# A New Scale Space Total Variation Algorithm for Limited Angle Tomography

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## Introduction

**Limited angle tomography**

- **Definition:** Scan angle $\beta_{\max} < \pi + 2\,\gamma_{\max}$, here $\beta = [10°,\ 170°]$
- **Challenge:** Data incompleteness causing artifacts
- **Technique:** Iterative reweighted total variation (wTV) [1,2]
- **Limitation:** Low frequency streaks remain
- **Proposed:** Scale space total variation (ssTV)

## Materials and Methods

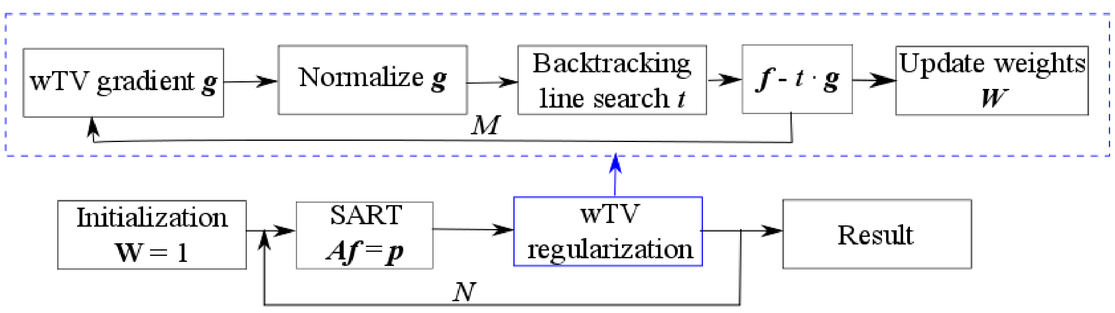
- **Iterative reweighted total variation (wTV) :**

$$\min ||\boldsymbol{f}||_{\mathrm{wTV}} \quad \text{subject to} \quad \boldsymbol{Af} = \boldsymbol{P}.$$

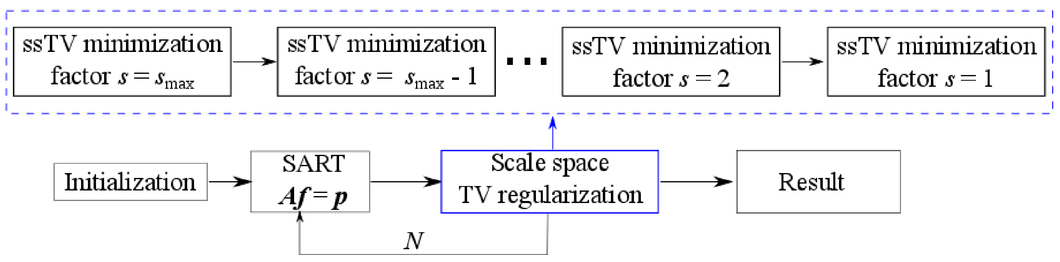
$$\text{Define } ||\boldsymbol{f}||_{\mathrm{wTV}} = \sum_{x,y,z} \boldsymbol{W}_{x,y,z} \left\|(\mathcal{D}\boldsymbol{f})_{x,y,z}\right\|_2,$$

where $\boldsymbol{W}_{x,y,z} = \frac{1}{\left\|(\mathcal{D}\boldsymbol{f})_{x,y,z}\right\|_2 + \epsilon}$, $\mathcal{D}\boldsymbol{f}$ is the gradient image, $\epsilon = 0.001$.

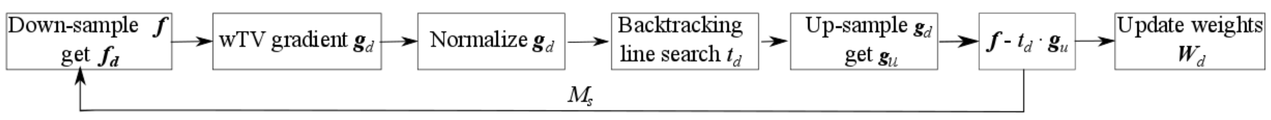
$$\text{Define the gradient of wTV as } \mathbf{g}_{x,y,z} = \frac{\partial ||\boldsymbol{f}||_{\mathrm{wTV}}}{\partial \boldsymbol{f}_{x,y,z}}.$$




- **Scale space total variation (ssTV):**
  - **Idea:** Down-/upsampling with varying scaling factors $s$ along direction perpendicular to streaks (**anisotropy**)
  - **Outer loop** (alternate data fidelity and ssTV minimization):




  - **Inner loop** (gradient descent for ssTV minimization):




## Discussion and Conclusion

- **Coarse scale** reduces **low frequency streaks** better
- **Fine scale** required to reduce noise and **high frequency streaks**
- **Scale space** successfully **combines the benefits** of both
- **Convenient** to implement (additional down-/upsampling only)

## References

[1] E. Candes et al., "Enhancing sparsity by reweighted /1 minimization," *The Journal of Fourier Analysis and Applications*, 2008.
[2] Y. Huang et al., "A new weighted anisotropic total variation algorithm for limited angle tomography," in *International Symposium on Biomedical Imaging*. IEEE, 2016.

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**Disclaimer**
The concepts and information presented in this paper are based on research and are not commercially available.

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## Results

- **Phantom study:**

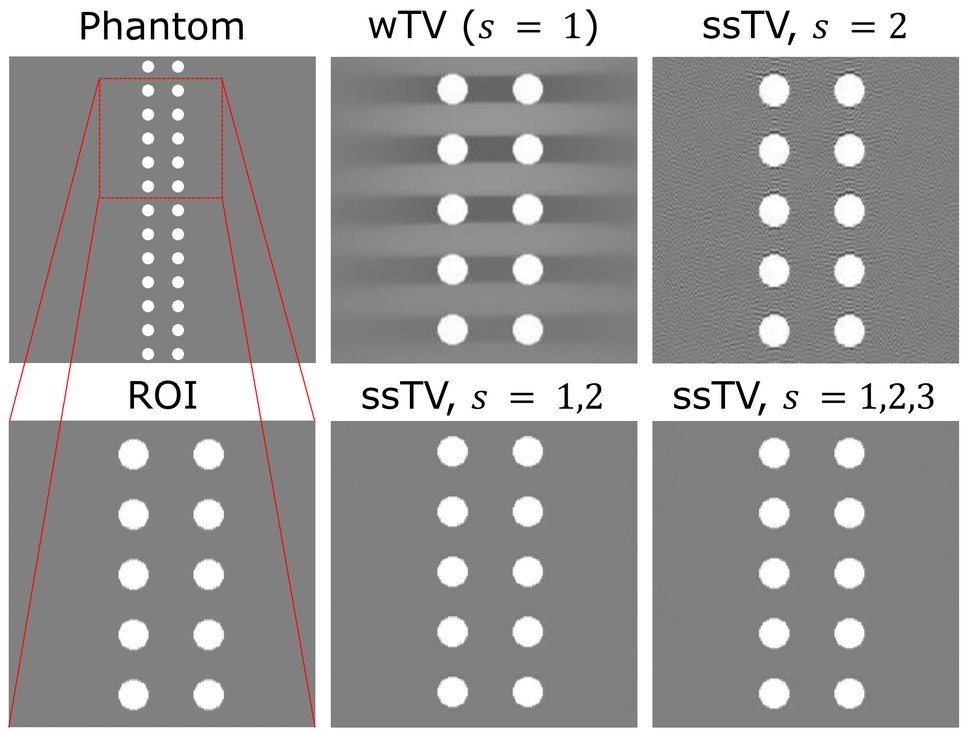


**Fig. 1:** Numerical phantom and reconstructions (ROIs) with different scaling factors.

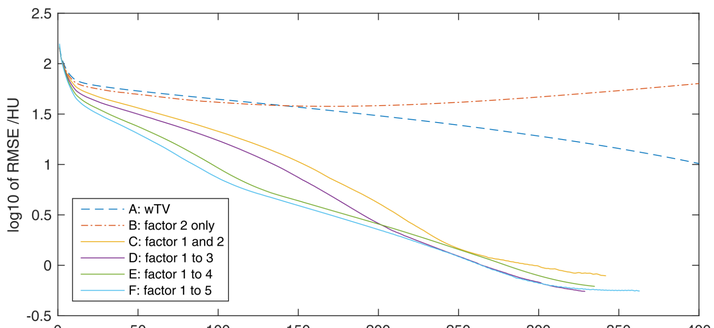


**Fig. 2:** Comparison of different scaling factors.

- **Results on clinical data:**

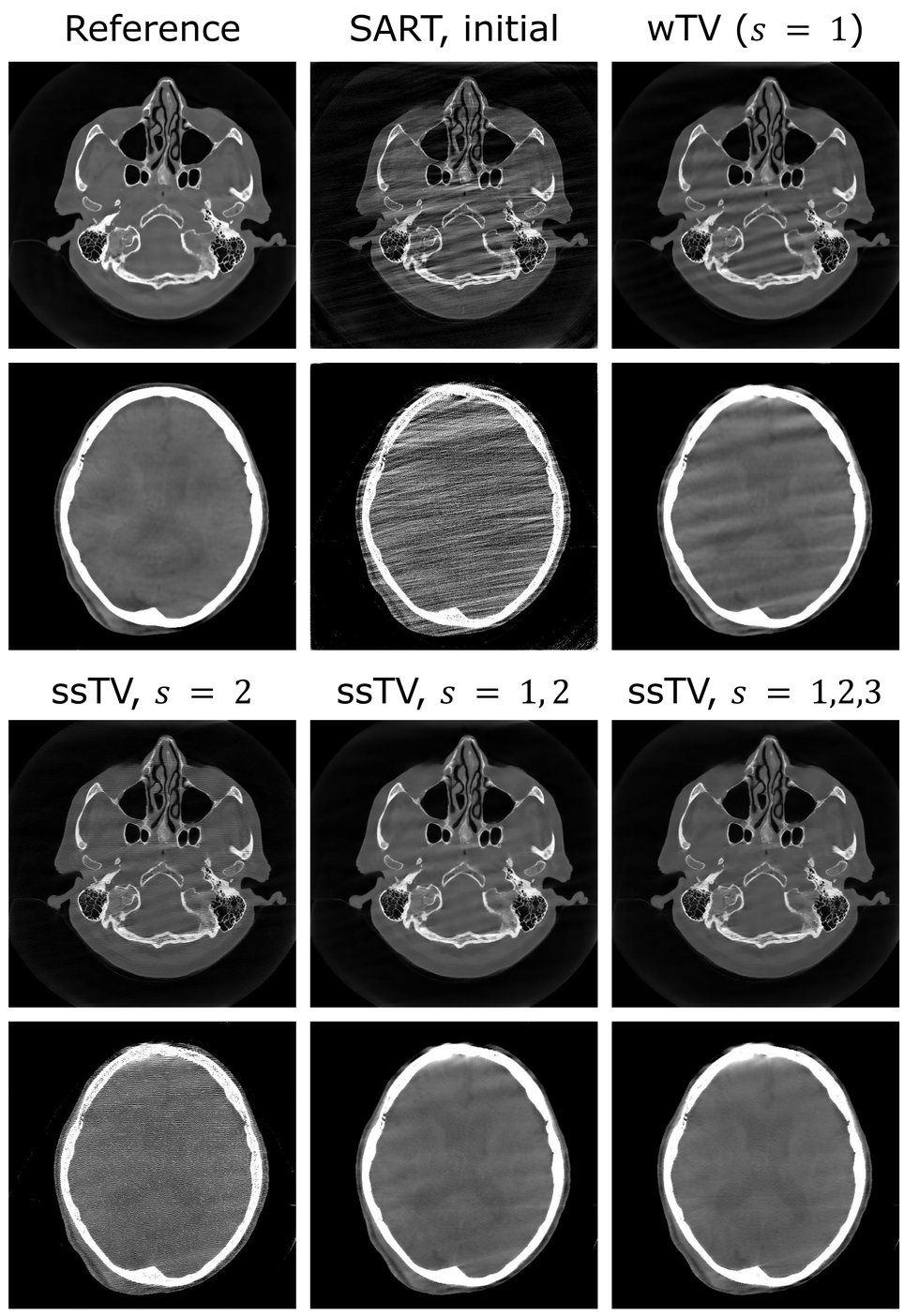


**Fig. 3:** Two slices of the reconstructed 3-D head dataset with different algorithms.