# Image Quality Analysis of Limited Angle Tomography Using the Shift-Variant Data Loss Model


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## Introduction

**Limited angle tomography**

- Definition: scan angle $< \pi + 2\gamma_{\text{max}}$
- Challenge: data incompleteness, thus streak artefacts
- Reconstruction: iterative reconstruction with total variation regularization
- Our task: Analyze the reconstructed image quality

## Materials and Methods

### The Shift-Variant Data Loss Model [1] (SVDL)

- SVDL describes data loss in frequency domain using local Fourier transform

$$\eta_1 = \arctan\left(\frac{y + D\sin\beta_{\text{min}}}{D\cos\beta_{\text{min}} - x}\right), \eta_2 = \arctan\left(\frac{x - D\cos\beta_{\text{max}}}{y + D\sin\beta_{\text{max}}}\right) + \frac{\pi}{2} \quad (1)$$

- Double wedge data loss area in frequency domain (Fig. 1)

### Iterative Reweighted Total Variation Algorithm [2] (wTV)

- Optimization model (Fig 2):

$$\min_{\boldsymbol{f}} ||\boldsymbol{f}||_{\text{wTV}} \quad \text{subject to} \quad \boldsymbol{A}\boldsymbol{f} = \boldsymbol{p} \quad (2)$$

$$||\boldsymbol{f}||_{\text{wTV}} = \sum_{x,y} \boldsymbol{W}_{x,y} ||(\mathcal{D}\boldsymbol{f})_{x,y}||_2 \qquad \text{where } \boldsymbol{W}_{x,y} = \frac{1}{||(\mathcal{D}\boldsymbol{f})_{x,y}||_2 + \epsilon} \quad (3)$$

$$\boldsymbol{g}_{x,y} = \frac{\partial ||\boldsymbol{f}||_{\text{wTV}}}{\partial \boldsymbol{f}_{x,y}} \quad (4)$$

### Simulated Phantom Design

- Phantom 1: circular areas and lines at different positions
- Phantom 2: rectangles with different sizes
- Phantom 3: rectangles with different orientation relative to scan trajectory

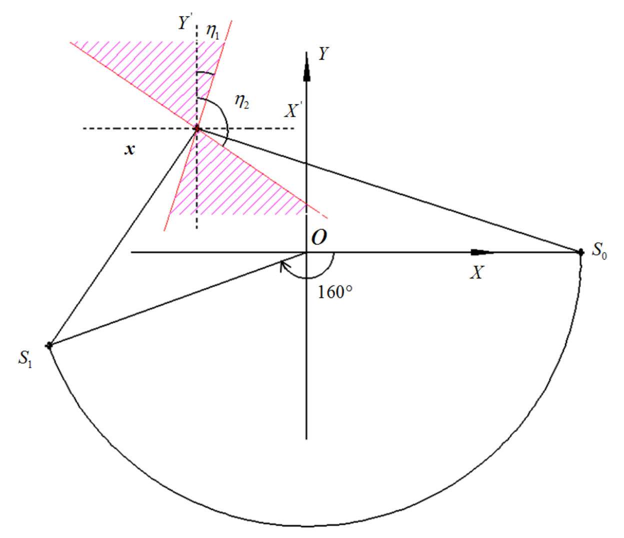

Fig. 1: SVDL model

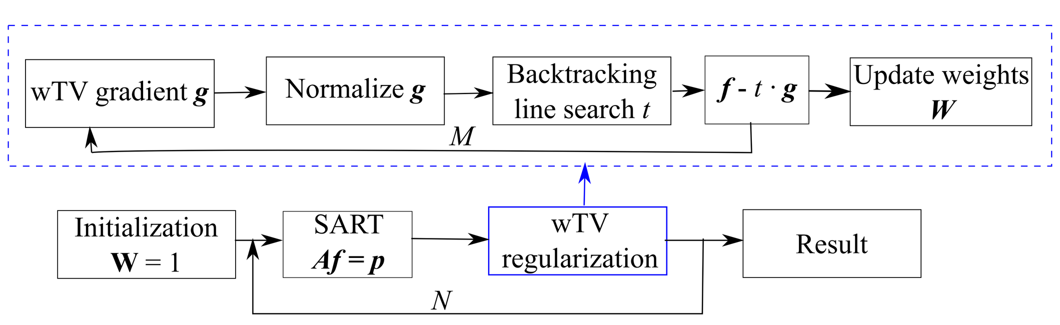


Fig. 2: wTV algorithm

## Results and Discussions

### Shape and Position Dependency

The line structure at the top position is missing while all other structures are reconstructed very well.

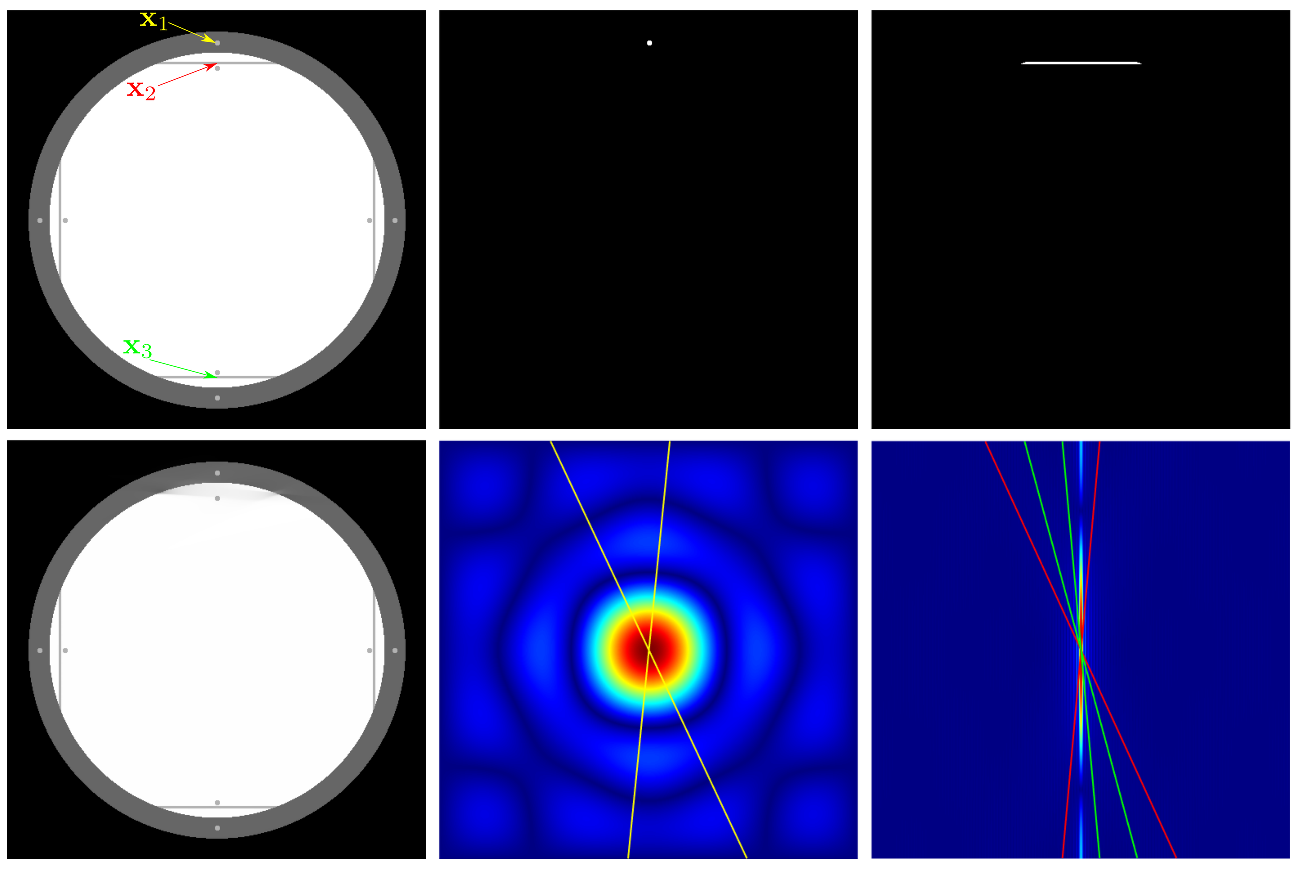

Fig. 3: Top left to right: phantom, top circular area, top line; bottom left to right: reconstruction, Fourier transform of top circular area, Fourier transform of top line. The yellow, red and green lines correspond to the double wedge boundaries of points $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$, respectively.

### Size Dependency

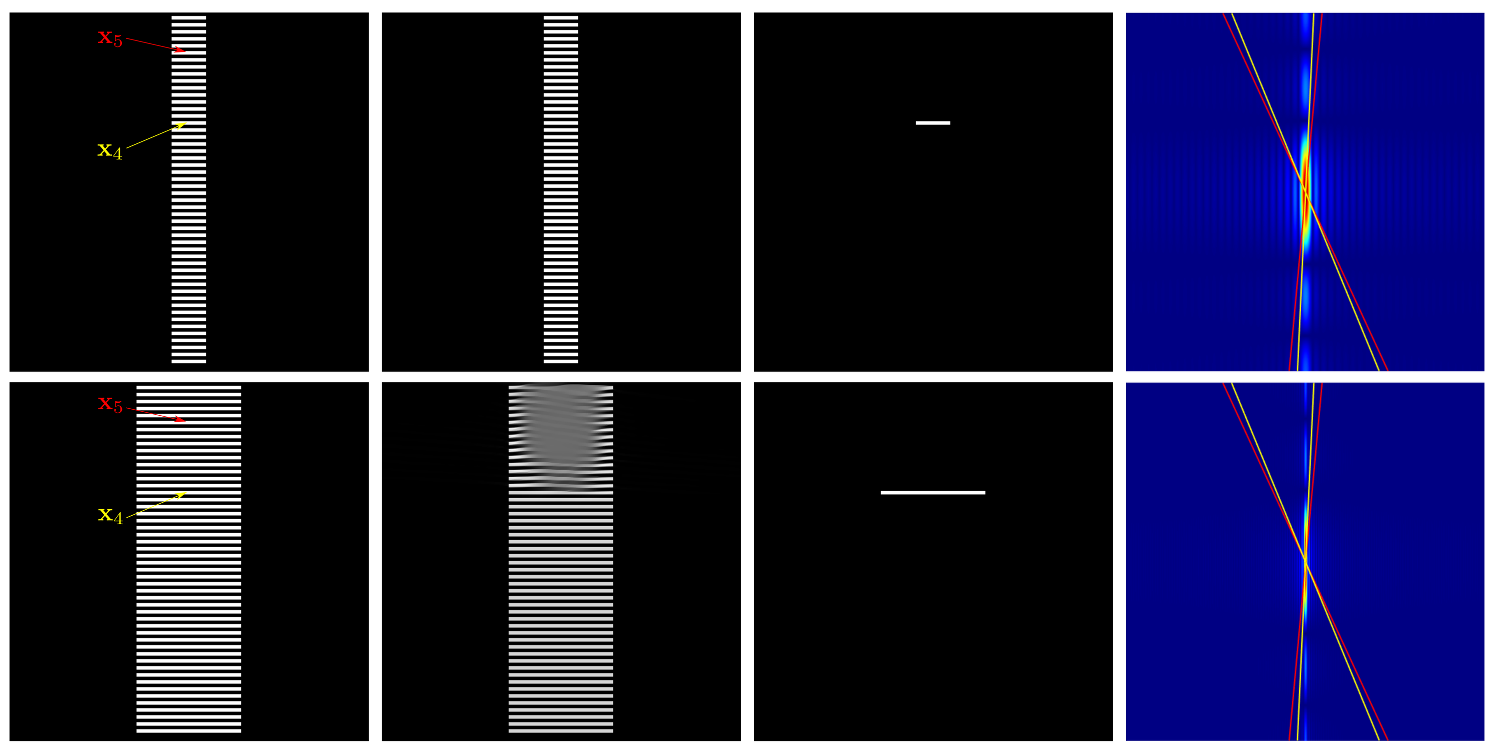

Fig. 4: Left to right: phantom, reconstruction, one rectangle, Fourier transform of the rectangle. The yellow and red lines correspond to the data loss double wedge boundaries of points $\boldsymbol{x}_4$ and $\boldsymbol{x}_5$.

### Orientation Dependency

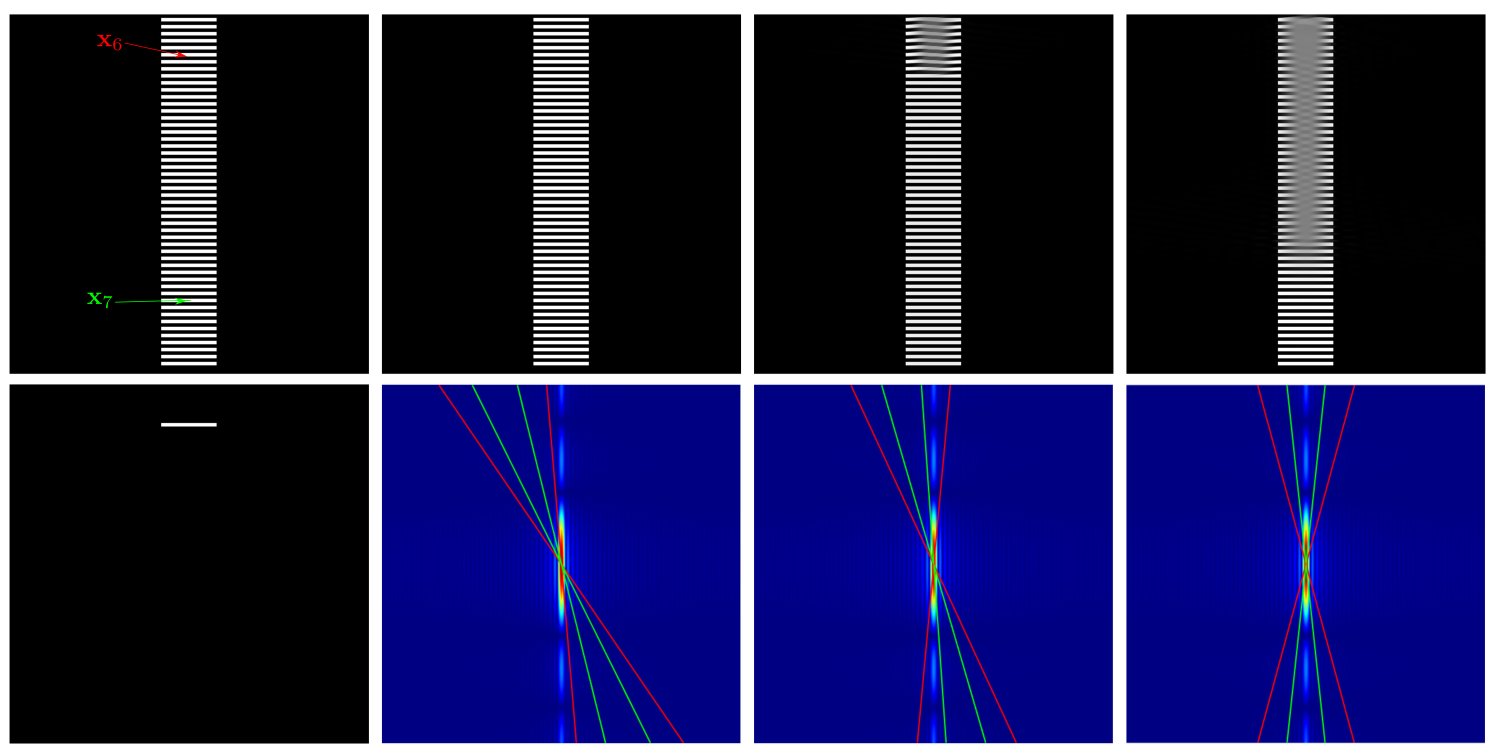

Fig. 5: Top: phantom and its reconstructions in trajectories $(\beta_{\text{min}}, \beta_{\text{max}}) = (-10^\circ, 150^\circ)$, $(0^\circ, 160^\circ)$ and $(10^\circ, 170^\circ)$, respectively. Bottom: one rectangle and the data loss double wedges in different trajectories.

## Conclusion

Reconstructed image quality depends on:

- Relationship of trajectory and object structure
  - Position
  - Shape
  - Size
  - Orientation
- Acquired frequency components

## References


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[3] Maier A. et al., CONRAD - A software framework for cone-beam imaging in radiology. Med Phys.2013;40(11):111914.


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The concepts and information presented in this paper are based on research and are not commercially available.


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