Mitigating Medialness Responses from Non-tubular Structures Using Entropy

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Purpose

Vessel segmentation and centerline extraction is relevant in clinical practice [1, 2]. Recent methods rely on minimum cost path search in a grid storing the edge cost values. Cost values are usually calculated using vesselness or medialness filters [1]. The medialness filter is contrast independent and does not require thorough parameter tuning. However, it yields high responses also at non-circular edges of bright structures, such as bones.

We propose to extend the contrast independent medialness filter introduced in [1] with a weighting factor based on the normalized Shannon entropy in order to suppress high responses from non-circular edges.

Methods

The medialness measure is a gradient-based, multi-scale method targeted at the detection of circular structures. Put concisely, the medialness response $m(\boldsymbol{x}_0)$ is highest if $\boldsymbol{x}_0$ is the center of a circle in any cross-sectional plane through $\boldsymbol{x}_0$ that is bright with respect to the background. The medialness measure is given by

$$m(\boldsymbol{x}_0) = \max_R\{\tilde{m}(\boldsymbol{x}_0, R)\} = \max_R\{\frac{1}{N}\sum_{i=0}^{N-1} E(\boldsymbol{y}_i(R))\}, \quad (1)$$

where $\boldsymbol{y}_i(R) = \boldsymbol{x}_0 + R\boldsymbol{u}(\alpha_i)$ are samples at N angles α_i, $E(\boldsymbol{y}_i(R)) \propto \max_\sigma\{\nabla_\sigma I(\boldsymbol{y}_i(R))\}$ is the normalized, contrast independent edge response of image $I(\boldsymbol{x})$ at scale σ [1], and R is the radius of the structure. The line $\boldsymbol{u}(\alpha_i) = \sin(\alpha_i) \cdot \boldsymbol{u}_1 + \cos(\alpha_i) \cdot \boldsymbol{u}_2$ lies on a cross-sectional image plane through $\boldsymbol{x}_0$ that is spanned by the basis vectors $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$.

The medialness measure yields high responses not only for circular structures but also for other objects with strong negative gradients.

In order to suppress high responses from non-circular structures while maintaining contrast independence, we propose to use the normalized Shannon entropy as a weighting factor.

In case of a circular structure, we can assume that the edge responses of sample points $E(\boldsymbol{y}_i(R))$ on a circle with radius R have approximately the same magnitude for all angular samples α_i, as we expect similar gradients at its boundaries. Then, the magnitude of the edge responses over all angular samples can be interpreted as a distribution, allowing the computation of its entropy. The entropy is related to the randomness of a distribution and is highest in case of uniform distributions [3]. To preserve normalization of the medialness, we use the normalized Shannon entropy $H(\boldsymbol{x}_0, R) = -\frac{1}{\ln(N)}\sum_{i=0}^{N-1} E\big(\boldsymbol{y}_i(R)\big)\ln(E(\boldsymbol{y}_i(R)))$ of all samples $E(\boldsymbol{y}_i(R))$ with radius R, fulfilling $\sum_{i=0}^{N-1} E\big(\boldsymbol{y}_i(R)\big) = 1$. This leads to entropies close to 1 if all angular samples yield edge responses with similar magnitude and values close to zero otherwise. For the purpose of increasing the effect of punishing non-circular structures, the quadratic entropy is multiplied to the radius-dependent medialness $\tilde{m}(\boldsymbol{x}_0, R)$ yielding the entropy-supported medialness

$$\hat{m}(\boldsymbol{x}_0) = \max_R\{\tilde{m}(\boldsymbol{x}_0, R) \cdot H^2(\boldsymbol{x}_0, R)\}. \quad (2)$$

We expect this extension to leave the response in the center of a circle largely unchanged but to mitigate the response away from the center and at non-circular boundaries.

In order to assess the performance of the novel measure quantitatively, we conduct a phantom study using a volumetric B-spline phantom based on the XCAT [4] showing the thorax with contrasted coronary arteries at diastole. Then response volumes are calculated for both the native medialness according to [1] and the entropy-supported medialness proposed here.

Finally, the average response for both cases is calculated along the arteries' centerlines, within the arteries, and within the whole volume.
Moreover, both medialness filters are applied to a four-chamber enhanced CT angiography scan of a 45 year old female.

Results

The results of the phantom study are summarized in Table 1. As expected, the average response along the centerlines and inside the arteries is only marginally affected. The mean entropy-supported medialness response of the whole volume, however, is two times smaller than for the native medialness filter. The results indicate that responses from non-tubular structures are suppressed effectively in the proposed approach. This observation is confirmed qualitatively when considering volume renderings of the response volume of the real CT angiography data set that are shown in Fig. 1.

Conclusion

The entropy-supported medialness successfully suppresses responses for structures without circular cross section such as edges while leaving responses of tubular structures such as vessels largely unaffected. The preliminary results presented here encourage a more thorough evaluation on publically available data sets [2]. We believe that the entropy-supported medialness may outperform the native medialness in scenarios where streaking artifacts degrade the image quality [5].

References

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Table 1. Quantitative results of the phantom study. The results are stated as the average response together with the corresponding standard deviation.

	Along centerline	Within arteries	Whole volume
Native	0.95 ± 0.057	0.79 ± 0.12	0.040 ± 0.082
Entropy-supported	0.95 ± 0.059	0.78 ± 0.14	0.020 ± 0.049

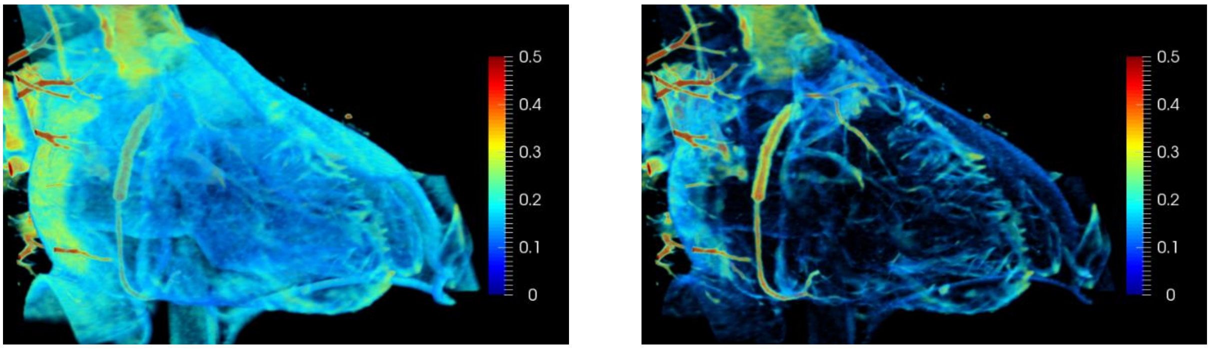

Fig. 1. Volume renderings of response volumes obtained with the native medialness filter (left) and the entropy-supported medialness (right) using the same transfer function.