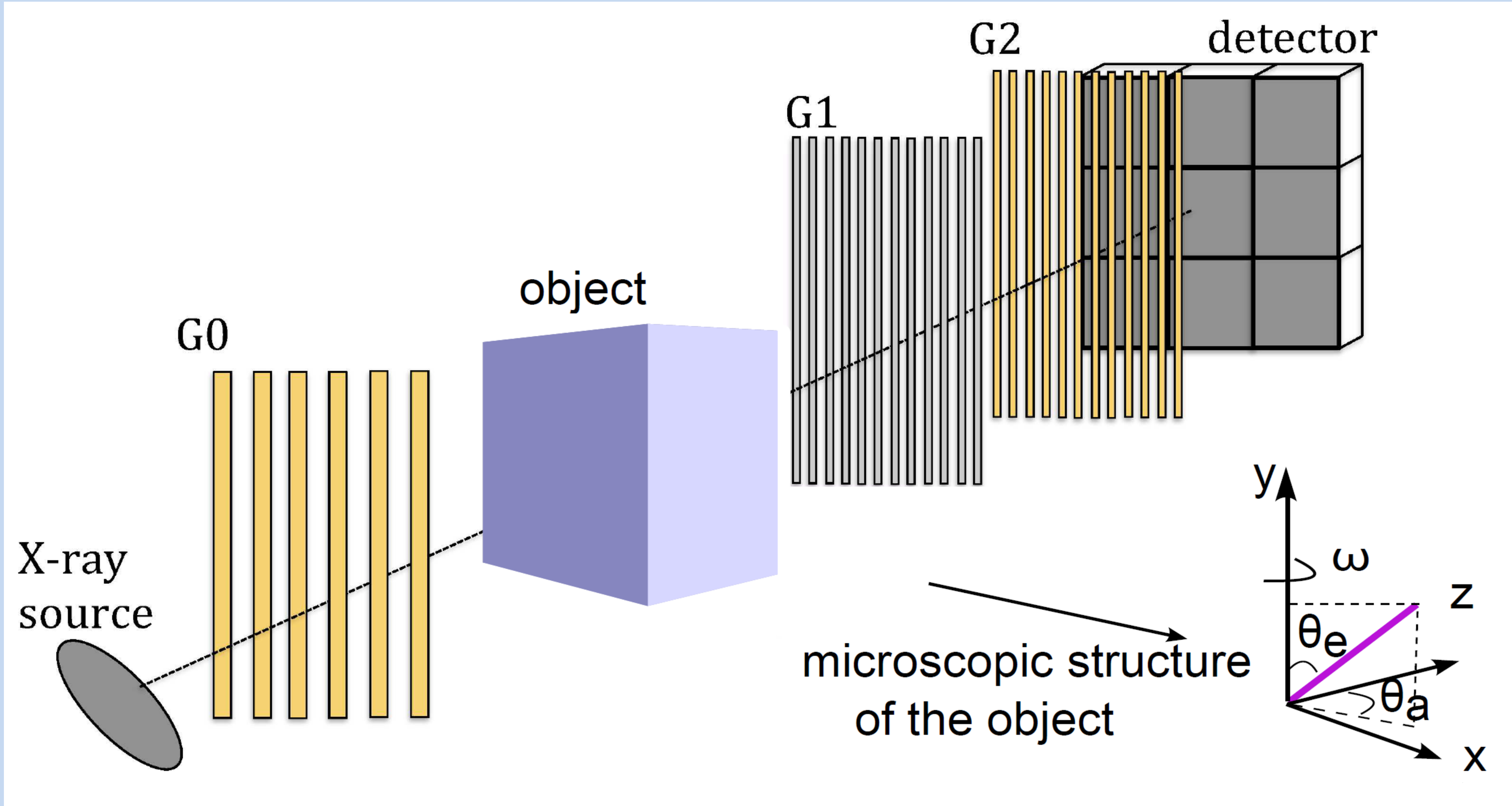

Figure 1: Talbot-Lau interferometer and coordinate system of the object.

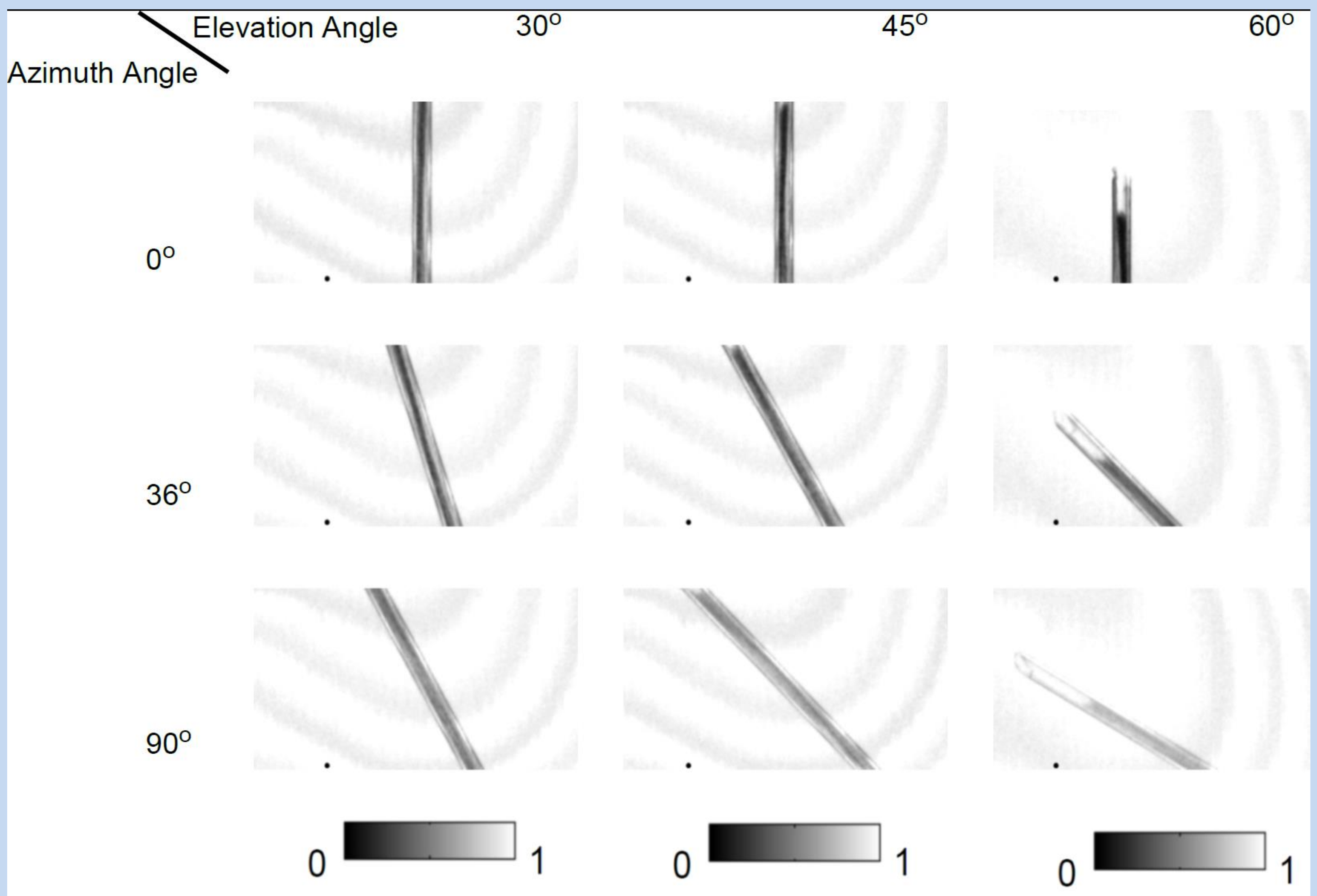

Figure 2: Dark-field images of a single carbon fiber.

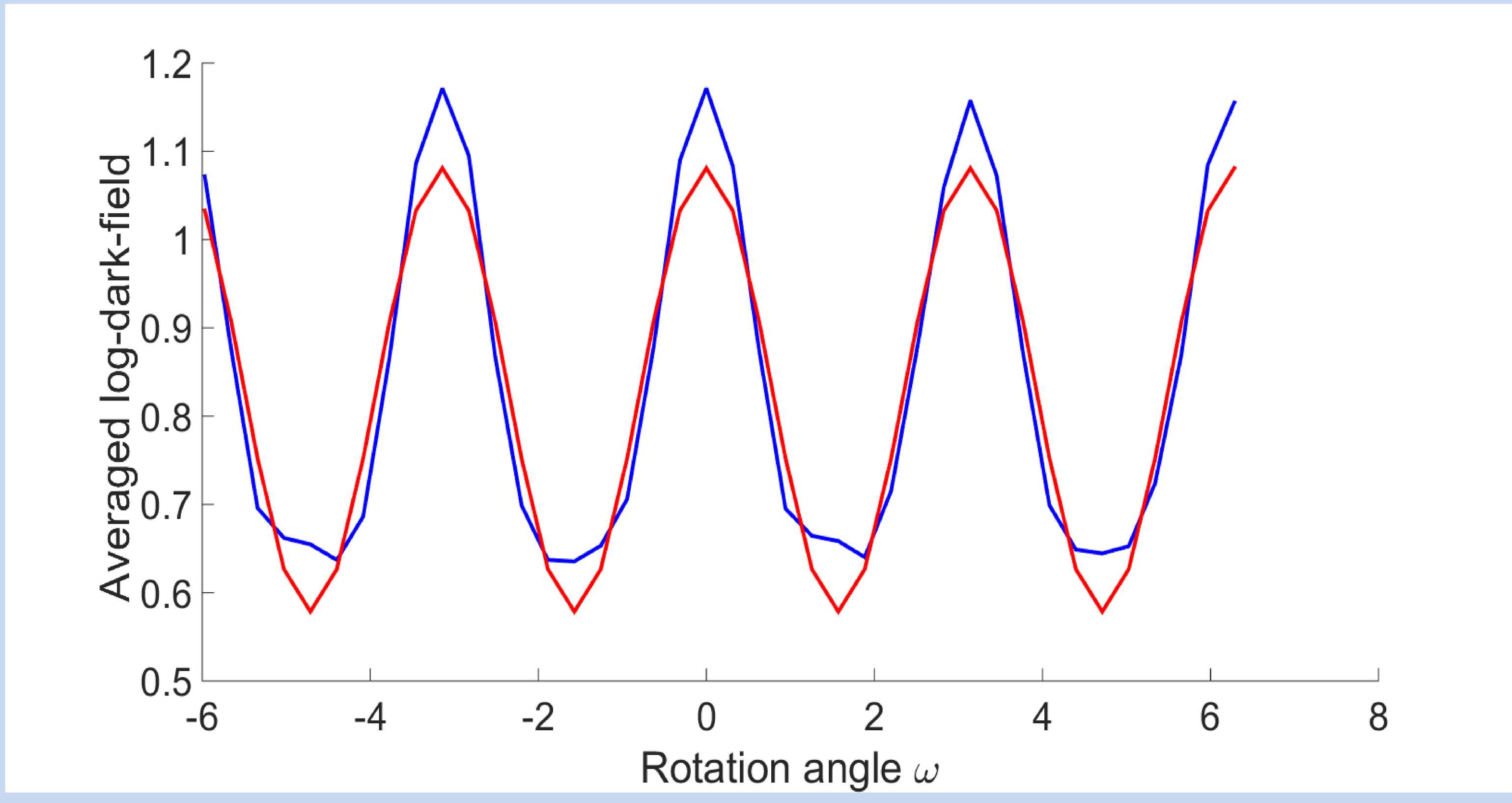

Figure 3: Comparison of simulated average log-dark-field of the carbon bunch at elevation angle 45° (red line) and the experimental log-dark-field of the same example (blue line).

Discussions

- A 3-D scattering model is proposed and validated
- Major step towards reconstructing 3-D orientations
- Inverse problem of reconstruction based on this model will be investigated

References

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