Convex Temporal Regularizers in Cardiac C-arm CT

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Clinical Setting

- Interventional 4-D (3-D+t) C-arm CT of heart chambers
- Goal: Analysis of cardiac function

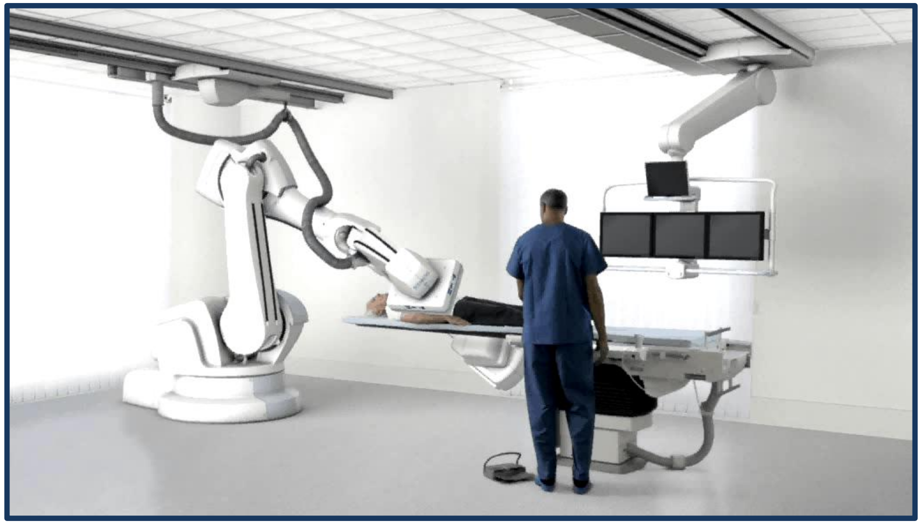


Fig.: Artis zeego multi-axis C-arm system, Siemens Healthcare GmbH, Forchheim, Germany.

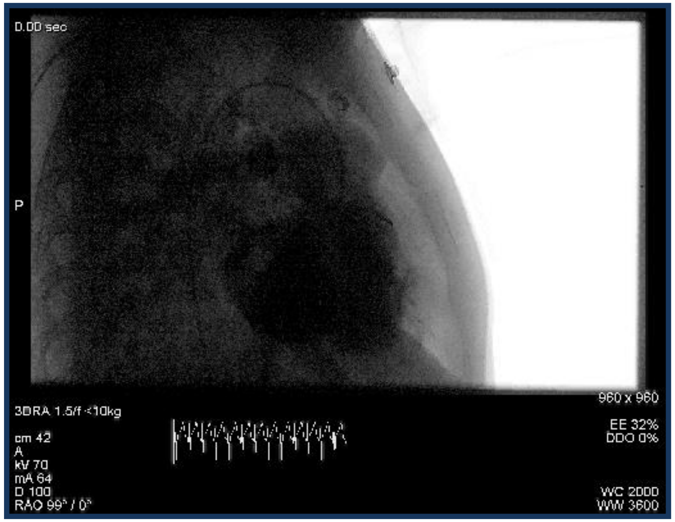


Fig.: Rotational angiogram, courtesy of Dr. Gregor Krings, University Medical Center Utrecht, Netherlands.

Gated Reconstruction

Retrospective electrocardiography (ECG) gating [1]:

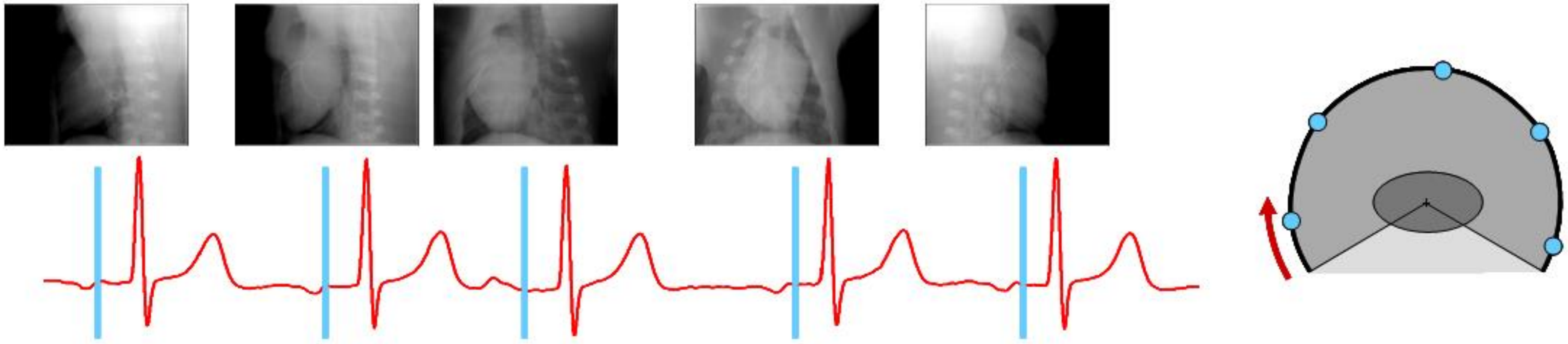


Fig.: Projection images from a C-arm sweep belonging to the same relative heart phase.

[1] Desjardin et al.: **ECG-gated Cardiac CT**, *Am J Roentgenol*, 2004

4-D Reconstruction

- Strong angular undersampling of individual phases
- But: Short scan available (albeit inconsistent)
  - Motion compensation [1,2]
  - Static guidance image for regularization [3]
  - Temporal smoothing / regularization [2,4]

[1] Müller et al.: **Image artefact propagation in motion estimation and reconstruction in interventional cardiac C-arm CT**, *PMB*, 2014

[2] Taubmann et al.: **Coping with real world data: Artifact reduction and denoising for motion-compensated cardiac C-arm CT**, *Med Phys*, 2016

[3] Chen et al.: **Prior image constrained compressed sensing (PICCS): A method to accurately reconstruct dynamic CT images from highly undersampled projection data sets**, *Med Phys*, 2008

[4] Mory et al.: **Cardiac C-arm computed tomography using a 3D + time ROI reconstruction method with spatial and temporal regularization**, *Med Phys*, 2014

Gated Iterative Reconstruction

1. Assign data to phases $t \in \{1, 2, \ldots, N_{\text{phases}}\}$
2. Initialize $\boldsymbol{I}^t = \mathbf{0}$
3. Main loop
   i. SART update
   ii. Spatial total variation (TV) reduction
   iii. One of three convex temporal regularizers:

Temporal Rank-Regularization	Temporal Total Variation Regularization	Temporal Tight Frame Regularization

Temporal Rank-Reducing Regularization

- Phases $\boldsymbol{I}^t$ not independent
  - Same object in different states
  - Underlying variation governed by few components

- Simplified assumption of few linear components:

$$\boxed{\boldsymbol{I}^{t_i}} = a \cdot \boxed{\boldsymbol{I}^{t_j}} + b \cdot \boxed{\boldsymbol{I}^{t_k}} + \cdots$$

- Principle: Rank reduction
  - But: Rank constraint yields NP-hard reconstruction problem
  - Best convex approximation $\rightarrow$ nuclear norm (sum of singular values)

Temporal Rank-Reducing Regularization

- Images arranged column-wise in a matrix

$$\boldsymbol{I} = [\boldsymbol{I}^1, \boldsymbol{I}^2, \dots, \boldsymbol{I}^{N_{\text{phases}}}] \in \mathbb{R}^{N_{\text{voxels}} \times N_{\text{phases}}}$$

- “Closest” images with reduced nuclear norm $\|\cdot\|_*$

$$\operatorname{prox}_{\|\|_*,\lambda_{\text{NN}}}(\boldsymbol{I}) = \operatorname*{argmin}_{\boldsymbol{I}'} \frac{1}{2}\|\boldsymbol{I} - \boldsymbol{I}'\|_{\text{F}}^2 + \lambda_{\text{NN}}\|\boldsymbol{I}'\|_*$$

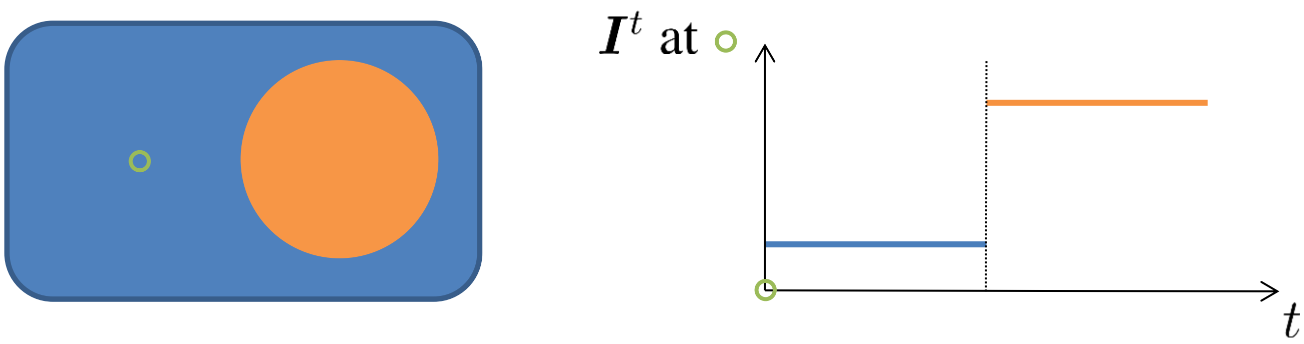
- Closed-form solution using SVD $\boldsymbol{I} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^\top$ and “soft thresholding”

$$\boldsymbol{I} \leftarrow \boldsymbol{U}(\boldsymbol{\Sigma} - \mathbf{1} \cdot \lambda_{\text{NN}})_+ \boldsymbol{V}^\top, \qquad (\cdot)_+ \equiv \max(\cdot, 0)$$

Temporal Total Variation (tTV) Regularization

- Medical images as cartoons (piecewise constant)
  - Ideal model: Reduce “jumps,” i.e. L^0 pseudo-norm of gradient
  - In practice: L^1 norm as its convex envelope ($\rightarrow$ total variation)

- Natural extension to temporal dimension:



Temporal Total Variation (tTV) Regularization

- Temporal gradient (forward differences) as sparsifying transform

$$\mathcal{D}_t \boldsymbol{I} = \left[\boldsymbol{I}^2 - \boldsymbol{I}^1, \boldsymbol{I}^3 - \boldsymbol{I}^2, \ldots, \boldsymbol{I}^1 - \boldsymbol{I}^{N_\text{phases}}\right]$$

- Iterative descent along negative gradient of TV norm w.r.t. image

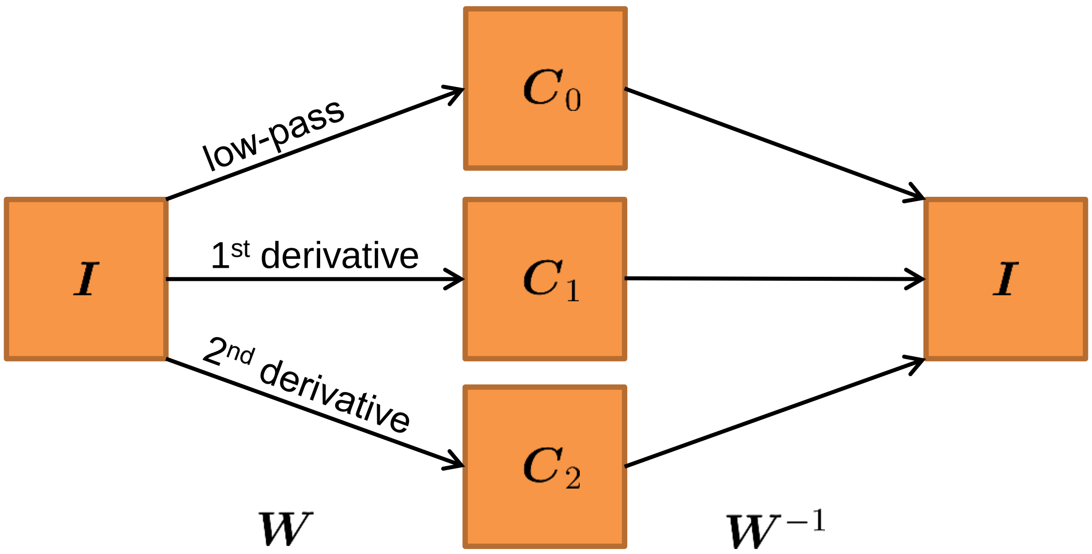
$$-\frac{\partial}{\partial \boldsymbol{I}} \|\boldsymbol{I}\|_\text{tTV} = -\frac{\partial}{\partial \boldsymbol{I}} \|\operatorname{vec}(\mathcal{D}_t \boldsymbol{I})\|_1$$ (approx. by “corner rounding”)

- Convex combination of result $\boldsymbol{I}_\text{tTV}$ to control trade-off

$$\boldsymbol{I} \leftarrow \boldsymbol{I} + \lambda_\text{tTV} \left(\boldsymbol{I}_\text{tTV} - \boldsymbol{I}\right)$$

Temporal Tight Frame (TF) Regularization

- Wavelet-based sparsifying transform as a popular alternative
- Decompose, reduce high-frequency energies, recompose:



Temporal Tight Frame (TF) Regularization

- Piecewise linear tight frame

$$\boldsymbol{h}_0 = \frac{1}{4}[1, 2, 1],\ \boldsymbol{h}_1 = \frac{\sqrt{2}}{4}[1, 0, -1],\ \boldsymbol{h}_2 = \frac{1}{4}[-1, 2, -1]$$

- Convolution along temporal dimension

$$\boldsymbol{C} = \boldsymbol{W}\boldsymbol{I} = \{\boldsymbol{C}_i : \boldsymbol{C}_i = \boldsymbol{h}_i *_t \boldsymbol{I}\}$$

- Reduction of high-pass energies $\|\boldsymbol{C}\|_{\mathrm{TF}}$

$$\|\boldsymbol{C}\|_{\mathrm{TF}} = \|\,\mathrm{vec}(\textstyle\sum_{i=1}^{2}(\boldsymbol{C}_i)^2)^{\frac{1}{2}}\|_1$$

Temporal Tight Frame (TF) Regularization

- “Closest” coefficient images with reduced $||\boldsymbol{C}||_{\mathrm{TF}}$

$$\mathrm{prox}_{||\,||_{\mathrm{TF}},\lambda_{\mathrm{TF}}}(\boldsymbol{C}) = \underset{\boldsymbol{C}'}{\mathrm{argmin}} \sum_{i=0}^{2} \frac{1}{2} ||\boldsymbol{C}'_i - \boldsymbol{C}_i||_{\mathrm{F}}^2 + \lambda_{\mathrm{TF}} ||\boldsymbol{C}'||_{\mathrm{TF}}$$

- Closed-form solution (“vector shrinkage”)

$$\boldsymbol{C}_{i,j}^t \leftarrow \begin{cases} \boldsymbol{C}_{i,j}^t & \text{if } i = 0, \\ \boldsymbol{C}_{i,j}^t \cdot \left(1 - \lambda_{\mathrm{TF}} \cdot \left(\sum_{i=1}^{2} (\boldsymbol{C}_{i,j})^2\right)^{-\frac{1}{2}}\right)_+ & \text{else} \end{cases}$$

- Update step

$$\boldsymbol{I} \leftarrow \boldsymbol{W}^{-1} \mathrm{prox}_{||\,||_{\mathrm{TF}},\lambda_{\mathrm{TF}}}(\boldsymbol{W}\boldsymbol{I})$$

Data

- Dynamic heart phantom data set [1,2]
  - Projections simulated using polychromatic X-ray spectrum
  - Ground truth reconstruction from projections of static phantom
  - 10 heartbeats

- Clinical patient data set
  - 13 heartbeats (140 bpm through external pacing)
  - 133 projection images (5 s scan, approx. 30 Hz)
  - 8 phases, 256^3 voxels (1.0 mm/voxel isotropic)

[1] Segars et al.: **4D XCAT phantom for multimodality imaging research**, *Med Phys*, 2010

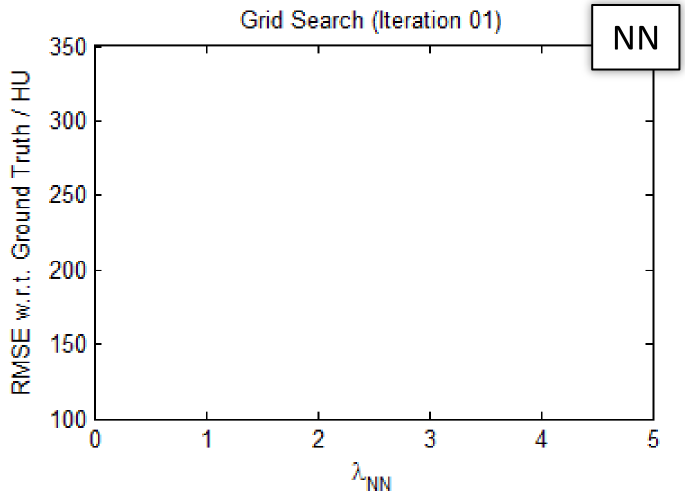
[2] Maier et al.: **Fast Simulation of X-ray Projections of Spline-based Surfaces using an Append Buffer**, *PMB*, 2012

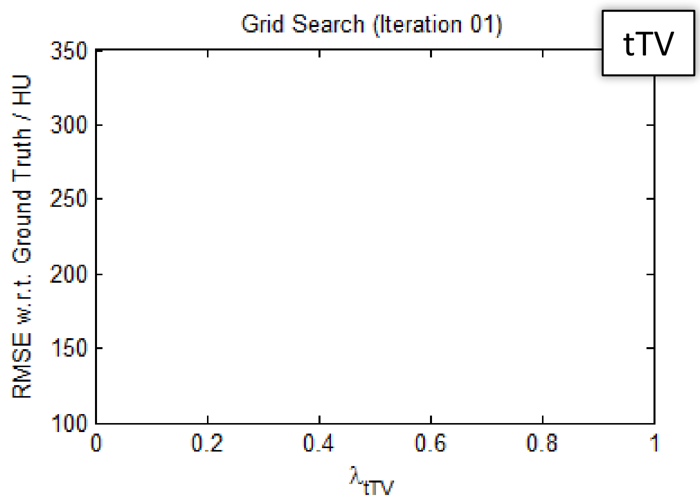
Evaluation

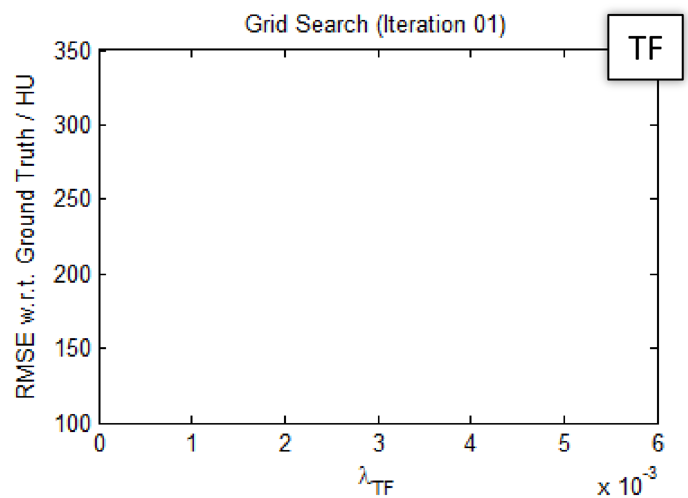
- Phantom study (quantitative, qualitative)
  - RMSE w.r.t. ground truth to assess convergence
  - Calculation restricted to ROI tightly enclosing the heart
  - Find “interesting” range of regularizer strength λ
- Clinical data (qualitative)
  - Similar behavior in real data? *(aka: does it work?)*
  - Is parameterization (somewhat) transferable?

Results

- Higher λ: Faster start, earlier saturation

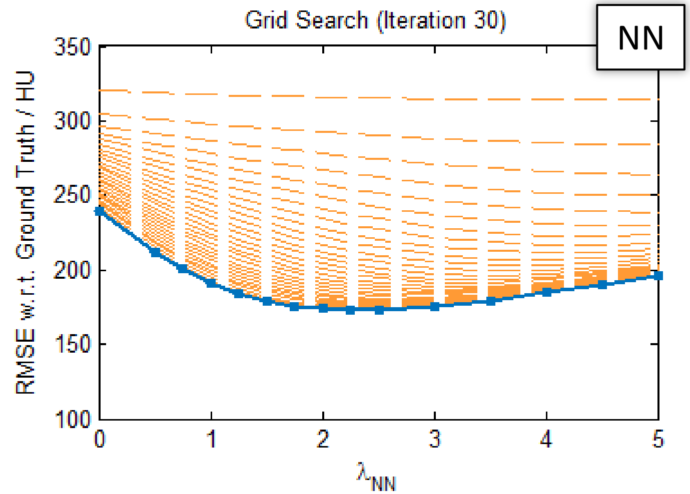


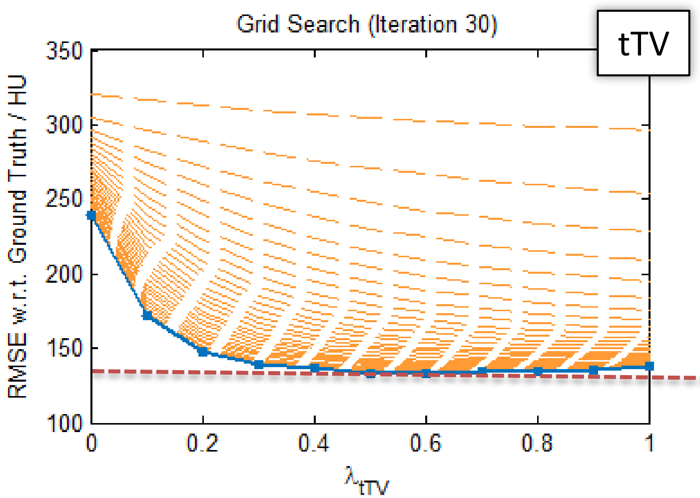


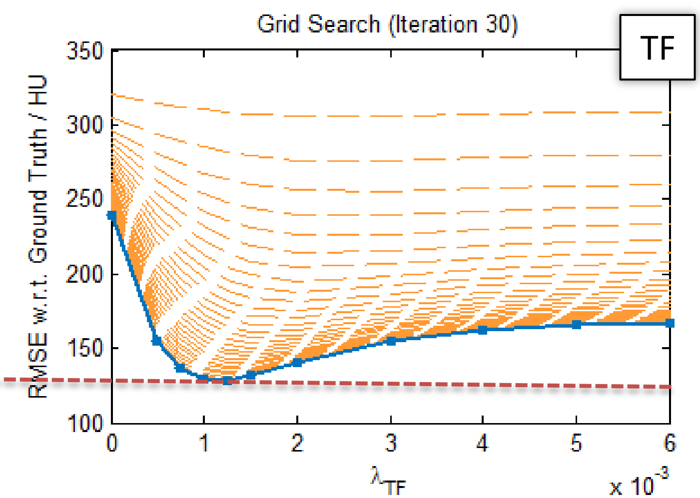


Results

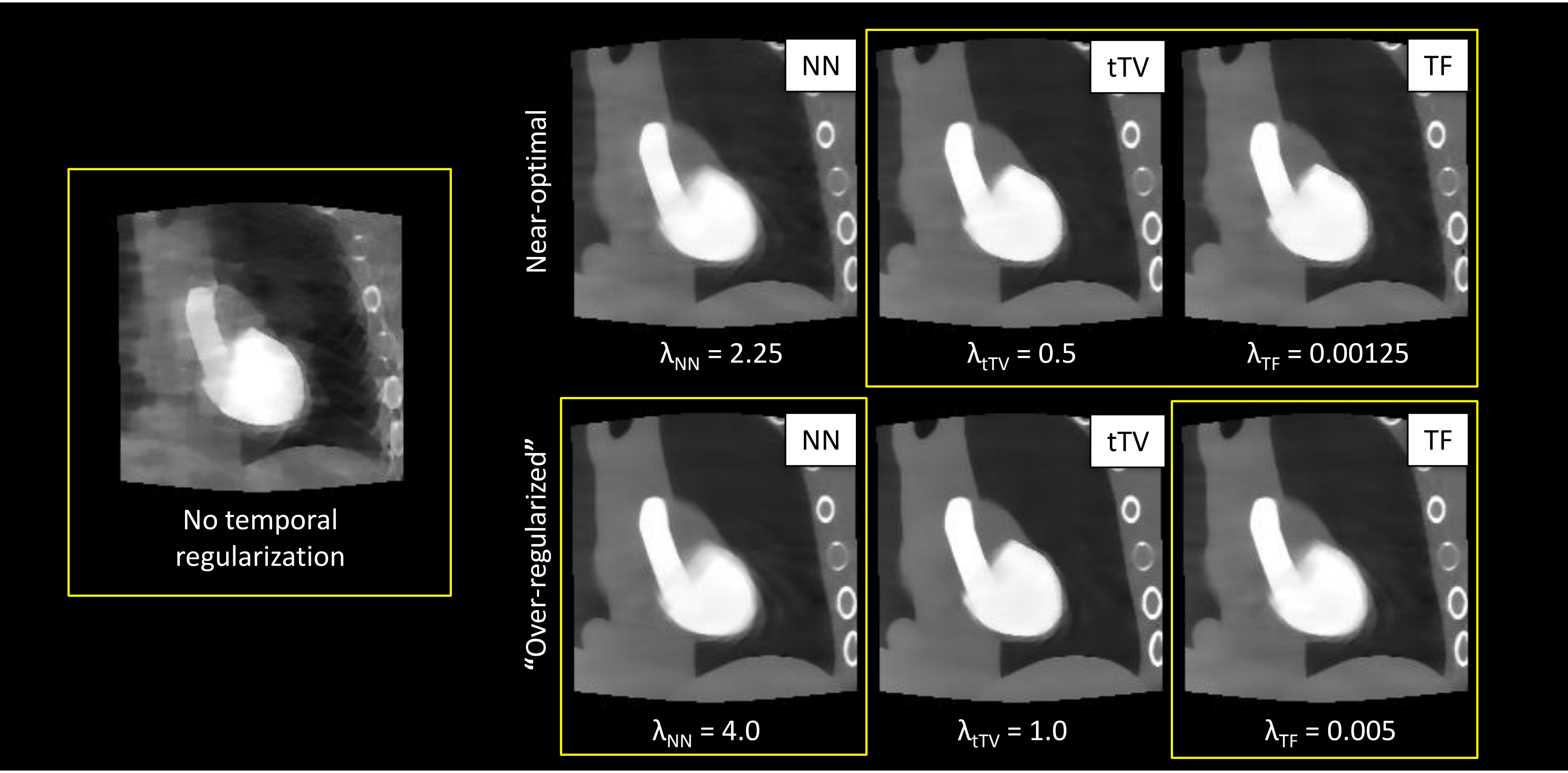
- Higher λ: Faster start, earlier saturation
- TF, tTV: Similar values near optimum
- “Valley” more pronounced for TF
- Rank-regularization least effective, yet still better than none





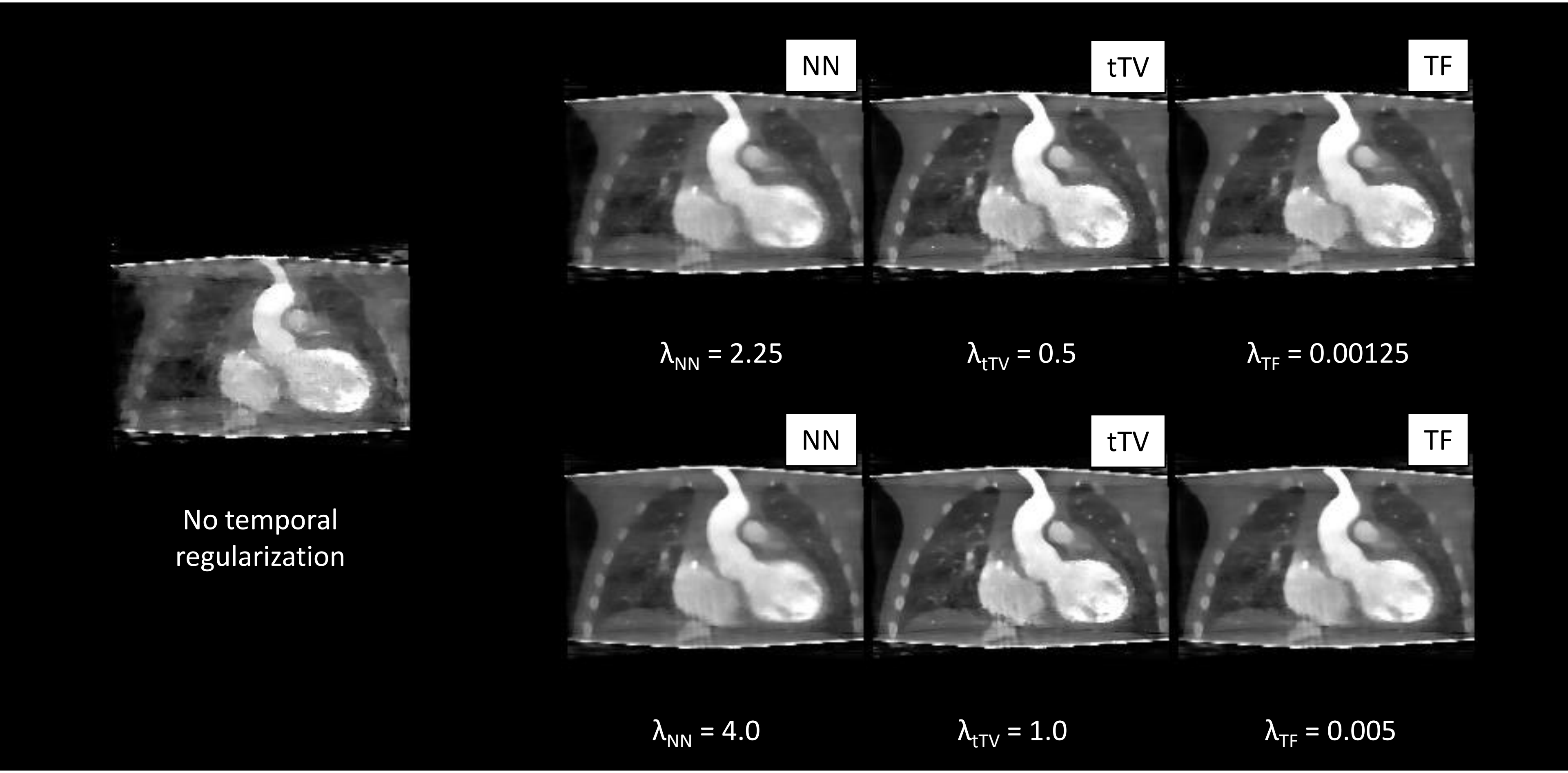


Results



No temporal regularization
Near-optimal
"Over-regularized"
NN
tTV
TF
λNN = 2.25
λtTV = 0.5
λTF = 0.00125
λNN = 4.0
λtTV = 1.0
λTF = 0.005

Results



No temporal regularization
NN
tTV
TF
λ_{NN} = 2.25
λ_{tTV} = 0.5
λ_{TF} = 0.00125
NN
tTV
TF
λ_{NN} = 4.0
λ_{tTV} = 1.0
λ_{TF} = 0.005

Discussion

- Poor performance of variant with no temporal regularization
  - Massive angular undersampling (~10-13 distinct views per phase)
  - Challenging even for compressed sensing methods

- Mediocre results for rank-regularization
  - Linear model too simplistic for our use case
  - Possibly useful at early stage for outlier suppression?

- *So, does TV reign supreme?*
  - Most forgiving w.r.t. parameter choice
  - Synergy with spatial TV?
  - But: No closed-form solution of proximal operator

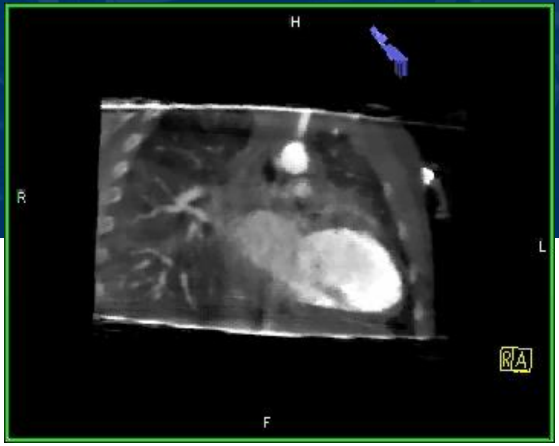
Conclusion

- Three temporal regularizers compared side-by-side:

Temporal Rank-Regularization	Temporal Total Variation Regularization	Temporal Tight Frame Regularization

  - Each better than spatial regularization only
  - Varying levels of effectiveness and parameter sensitivity
  - Similar behavior in phantom and real data (tentatively)

- Upcoming:
  - Convex optimization based on primal-dual splitting
  - Task-based assessment



Thanks for your attention!

Questions?

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