A New Scale Space Total Variation Algorithm for Limited Angle Tomography

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Introduction

Limited angle tomography

- **Definition:** Scan angle \( \beta_{\text{max}} < \pi + 2 \gamma_{\text{max}} \), here \( \beta = [10^\circ, 170^\circ] \)
- **Challenge:** Data incompleteness causing artifacts
- **Technique:** Iterative reweighted total variation (wTV) [1,2]
- **Limitation:** Low frequency streaks remain
- **Proposed:** Scale space total variation (ssTV)

Materials and Methods

- **Iterative reweighted total variation (wTV):**
  \[
  \min ||f||_{wTV} \quad \text{subject to} \quad Af = P.
  \]
  Define \( ||f||_{wTV} = \sum_{x,y,z} W_{x,y,z} ||\partial f||_{L_2} \) ,
  where \( W_{x,y,z} = \frac{1}{||\partial f||_{L_2} + \epsilon} \) is the gradient image, \( \epsilon = 0.001 \).
  Define the gradient of wTV as \( g_{x,y,z} = \frac{\partial ||f||_{wTV}}{\partial f_{x,y,z}} \)

- **Scale space total variation (ssTV):**
  - **Idea:** Down-/upsampling with varying scaling factors \( s \) along direction perpendicular to streaks (anisotropy)
  - **Outer loop** (alternate data fidelity and ssTV minimization):
    - ssTV minimization factor \( s = 2 \) / ssTV minimization factor \( s = 2 \) / ssTV minimization factor \( s = 1 \)
  - **Inner loop** (gradient descent for ssTV minimization):
    - Downsample \( f \) to get \( f_s \)
    - wTV gradient \( g \)
    - Normalize \( g \)
    - Backtracking line search \( r \)
    - Update weight \( W \)

Discussion and Conclusion

- **Coarse scale** reduces low frequency streaks better
- **Fine scale** required to reduce noise and high frequency streaks
- **Scale space** successfully combines the benefits of both
- **Convenient** to implement (additional down-/upsampling only)

References


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Disclaimer
The concepts and information presented in this paper are based on research and are not commercially available.

Results

- **Phantom study:**
  - Phantom
  - wTV (\( s = 1 \))
  - ssTV, \( s = 2 \)
  - ROI
  - ssTV, \( s = 1,2 \)
  - ssTV, \( s = 1,2,3 \)

Fig. 1: Numerical phantom and reconstructions (ROIs) with different scaling factors.

Fig. 2: Comparison of different scaling factors.

Fig. 3: Two slices of the reconstructed 3-D head dataset with different algorithms.