**RESEARCH ARTICLE** 

# Highly undersampled peripheral Time-of-Flight magnetic resonance angiography: optimized data acquisition and iterative image reconstruction

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#### Abstract

*Object* The aim of this study was to investigate the acceleration of peripheral Time-of-Flight magnetic resonance angiography using Compressed Sensing and parallel magnetic resonance imaging (MRI) while preserving image quality and vascular contrast.

*Materials and methods* An analytical sampling pattern is proposed that combines aspects of parallel MRI and Compressed Sensing. It is used in combination with a dedicated Split Bregman algorithm. This approach is compared with current state-of-the-art patterns and reconstruction algorithms. *Results* The acquisition time was reduced from 30 to 2.5 min in a study using ten volunteer data sets, while showing improved sharpness, better contrast and higher accuracy compared to state-of-the-art techniques.

*Conclusion* This study showed the benefits of the proposed dedicated analytical sampling pattern and Split Bregman algorithm for optimizing the Compressed Sensing reconstruction of highly accelerated peripheral Time-of-Flight data.

Keywords Iterative reconstruction  $\cdot$  Non-contrastenhanced MRA  $\cdot$  Peripheral angiography  $\cdot$  Compressed Sensing

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#### Introduction

The diagnosis of peripheral arterial occlusive disease requires comprehensive imaging of a large field of view, typically including the abdominal aorta, and iliac, femoral and popliteal arteries down to the tibial and fibular arteries. In addition to digital subtraction angiography, which is considered the reference technique, contrast-enhanced magnetic resonance angiography (MRA) techniques are widely used in clinical practice. These require the injection of a gadolinium-based contrast agent, which can only be administered in patients with adequate renal function, often excluding those who need the scan most. Moreover, correct bolus timing is important and represents a source of potential errors. Time-of-Flight imaging is a non-invasive, non-contrast-enhanced MRA technique offering both high resolution and excellent imaging contrast. However, its main drawback is the long acquisition time [1], due to the large field of view combined with the 2D electrocardiogram (ECG)-triggered acquisition method that is commonly used.

Iterative image reconstruction methods, combined with a dedicated data sampling strategy, have shown potential to shorten acquisition times for non-contrast-enhanced MRA. Recently proposed magnetic resonance imaging (MRI) reconstruction techniques combine the concept of Compressed Sensing [2] with widely-used parallel MR reconstruction techniques such as SENSE or GRAPPA [3, 4].

Compressed sensing exploits known sparsity properties of the data in transform domains, in combination with incoherent k-space sampling and non-linear reconstruction algorithms. The choice of the sampling and the minimization algorithm are challenging, especially for high acceleration factors that are desired in many applications. In addition, noise amplification is a known problem in iterative SENSE

reconstructions [3, 5, 6], originating from linear dependent coil setups and the resulting ill-conditioned reconstruction matrices. The noise is more pronounced when using high acceleration factors. Therefore, regularization with prior knowledge terms such as isotropic Total Variation or Wavelet regularization is required. As a consequence, joint problems emerge that need to be solved using adapted optimization algorithms. These traditionally include gradient-based methods such as the Conjugate Gradient [7] or Newton methods. However, multi-step algorithms such as the Split Bregman algorithm that were recently adapted include  $L_1$ -regularized optimization problems [8, 9], and they have proven to be effective in a broad range of applications. In this family of algorithms, the optimization problem is not solved in its joint form, but split into multiple steps [10], hence allowing for dedicated minimization strategies.

This work proposes a novel combination of a MultI-Coil Compressed Sensing (MICCS) sampling and the Split Bregman algorithm. It is shown that this approach provides high acceleration factors of up to 12, while overcoming the problem of noise amplification and promoting sharpness in MRA images. The MICCS pattern incorporates aspects of sampling patterns as previously used in the context of the parallel MRI algorithm SENSE and Compressed Sensing, and uses interleaved calculations of the reference scans.

The purpose of this work is to compare this novel combination with other state-of-the-art schemes with different reconstruction algorithms. Our study aims to quantitatively show the benefits of the proposed acquisition and reconstruction strategy, using image-based criteria and the Vessel Sharpness Index to evaluate the diagnostic usability.

# Materials and methods

In the following section, the novel MICCS sampling pattern is introduced and the Compressed Sensing-SENSE problem using the Split Bregman algorithm is formulated.

### MICCS sampling pattern and coil sensitivities

The developed data acquisition strategy, referred to as the MultI Coil Compressed Sensing (MICCS) pattern, combines aspects of sampling schemes used for parallel MRI and Compressed Sensing techniques. These aspects are incorporated differently in the central and peripheral k-space regions.

The traditional sampling scheme for parallel MRI combines regular undersampling in the periphery with a fully sampled *k*-space center to provide low-frequency information for coil sensitivity maps. Both paradigms are combined in the proposed MICCS pattern. In the central

region, a regular undersampling with factor  $d \in \mathbb{Z}$  is used, which can be adapted to the number of independent coils in the acceleration direction. Shifts between neighboring slices, parametrized using the center offset  $o \in \mathbb{N}$ , allow for an interleaved generation of coil sensitivity information as described in the next section. In the *k*-space periphery, random, pseudo-random or Poisson-disc distributions were previously proposed [11] for Compressed Sensing.

For the MICCS pattern, the distance from one sampled line  $\chi$  to the next is calculated using the following analytical inverse root function *f*, where the parameters *a* and *b* are used to scale the slope and distance between sampled *k* -space points:

$$f(\chi, (a, b)) = \lceil (a \cdot \chi)^b \rceil \quad \text{with } a, b \in \mathbb{R}.$$
 (1)

The obtained total number of sampled lines and their distribution depends on the *k*-space size and the parameters for the center (d, o) and periphery (a, b). Typical settings are (d, o) = ([2, 4], [0, 3]) and (a, b) = ([0.01, 0.1], [3, 6]). The generation of the pattern as a combination of SENSE and Compressed Sensing features is illustrated in Fig. 1.

# Internal, interleaved and interpolated coil sensitivity calculation

The complex spatial coil sensitivity profiles, required for the SENSE algorithm, are described by  $\mathbf{c}_{(z,\nu)} \in \mathbb{R}^N$  for slice z and coil  $\gamma$ . They are typically obtained from lowfrequency k-space samples, representing, after Fourier transform and a filtering step, approximations of the coil images and thus the sensitivity of each coil. Two different methods to obtain these central lines are common, either by a separate external scan or internally from the image data. In the proposed method, the spatial adjacency of neighboring slices is exploited as shown in Fig. 2. The low frequency samples are completely obtained from the imaging data using an interleaved approach based upon variation of the offset o as introduced in the previous section. Choosing the offset o = 1 relative to adjacent slices leads to a shift of the central regular undersampling k-space pattern from slice to slice. The  $N_7$  2-D slice images are written as column vectors  $\mathbf{x}_z$  with  $z = 1, \ldots, N_z$  and are arranged in a common vector  $\mathbf{x} \in \mathbb{C}^{NN_z}$  with  $N = N_x N_y$ . The k-space data from  $N_{\nu}$  coils is likewise arranged in column vector  $\mathbf{m} \in \mathbb{C}^{N_{\gamma}N_kN_z}$  for all slices with  $N_k = N_{kx} \cdot N_{kz}$ . Thereby, the vectors **m** and **x** contain the linearized image pixels and k-space samples. The reconstruction problem is formulated for the entire 3D stack of slices using the encoding matrix  $\mathbf{E} \in \mathbb{C}^{N_{\gamma}N_kN_z \times NN_z}$ , incorporating the sampling information and the coil sensitivity map, as well as the inverse Fourier Transform coefficients.

**Fig. 1** Illustration of the MICCS pattern, made from combined requirements from SENSE and Compressed Sensing. The *k*-space is divided into central and peripheral regions. The *k*-space information used for coil sensitivity calculations is combined over adjacent slices



The data fidelity term is formulated as  $\frac{1}{2} || \mathbf{E} \mathbf{x} - \mathbf{m} ||_2^2$ . The assumption that medical images are piecewise constant and smooth is exploited using the sparsity of the data in the 2D Total Variation (TV) [12, 13] and the 2D Wavelet Transform (W) [7]. The joint minimization problem combines  $L_2$  and  $L_1$  terms, including the data fidelity term and the regularization terms:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \mathcal{L}_{\text{GB}}(\mathbf{x}) \text{ with } \mathcal{L}_{\text{GB}}(\mathbf{x})$$

$$= \underbrace{\frac{1}{2} \|\mathbf{E}\mathbf{x} - \mathbf{m}\|_{L_{2}}^{2}}_{\text{Data fidelity term}} + \underbrace{\lambda_{t} \|\nabla \mathbf{x}\|_{L_{2,1}}}_{\text{Total Variation}} + \underbrace{\lambda_{w} \|W(\mathbf{x})\|_{L_{1}}}_{\text{Wavelet transform}}, \qquad (2)$$

where  $\nabla \mathbf{x}$  denotes the 2D finite difference in the *x* and *y* direction, *W* the Wavelet decomposition operator, while the weights  $\lambda_t$  and  $\lambda_w$  regulate the influence of the respective terms.

# Split Bregman and Quasi Newton method

The joint unconstrained problem formulation in Eq. (2) can be solved using different algorithmic types. The methods originally proposed [7] use the joint problem formulation, which is then solved using gradient-based minimization algorithms such as the Quasi-Newton or Conjugate Gradient algorithms [14], seeking to minimize all terms at the same time.

Alternatively, the parts of the minimization problem can be split and treated individually, allowing a better-suited strategy for each of them. The recently developed Split Bregman algorithm [8] alternates between a gradient-based optimization for an  $L_2$  problem and shrinkage for the  $L_1$ sub-problems. The minimization problem in Eq. (2) is used as a basis for the Split Bregman algorithm. The introduction of the additional variables  $\mathbf{d}_w, \mathbf{d}_x$  and  $\mathbf{d}_y \in \mathbb{C}^N$  yields the formulation with penalty terms



Fig. 2 Illustration of the calculation of the coil sensitivity maps: using the interleaved approach (*top row*), an individual coil calculation for each slice (*second row*), the difference between both using the same scaling *third row*, and a factor of 10 in scaling (*bottom row*)

$$(\hat{\mathbf{x}}, \hat{\mathbf{d}}_{x}, \hat{\mathbf{d}}_{y}, \hat{\mathbf{d}}_{w}) = \underset{\mathbf{x}, \mathbf{d}_{x}, \mathbf{d}_{y}, \mathbf{d}_{w}}{\operatorname{argmin}} \mathcal{L}_{SB}(\mathbf{x}, \mathbf{d}_{x}, \mathbf{d}_{y}, \mathbf{d}_{w}) \text{ with}$$

$$\mathcal{L}_{SB}(\mathbf{x}, \mathbf{d}_{x}, \mathbf{d}_{y}, \mathbf{d}_{w}) = \frac{1}{2} \|\mathbf{E}\mathbf{x} - \mathbf{m}\|_{L_{2}}^{2}$$

$$+ \lambda_{t} \sum_{l=1}^{N} \sqrt{(\mathbf{d}_{x})_{l}^{2} + (\mathbf{d}_{y})_{l}^{2}} + \lambda_{w} \|\mathbf{d}_{w}\|_{L_{1}}$$

$$+ \frac{\alpha \lambda_{t}}{2} \left( \|\mathbf{d}_{x} - \nabla_{x}\mathbf{x}\|_{L_{2}}^{2} + \|\mathbf{d}_{y} - \nabla_{y}\mathbf{x}\|_{L_{2}}^{2} \right)$$

$$+ \frac{\alpha \lambda_{w}}{2} \|\mathbf{d}_{w} - W(\mathbf{x})\|_{L_{2}}^{2},$$
(3)

Fig. 3 Schematic representation of the joint gradient-based and Split Bregman-based algorithm



where  $\alpha \in \mathbb{R}$  with  $\alpha > 0$ .

The decoupling of  $\mathbf{d}_x$  and  $\mathbf{d}_y$  in the isotropic Total Variation term is not possible [15], and so a joint formulation is used in Eq. (3). The problem is then reformulated using the Split Bregman algorithm with the penalty terms and residual errors  $\mathbf{b}_x, \mathbf{b}_y, \mathbf{b}_w \in \mathbb{C}^N$  as explained in the "Appendix" in more detail. The minimization problem can be split into sub-problems allowing the de-coupling of the  $L_1$  and  $L_2$  parts. The  $L_2$  problem, formulated with the objective function  $\mathcal{L}_{SB-L2}$  is solved using a gradientbased optimization technique with  $N'_i$  iterations. The isotropic Total Variation is minimized using the generalized shrinkage formula [8], while the Wavelet term is minimized with soft thresholding. In contrast to the joint problem, numerical calculation of the gradients of the Total Variation norm, the Wavelet L<sub>1</sub> norm, as well as the corner smoothing parameter  $\tau$ , are not required. This is beneficial, as the choice of adapted  $\tau$  constitutes a tradeoff between convergence speed and the quality of the obtained solution [7, 16].

The stated steps, minimization of the L<sub>2</sub> term, generalized Total Variation thresholding and Wavelet soft thresholding, are repeated in each iteration *j* for  $j \in \{1, ..., N_j\}$ . Figure 3 illustrates both techniques schematically, and the corresponding pseudo codes are shown in Algorithm 1.

# Experimental setup

Specifically for the following experiments,  $3 \times 1$  or  $2 \times 3$ independent left-right elements of the spine and  $1 \times 4$  or  $2 \times 4$  independent elements of the peripheral coil were used, depending on the table position. The acquisition was segmented such that 23 lines were acquired in one shot. To evaluate the image quality achieved with the presented

Algorithm 1: Split Bregman algorithm	
Require: u, m, $x^0$ , $\lambda_t$ , $\lambda_w$ , $\mathcal{L}_{GB}$ , $N_i$ Calculate combined coil profiles $c_{(\gamma,\iota)}$ while $i < N_i$ do	Require: u, m, x <sup>0</sup> , $\lambda_t$ , $\lambda_w$ , $\mathcal{L}_{SB}$ , $N_j$ , $N'_i$ while $j < N_j$ do while $i < N'_i$ do
Perform optimizer step (objective function $\mathcal{L}_{GB}$ ) Calculate the data fidelity term Calculate Total Variation $L_{2,1}$ term Calculate Wavelet $L_1$ term Update $\mathbf{x}^i$ end while	Perform optimizer step (objective function $\mathcal{L}_{SB-L2}$ ) Calculate the data fidelity term Calculate Total Variation $L_{2,1}$ term Calculate Wavelet $L_1$ term Update $\mathbf{x}^{i'}$ <b>end while</b> Perform the shrinkage operations
return x	$(\mathbf{d}_x^{j+1}, \mathbf{d}_y^{j+1}) = s_1(\mathbf{x}^{j+1}, \mathbf{b}_x^j, \mathbf{b}_y^j, 1/\alpha)$ $\mathbf{d}_w^{j+1} = s_2(\mathbf{x}^{j+1}, \mathbf{b}_w^j, 1/\alpha)$ Update the residual errors $\mathbf{b}_x^{j+1} = \mathbf{b}_x^j + \nabla_x \mathbf{x}^{j+1} - \mathbf{d}_x^{j+1}$ $\mathbf{b}_y^{j+1} = \mathbf{b}_y^j + \nabla_y \mathbf{x}^{j+1} - \mathbf{d}_y^{j+1}$ $\mathbf{b}_w^{j+1} = \mathbf{b}_w^j + W(\mathbf{x}^{j+1}) - \mathbf{d}_w^{j+1}$ end while return $\mathbf{x}$

algorithms, the data sets were fully sampled so that they could be used as a reference and to render retrospective undersampling possible. Two different types of data sets were acquired, referred to as protocols I and II. With protocol I, data sets of four slices were acquired from the region below the bifurcation of the popliteal artery into the anterior and posterior tibial and the origin of the peritoneal artery. An in-plane field of view of 448 mm  $\times$  448 mm and matrix size of  $460 \times 460$  were used. With protocol II, data sets of the entire upper and lower peripheral vasculature were acquired in three bed positions with 128 slices each. The field of view was 448 mm  $\times$  168 mm, which is sufficient for covering the volunteers' anatomy. With a matrix size of 448  $\times$  184, the in-plane resolution was 1 mm  $\times$  0.9 mm. For both protocols, TE/TR were 5/34.7 ms and the slice thickness was 2 mm.

#### Implementation

The algorithms were implemented in a C++ framework offering both the connection to the scanner manufacturer's reconstruction pipeline and the possibility for operation as a stand-alone platform. In the present work, a limited-memory Broyden–Fletcher–Goldfarb–Shanno (IBGFS) Quasi Newton algorithm was chosen as the gradient-based optimization method for the joint problem formulation. Its low memory usage, combined with the low number of required function evaluations, is beneficial for large-scale optimization problems [14].

The parameters given for the joint problem solved with gradient-based methods in Eq. (2) included  $N_i$ ,  $\lambda_t$  and  $\lambda_w$ . For the Split Bregman algorithm, the minimization problem of interest was formulated in Eq. (3). The relevant parameters were the number  $N_j$  of Split Bregman iterations, the number of Quasi Newton iterations  $N'_i$  used to solve the L<sub>2</sub> problem and the regularization weights. To limit redundancy, the weight  $\alpha$  was chosen as  $\alpha = 2.0$ . The Total Variation and Wavelet weights were referred to by  $\lambda_t^{SB}$  and  $\lambda_w^{SB}$ . These parameters were chosen individually for each method by limiting the respective normalized Root Mean Square Error (NRMSE), but fixed for all data sets so that robust reconstruction could be provided.

#### Reconstruction experiments

The purpose of the reconstruction experiments was to investigate both the proposed sampling strategy and the choice of the reconstruction algorithm.

To this end, all protocol I data sets were undersampled with six different patterns, all using the same acceleration factor of 6, corresponding to 76 phase encoding lines of 460. These patterns varied in the way the *k*-space periphery was sampled, as well as in the size and chosen

undersampling factor of the central region. Alongside full sampling, the proposed MICCS pattern with a central region consisting of 10 % of the k-space size sampled with a factor of 3, and two patterns with a regularly sampled periphery and two with randomly sampled periphery, were used. Thereby, the central region of the regular patterns was fully sampled; its size corresponded to 8 % of the k-space for the first pattern (Regular 1), and to 10 % of the k-space for the second pattern (Regular 2). The central size of both random patterns corresponded to 10 % of the k-space size, but while the first was regularly undersampled by a factor of 3, matching the MICCS pattern (Random 1), the second was fully sampled in the center (Random 2). The number of remaining lines in the periphery was chosen to match the desired target undersampling factor. These analyzed patterns are illustrated schematically on the left side of Fig. 4. The reconstruction algorithm used was the iterative SENSE algorithm solved with the gradient-based algorithm with  $N_i = 8$  iterations.

In experiment II, the MICCS pattern was applied to the presented application of interest in combination with the joint gradient-based and Split Bregman algorithms. The raw data acquired with protocol I and undersampled with the MICCS pattern using a factor of 12, corresponding to 39 lines out of 460, was reconstructed using a nonregularized iterative SENSE algorithm. In addition, two different reconstruction algorithms were applied: a gradient-based solver operating on the joint unconstrained function (See Algorithm 1, left column) with  $N_i = 8$ , and the proposed split formulation with the Split Bregman algorithm (See Algorithm 1, right column) with  $N'_i = 3$ and  $N_j = 5$ .

Both reconstruction pathways were combined with Total Variation regularization, Wavelet regularization and the combination of both. The regularization weights  $\lambda_t$  and  $\lambda_w$ , respectively  $\lambda_t^{SL}$  and  $\lambda_w^{SL}$ , and the iteration numbers were optimized for minimal NRMSE for each of the combinations, and were kept fixed over all data sets. They were chosen as  $\lambda_t/\lambda_w = 0.002/0.0$ ,  $\lambda_t/\lambda_w = 0.0/0.001$  and  $\lambda_t/\lambda_w = 0.002/0.001$ . The chosen iteration numbers were  $N_i = 5$  for iterative SENSE,  $N_j/N_i' = 5/3$  for Split Bregman, and  $N_i = 8$  for the joint gradient based reconstruction.

To demonstrate the diagnostic usability, the data sets acquired with protocol II, including the data from the entire lower peripheral vasculature, were reconstructed with the same settings. Maximum intensity projections were generated in coronal orientation to evaluate the visualization of the vessels along the entire leg.

The parameterization of the reconstructions was chosen such that the NRMSE was minimized for each technique, but kept fixed for all volunteers. The number of iterations was chosen based on a trade-off between run-time and the convergence of the data fidelity term.



Fig. 4 Schematic illustration of the different patterns with an acceleration factor of 6, corresponding to 76 lines out of 460. Aside from the full sampling in the *first row* and the proposed MICCS pattern in the *second row*, two regular undersampled patterns with undersampled and fully sampled center, as well as two randomly undersampled

#### Data analysis

All data sets were acquired fully sampled to allow for analytical quantitative comparison of the iterative result vectors  $\mathbf{x}$  against the fully sampled directly reconstructed reference  $\mathbf{r}$  using the normalized Root Mean Square Error (NRMSE) and the Structural Similarity metric (SSIM [17]).

Both NRMSE and Structural Similarity were calculated for the stack of 2D images. For the Contrast-to-Noise Ratio and Vessel Sharpness Index, the posterior tibial and the femoral artery were chosen as vascular regions of interest  $(\mathbf{x}_{\nu})$ ; a pixel region of  $10 \times 10$  pixels within the soleus muscle was used to determine tissue signal  $(\mathbf{x}_t)$ .

$$\begin{split} \text{NRMSE}(\mathbf{r}, \mathbf{x}) &= \frac{1}{N_l} \|\mathbf{r} - \mathbf{x}\|_{L_2},\\ \text{SSIM}(\mathbf{r}, \mathbf{x}) &= \frac{1}{N_t} \left( \frac{2\mu(\mathbf{r})\mu(\mathbf{x}) + c_1}{\mu(\mathbf{x})^2 + \mu(\mathbf{r})^2 + c_1} + \frac{cov(\mathbf{x}, \mathbf{r}) + c_2}{\sigma(\mathbf{r})^2 + \sigma(\mathbf{x})^2 + c_2} \right) \text{ and}\\ \text{CNR}(\mathbf{x}_{\nu}, \mathbf{x}_t) &= \frac{1}{N_t} \left( \frac{\mu(\mathbf{x}_{\nu}) - \mu(\mathbf{x}_b)}{\sqrt{0.5(\sigma(\mathbf{x}_{\nu})^2 + \sigma(\mathbf{x}_b)^2)}} \right), \end{split}$$

where  $\sigma$  and  $\mu$  denote standard deviation and mean value. As proposed by Li et al. [18], the Vessel Sharpness Index was evaluated on different vessels in subsequent slices, by patterns with undersampled and fully sampled center, are shown. Representative image results for the in vivo study corresponding to these patterns are shown for volunteers 1 and 2, along with a *magnified view* 

calculating the location of the 20 % and 80 % quartiles of the maximal vascular signal and the inverse distance.

#### Results

Experiment I: Evaluation of the MICCS sampling strategy

Quantitative results for the different pattern choices are given in Table 1, and representative image results for two volunteers are illustrated in Fig. 4. The image results showed an increased vessel sharpness and generally enhanced image quality of the proposed MICCS pattern in comparison to the alternative patterns. The arrows indicate small vascular structures, which were well depicted with the MICCS, but barely visible using the regular or random undersampling patterns. Furthermore, noise and aliasing artifacts were visibly reduced for MICCS and the results almost entirely recovered the reference results. This visual result of better image quality corresponded well with the quantitative results of Table 1. In the following, the best non-MICCS pattern was evaluated for each quantitative measure and compared to the results from the MICCS pattern.

Normalized Root Mean Square Error (NRMSE)	Structural Similarity	Contrast-to-Noise Ratio	Vessel Sharpness
$0.000 \pm 0.000$	$1.000 \pm 0.000$	34.714 ± 19.416	$0.279 \pm 0.046$
$0.029 \pm 0.003$	$0.959 \pm 0.013$	$31.707 \pm 14.910$	$0.239 \pm 0.039$
$0.070 \pm 0.012$	$0.793 \pm 0.047$	$19.881\pm 6.885$	$0.173 \pm 0.046$
$0.061 \pm 0.012$	$0.830 \pm 0.048$	$21.674 \pm 8.461$	$0.198 \pm 0.050$
$0.050 \pm 0.006$	$0.854 \pm 0.039$	$25.112\pm9.832$	$0.177 \pm 0.044$
$0.056 \pm 0.008$	$0.882\pm0.024$	$22.378 \pm 7.422$	$0.171 \pm 0.041$
	Normalized Root Mean Square Error (NRMSE) $0.000 \pm 0.000$ $0.029 \pm 0.003$ $0.070 \pm 0.012$ $0.061 \pm 0.012$ $0.050 \pm 0.006$ $0.056 \pm 0.008$	Normalized Root Mean Square Error (NRMSE)Structural Similarity $0.000 \pm 0.000$ $1.000 \pm 0.000$ $0.029 \pm 0.003$ $0.959 \pm 0.013$ $0.070 \pm 0.012$ $0.793 \pm 0.047$ $0.061 \pm 0.012$ $0.830 \pm 0.048$ $0.050 \pm 0.006$ $0.854 \pm 0.039$ $0.056 \pm 0.008$ $0.882 \pm 0.024$	Normalized Root Mean Square Error (NRMSE)Structural SimilarityContrast-to-Noise Ratio $0.000 \pm 0.000$ $1.000 \pm 0.000$ $34.714 \pm 19.416$ $0.029 \pm 0.003$ $0.959 \pm 0.013$ $31.707 \pm 14.910$ $0.070 \pm 0.012$ $0.793 \pm 0.047$ $19.881 \pm 6.885$ $0.061 \pm 0.012$ $0.830 \pm 0.048$ $21.674 \pm 8.461$ $0.050 \pm 0.006$ $0.854 \pm 0.039$ $25.112 \pm 9.832$ $0.056 \pm 0.008$ $0.882 \pm 0.024$ $22.378 \pm 7.422$

Table 1 Quantitative image and Vessel Sharpness Index evaluation for the in vivo pattern study

The reference is used to calculate the NRMSE and the Structural Similarity for all patterns. The Contrast-to-Noise Ratio between vessel and background, and vessel and tissue, as well as the Vessel Sharpness Index, are evaluated for all seven reconstruction results

 Table 2 Quantitative image and Vessel Sharpness Index evaluation for the in vivo algorithm study

Method	Normalized Root Mean Square Error (NRMSE)	Structural Similarity	Contrast-to-Noise Ratio	Vessel Sharpness
Reference	$0.000\pm0.000$	$1.000 \pm 0.000$	34.714 ± 19.416	$0.279 \pm 0.046$
Iterative SENSE	$0.0652 \pm 0.102$	$0.817\pm0.036$	$26.304 \pm 06.527$	$0.178 \pm 0.034$
Split Bregman (TV)	$0.0153 \pm 0.004$	$0.991 \pm 0.003$	$46.140 \pm 23.372$	$0.296 \pm 0.067$
Split Bregman (W)	$0.0715 \pm 0.020$	$0.788 \pm 0.002$	$24.326 \pm 05.253$	$0.186 \pm 0.034$
Split Bregman (TV+W)	$0.0152 \pm 0.004$	$0.991 \pm 0.003$	$45.950 \pm 22.714$	$0.300\pm0.072$
Joint grad. based (TV)	$0.0400 \pm 0.008$	$0.931 \pm 0.020$	$33.619 \pm 10.502$	$0.182\pm0.040$
Joint grad. based (W)	$0.051\pm0.009$	$0.885\pm0.028$	$28.630 \pm 13.286$	$0.164 \pm 0.056$
Joint grad. based (TV+W)	$0.0405 \pm 0.008$	$0.929 \pm 0.001$	$33.396 \pm 10.278$	$0.182\pm0.039$

The reference is used to calculate the NRMSE and the Structural Similarity for all patterns. The Contrast-to-Noise Ratio between vessel and background, and vessel and tissue, as well as the Vessel Sharpness Index, are evaluated for all seven reconstruction results

Evaluating the NRMSE, the Random 1 pattern reached the lowest value among the non-MICCS patterns, with NRMSE =  $0.050 \pm 0.006$ . The MICCS pattern decreased this NRMSE by 42 % to  $0.029 \pm 0.003$ .

Regarding the Structural Similarity, the Random 2 pattern performed best among the non-MICCS patterns (SSIM =  $0.882 \pm 0.024$ ), which was increased with MICCS by 8.7 % to 0.959  $\pm$  0.013, again signifying a closer fit to the reference reconstructions.

The Contrast-to-Noise Ratio values for MICCS reached values of  $32.401 \pm 18.225$ , which corresponded to an increase of 22 % in relation to the best non-MICCS pattern, Random 1. The Contrast-to-Noise Ratio values, however, varied significantly over the data sets.

While the best non-MICCS patterns for NRMSE, Structural Similarity and Contrast-to-Noise Ratio were based on random sampling, the Vessel Sharpness Index was highest for regularly sampled patterns with  $0.198 \pm 0.050$  (Regular 2). The MICCS Vessel Sharpness Index was increased by 20 % to  $0.239 \pm 0.039$  and reached nearly that of the reference ( $0.279 \pm 0.046$ ). Further statistical analysis results are presented in the box plots in Fig. 6, showing the decreased interquartile range for MICCS for the NRMSE and the Structural Similarity. Experiment II: Evaluation of the MICCS pattern in combination with the Split Bregman algorithm

After the evaluation of the pattern in the previous section, the results of the combination from the MICCS pattern and the Split Bregman algorithm are presented using different combinations of regularization terms.

Table 2 summarizes the calculated quantitative results for the algorithm study, comparing both Split Bregman and gradient-based method with different regularization terms. The best results were obtained using the Split Bregman algorithm with both Wavelet and Total Variation regularization. The NRMSE was reduced by 76 % compared to the iterative SENSE method and by 40.5 % compared to the gradient-based method using the same regularization terms. The mean of the Contrast-To-Noise Ratio was increased by roughly 30 %, and Vessel Sharpness Index by 64 % compared to the joint gradient-based results. Regarding the regularization terms in combination with the Split Bregman algorithm, results with Total Variation regularization were very similar, independent of the addition of a Wavelet regularization term. The errors for the reconstruction results with only Wavelet regularization were higher, and the Contrast-To-Noise Ratio and Structural Similarity



**Fig. 5** Results of volunteers 1 and 2 using the reference reconstruction, the iterative SENSE algorithm, the Split Bregman algorithm and the joint gradient based algorithm with three different regularization

combinations. The data is undersampled using the MICCS pattern, selecting 39 phase-encoding lines from 460

Fig. 6 Statistical analysis a for the pattern experiment comparing the proposed MICCS pattern performance to the reference and four sampling strategies with similar acceleration; and b for the algorithmic experiment comparing the Split Bregman result with the reference, the iterative SENSE and the joint gradient-based results



were lower, even compared to the joint gradient-based results. In general, the differences between regularization terms were less pronounced in the gradient-based reconstructions. These results were reflected by the imagebased findings of two representative examples as shown in Fig. 5, depicting axial slices for each of the eight discussed reconstruction techniques. The Split Bregman results with Total Variation show a suppressed background noise, and accurate delineation of the vascular structures and sharp vessel borders. Finally, the maximum intensity projection results for protocol II in Fig. 7 underline the diagnostic usability of the proposed method. The results for the entire lower vasculature of volunteers 2 and 3 are shown, as obtained via the reference method and the proposed accelerated technique in Fig. 7. The smaller vessels were accurately depicted, as can easily be observed with the arrows indicating the perforating branches of the femoral artery. The vessels of the lower leg, and the anterior, posterior and peritoneal artery are indicated with red arrows. Fig. 7 Example image results for volunteers 2 and 3, comparing the fully sampled reference and the MICCS pattern + Split Bregman algorithm reconstruction. The coronal Maximum Intensity Projections are displayed with inverted intensity scale. The shifts in imaging position are indicated in *blue*, the *arrows* indicate the precise depiction of the smaller vessels, despite shorter acquisition time



The box-plot results for all 10 volunteer cases can be seen in Fig. 6b, showing the lower NRMSE and higher Structural Similarity for both the Split Bregman and gradient-based methods, but also the smaller interquartile range for the Split Bregman method (Fig. 7).

# Discussion

The scans for the entire lower vasculature with protocol II had a limited field of view, as specified in the "Experimental setup" section, to allow acquisition of the fully sampled scans in an acceptable time. This required very accurate positioning for each of the acquired steps. This resulted in a reduction of the field of view to 37.5 % of the previously used square field of view. The reported acceleration factor was therefore lower in absolute values than that previously chosen. Further parameter studies, mainly addressing the required number of iterations for each reconstruction, would be beneficial. Specific regularization strategies, such as multi-slice approaches [19], are feasible and could further reduce the amount of data required. The proposed technique should be applicable to other 2D MRA techniques, such as Quiescent Interval Single-Shot [20]. The Quiescent Interval Single-Shot technique is a single-shot acquisition, and typically one slice is acquired per heart beat. As a result, the presented reduction of sampled k-space data would not directly result in a reduced examination time. The reconstruction times for the proposed algorithm can be greatly improved using multi-threading or graphics processing unit (GPU)-based methods.

### Conclusion

The proposed data sampling strategy combines elements of SENSE and Compressed Sensing sampling strategies in different regions of k-space. The experiments presented here show, that this data sampling strategy allowed for high undersampling factors in peripheral non-contrast-enhanced MRA, with higher image quality and vessel sharpness compared to those obtained with further Compressed Sampling and SENSE sampling patterns at the same acceleration factor.

In combination with the Split Bregman method, very good results regarding image quality and vessel sharpness were achieved for all volunteers. The use of the Split Bregman algorithm led to a lower noise amplification by maintaining a good data fit and reducing ghost artifacts, even for high acceleration factors.

In summary, the study of different undersampling patterns and the combination with the Split Bregman algorithm showed significant benefits for the combination of the proposed MICCS pattern and the Split Bregman algorithm. The major advantages were the improved results for sharpness, NRMSE and SSIM, as well as the analytical and flexible generation of the MICCS pattern.

**Conflict of interest** The author Jana Hutter declares that she has no conflict of interest. Christoph Forman, Robert Grimm and Peter Schmitt are employees of Siemens AG, Healthcare Sector, Magnetic Resonance, Application Development.

**Ethical standards** The manuscript does not contain clinical studies or patient data. All volunteers gave their informed consent prior to inclusion in the study.

# Appendix: Formulation of the split problem sub-problems

Starting from Eq. (3), the inclusion of the residual errors  $\mathbf{b}_x, \mathbf{b}_y, \mathbf{b}_w \in \mathbb{C}^N$  yields the formulation in two steps, given in Eqs. (4–5).

$$\begin{aligned} (\hat{\mathbf{x}}, \hat{\mathbf{d}}_{x}, \hat{\mathbf{d}}_{y}, \hat{\mathbf{d}}_{w}) &= \operatorname*{argmin}_{\mathbf{x}, \mathbf{d}_{w}, \mathbf{d}_{x}, \mathbf{d}_{y}} \frac{1}{2} \| \mathbf{E}\mathbf{x} - \mathbf{m} \|_{L_{2}}^{2} \\ &+ \lambda_{t} \sum_{\iota=1}^{N} \sqrt{(\mathbf{d}_{x})_{\iota}^{2} + (\mathbf{d}_{y})_{\iota}^{2}} + \lambda_{w} \| \mathbf{d}_{w} \|_{L_{1}} \\ &+ \frac{\alpha \lambda_{t}}{2} \left( \left\| \mathbf{d}_{x} - \nabla_{x}\mathbf{x} - \mathbf{b}_{x}^{j} \right\|_{L_{2}}^{2} + \left\| \mathbf{d}_{y} - \nabla_{y}\mathbf{x} - \mathbf{b}_{y}^{j} \right\|_{L_{2}}^{2} \right) \\ &+ \frac{\alpha \lambda_{w}}{2} \left\| \mathbf{d}_{w} - W(\mathbf{x}) - \mathbf{b}_{w}^{j} \right\|_{L_{2}}^{2} \end{aligned}$$
(4)

$$\mathbf{b}_{x}^{j+1} = \mathbf{b}_{x}^{j} + \nabla_{x} \mathbf{x}^{j+1} - \mathbf{d}_{x}^{j+1}, \\
 \mathbf{b}_{y}^{j+1} = \mathbf{b}_{y}^{j} + \nabla_{y} \mathbf{x}^{j+1} - \mathbf{d}_{y}^{j+1}, \text{ and} \\
 \mathbf{b}_{w}^{j+1} = \mathbf{b}_{w}^{j} + W(\mathbf{x}^{j+1}) - \mathbf{d}_{w}^{j+1}.$$
(5)

The minimization problem for the L<sub>2</sub> component equals

$$\begin{aligned} (\mathbf{x}^{j+1}) &= \operatorname*{argmin}_{\mathbf{x}} \mathcal{L}_{\mathrm{SB-L2}}(\mathbf{x}, \mathbf{d}_{x}^{j}, \mathbf{d}_{y}^{j}, \mathbf{b}_{w}^{j}, \mathbf{b}_{x}^{j}, \mathbf{b}_{y}^{j}, \mathbf{b}_{w}^{j}) \text{ with} \\ \mathcal{L}_{\mathrm{SB-L2}}(\mathbf{x}, \mathbf{d}_{x}^{j}, \mathbf{d}_{y}^{j}, \mathbf{d}_{w}^{j}, \mathbf{b}_{x}^{j}, \mathbf{b}_{y}^{j}, \mathbf{b}_{w}^{j}) &= \frac{1}{2} \|\mathbf{E}\mathbf{x} - \mathbf{m}\|_{L_{2}}^{2} \\ &+ \frac{\alpha \lambda_{t}}{2} \left( \left\| \mathbf{d}_{x}^{j} - \nabla_{x}(\mathbf{x}) - \mathbf{b}_{x}^{j} \right\|_{L_{2}}^{2} + \left\| \mathbf{d}_{y}^{j} - \nabla_{y}(\mathbf{x}) - \mathbf{b}_{y}^{j} \right\|_{L_{2}}^{2} \right) \\ &+ \frac{\alpha \lambda_{w}}{2} \left\| \mathbf{d}_{w}^{j} - W(\mathbf{x}) - \mathbf{b}_{w}^{j} \right\|_{L_{2}}^{2} \end{aligned}$$
(6)

The sub-problems for the wavelet and total variation terms are formulated as

$$(\mathbf{d}_{x}^{j+1}, \mathbf{d}_{y}^{j+1}) = \underset{\mathbf{d}_{x}, \mathbf{d}_{y}}{\operatorname{argmin}} \frac{\alpha \lambda_{t}}{2} \left\| \mathbf{d}_{x} - \nabla_{x}(\mathbf{x}^{j+1}) - \mathbf{b}_{x}^{j} \right\|_{L_{2}}^{2} + \frac{\alpha \lambda_{t}}{2} \left\| \mathbf{d}_{y} - \nabla_{y}(\mathbf{x}^{j+1}) - \mathbf{b}_{y}^{j} \right\|_{L_{2}}^{2} + \lambda_{t} \sum_{\iota=1}^{N} \sqrt{(\mathbf{d}_{x})_{\iota}^{2} + (\mathbf{d}_{y})_{\iota}^{2}} \text{ and}$$

$$(7)$$

$$(\mathbf{d}_{w}^{j+1}) = \underset{\mathbf{d}_{w}}{\operatorname{argmin}} \frac{\alpha \lambda_{w}}{2} \left\| \mathbf{d}_{w} - W(\mathbf{x}^{j+1}) - \mathbf{b}_{w}^{j} \right\|_{L_{2}}^{2} + \lambda_{w} \|\mathbf{d}_{w}\|_{L_{1}}.$$
(8)

# References

- Meaney JFM (2003) Magnetic resonance angiography of the peripheral arteries: current status. Eur Radiol 13(4):836–852
- Donoho DL (2006) Compressed sensing. IEEE Trans Inf Theory 52(4):1289–1306

- Pruessmann KP, Weiger M, Scheidegger MB, Boesiger P (1999) SENSE: sensitivity encoding for fast MRI. Magn Reson Med 42(5):952–962
- Griswold MA, Jakob PN, Heidemann RM, Nittka M, Jellus V, Wang J, Kiefer B, Haase A (2002) Generalized autocalibrating partially parallel acquisitions (GRAPPA). Magn Reson Med 47:1202–1210
- Qu P, Luo J, Zhang B, Wang J, Shen GX (2007) An improved iterative SENSE reconstruction method. Concept Magn Reson B 31B:44–50
- Liu B, King K, Steckner M, Xie J, Sheng J, Ying L (2009) Regularized sensitivity encoding (SENSE) reconstruction using Bregman iterations. Magn Reson Med 61(1):145–152
- Lustig M, Donoho D, Pauly JM (2007) Sparse MRI: the application of compressed sensing for rapid MR imaging. Magn Reson Med 58(6):1182–1195
- Goldstein T, Osher S (2009) The Split Bregman method for L1-regularized problems. SIAM J Imaging Sci 2:323–343
- Aelterman J, Luong H, Goossens B, Pizurica A, Philips W (2010) COMPASS: a joint framework for parallel imaging and compressive sensing in MRI. In: Proceedings of the IEEE international conference on image processing (ICIP 2010), Hong Kong, China, Sept 26–29 2010, pp 1653–1656
- Facchinei F, Pang J (2003) Finite-dimensional variational inequalities and complementarity problems, vol I and II. Springer, Berlin
- Lustig M, Alley M, Vasanawala S, Donoho DL, Pauly JM (2009) L1SPIR-iT: autocalibrating parallel imaging compressed sensing. In: Proceedings of the annual meeting ISMRM, Honolulu, USA, April 18–24 2009, p 334
- Osher R, Fatemi E (1992) Nonlinear total variation based noise removal algorithms. Phys D 60(1–2):259–268
- Block KT, Uecker M, Frahm J (2007) Undersampled radial MRI with multiple coils. Iterative image reconstruction using a total variation constraint. Magn Reson Med 57(6):1086–1098
- Nocedal F (1980) Updating quasi-Newton matrices with limited storage. Math Comput 35(151):773–782
- Plonka G, Ma J (2011) Curvelet–wavelet regularized split bregman iteration for compressed sensing. Int J Wavelets Multires 9(1):79–110
- Ramani S, Fessler J (2011) Parallel MR image reconstruction using augmented Lagrangian methods. IEEE Trans Med Imaging 30(3):694–706
- Wang Z, Bovik A, Sheikh H, Simoncelli E (2004) Image quality assessment: from error visibility to structural similarity. IEEE Trans Image Process 13(4):600–612
- Li D, Carr JC, Shea SM, Zheng J, Deshpande VS, Wielopolski PA, Finn JP (2001) Coronary arteries: magnetization-prepared contrast-enhanced three-dimensional volume-targeted breathhold MR angiography. Radiology 219(1):270–277
- Hutter J, Grimm R, Forman C, Hornegger J, Schmitt P (2012) Vessel adapted regularization for iterative reconstruction in MR angiography. In: Pipe J (ed) Proceedings of the 21st annual meeting of the ISMRM. Melbourne, Australia, May 5–11, 2012
- Edelman R, Sheehan J, Dunkle E, Schindler N, Carr J, Koktzoglou I (2010) Quiescent-interval single-shot unenhanced magnetic resonance angiography of peripheral vascular disease: technical considerations and clinical feasibility. Magn Reson Med 63(4):951–958