# Motion Estimation in Rotational Angiography with $\alpha$ -Expansion Moves

Lina Felsner, André Aichert, Mathias Unberath, and Andreas Maier

#### Abstract

Rotational coronary angiography allows for 3D reconstruction in interventional guidance of percutaneous coronary interventions. While gated reconstruction is able to account for cardiac motion, residual breathing motion must be compensated for. Epipolar consistency has been shown to accurately extract the craniocaudal displacement of the heart during contraction and breathing. However, optimization of consistency over detector shifts proves complicated as recurrent motion of similar structures produces multiple local minima in the objective function. This prevents the use of typical local non-linear optimization methods. Related work, therefore, relies on grid-search, which is prohibitively expensive in high-dimensional parameter space. This, each projection is optimized individually, making existing methods slow and prone to unnatural jumps in the estimated motion. Therefore, we propose a new method for motion estimation using a graph-based approach. We reformulate the objective function in terms of a labeling problem and use  $\alpha$ -expansion moves to solve it. For each label, representing a specific shift, a minimum cut of a graph represents the optimal assignment of that label to each projection versus its current estimate. Our approach is aware of the local neighborhood, which makes it more efficient to optimize, as well as more robust and elegant. It is also preferable to optimize all projections at once, in contrast to the order-dependent optimization with grid-search. The robustness and reliability of the optimization is evaluated on two phantom datasets with different heart and breathing motion. Our method is more stable with respect to the selection of parameter range and sequence of optimization. Using a graph based method we further gain some flexibility in how we choose the neighborhood term.

## I. INTRODUCTION

Rotational coronary angiography allows for 3D reconstruction and has, consequently, received considerable attention in the context of interventional guidance of percutaneous coronary interventions. However, movement of the heart due to contraction and imperfect breath hold impede uncompensated 3D reconstructions of sufficient quality [1], implying the need for motion management strategies. While cardiac motion is high frequency and can be handled with gating, respiratory motion is quasi non-recurrent in standard imaging protocols and, consequently, requires compensation. In previous work, we showed that sophisticated background subtraction enables estimation of craniocaudal shifts between radiographic images [2] by optimizing for epipolar consistency (EC) [3]. The method accurately extracts the craniocaudal displacement of the heart during contraction and, more importantly, respiratory motion from background subtracted rotational sequences.

However, the optimization proved complicated as recurrent motion of similar structures produces multiple local minima in the objective function. Fig. 1 shows the cost function for one view with respect to all other views over a vertical translation of  $\pm 150$  px. The highly non-convex behavior is preventing the use of typical local non-linear optimization methods. Previous studies, therefore, relied on grid-search [2]. Due to the high-dimensional search-space, however, grid-search is prohibitively expensive and projections can only be optimized sequentially, one at a time. In consequence, previous methods are prone to inconsistent jumps in the extracted signals.

In this work, we investigate graph-based optimization, specifically  $\alpha$ -expansion moves [4], that promise improved

\*All authors are with the Pattern Recognition Lab., Computer Science Department, Friedrich-Alexander University Erlangen-Nuremberg, Germany.

\*M. Unberath and A. Maier are with the Erlangen Graduate School in Advanced Optical Technologies, Germany.



Fig. 1. Cost function for one view with respect to all other views over a vertical translation of  $\pm 150$  px.

performance, while additionally allowing for the introduction of a smoothness prior. Each expansion move is formulated as a discrete partitioning problem on a graph, which is then solved efficiently using a minimum cut for all projections at a time. We quantitatively compare the proposed method to grid-search on two numerical phantom data sets with unique cardiac and respiratory motion.

### II. MATERIAL AND METHODS

The dominant component of the respiratory motion of the heart is craniocaudal translation [5], which is conveniently aligned with the vertical detector axis and model motion as a set of detector shifts, one for each of the *n* images  $\mathcal{V} = \{v_1, ..., v_n\}$ .

EC is based on the observation that pairs of epipolar lines contain redundant information. After background subtraction [6], patient motion is the main source of inconsistency in the scan. Therefore, we can estimate shifts of the projection images by minimizing the inconsistency between images [7].



Fig. 2. Motion estimation of two phantom datasets. Estimated shifts are plotted by projection index for grid-search and  $\alpha$ -expansion moves.

Graph Cuts are a powerful tool for energy minimization with a multitude of applications in computer vision. The generic energy function E comprises two parts: (i) a data term that measures the agreement of estimates and observed data that we chose as EC; (ii) a neighborhood constraint that prefers smooth motion over time:

$$E(\mathcal{V}) = E_{data}(\mathcal{V}) + \lambda \cdot E_{prior}(\mathcal{V}), \tag{1}$$

where  $\lambda$  is a regularization weight and

$$E_{data}(\mathcal{V}) = \sum_{\substack{i=1 \ j=1 \\ j \neq i}}^{n} \text{EC}(I_i, I_j, v_i, v_j), \qquad (2)$$

with the image data  $I_i$  and  $I_j$  shifted by  $v_i$  and  $v_j$ , respectively. The neighborhood constraint is a penalty function

$$E_{prior}(\mathcal{V}) = \sum_{\{i,j\} \in \mathcal{N}} |v_i - v_j|, \qquad (3)$$

for each pair of neighbors in the neighborhood  $\mathcal{N}$ . This reflects the expected motion pattern characterized by the continuous, smooth displacements of both the cardiac and respiratory motion.

We define the optimization problem over discrete shifts in a range of  $\pm r \cdot s$  and a spacing of s, which form a set of labels



Fig. 3. Consistency images for phantom II. The consistency metric EC is evaluated for each pair of projections. Left: Consistency before (below the green line) and after (above the green line) motion estimation. Right: Consistency after one optimization for grid-search (below the green line) and alpha-expansion moves (above the green line).

 $\mathcal{L} = \{v = i \cdot s \in \mathbb{R} \mid i \in \{-r, \dots, +r\} \in \mathbb{Z}\}.$  The solution is then an optimal labeling

$$\mathcal{V}^{\star} = \operatorname{argmin} E\left(\mathcal{V}\right). \tag{4}$$

The  $\alpha$ -expansion algorithm solves a sequence of binary problems for all expanding labels  $\alpha \in \mathcal{L}$ . For each label  $\alpha$ , a minimum cut of a graph represents the optimal assignment of  $\alpha$  to each projection versus its current estimate. This way, good approximations to high-quality solutions are found in practice [8]. Since a specific expansion move uses the most recent estimate, the method needs to be iterated to convergence.

*Experiments:* We present two exemplary realizations of heart and breathing motion, simulated with the XCAT phantom [9]. Circular short scans of  $200^{\circ}$  in 5 s with 128 projections of  $960 \text{ px} \times 960 \text{ px}$  at 0.308 mm pixel spacing are simulated at realistic noise levels. The source-detector and source-isocenter distance were 1200 mm and 800 mm, respectively. We applied virtual subtraction to all images [6] and used a generous range of  $\pm 150 \text{ px}$  at 1 px spacing for both grid-search and  $\alpha$ -expansion.

Since grid-search is applied to shifts of individual projections in sequence, the solution is not necessarily optimal w.r.t. the discretization of all shifts. Several iterations over all projections are therefore performed. The scaling parameter  $\lambda$ of the energy function in Eq. 1 was set to unity.

## **III. RESULTS AND DISCUSSION**

Results are presented in Fig. 2 for grid-search and  $\alpha$ -expansion moves for both datasets, respectively. Both methods are in good agreement after two iterations. In our experiments, only minor changes were observed after the second iteration. The reduction of the inconsistency after the motion estimation is visualized in Fig. 3. The consistency value EC is computed for all pairs of views before and after the optimization, respectively (Fig. 3, left). Due to the symmetry of the metric, only the pairs with i < j are evaluated, producing the triangular appearance.

The computational time for the motion estimation with  $\alpha$ -expansion moves and grid-search is similar. For one

 $\alpha$ -expansion moves it took about 0.4 seconds on a low-end mobile hardware.

While for phantom I, the motion estimation with grid-search after one optimization has no irregularities, the dependency of grid-search on the dataset and the pre-processing can clearly be recognized for phantom II. The second optimization for phantom II is crucial due to the jump in the first heart beat. The influence of the incorrect motion estimation can also be observed in the consistency image (Fig. 3, right).

 $\alpha$ -expansion consistently underestimates the signal peaks in the first iteration. The resulting plateaus are the reason for the added stability of the algorithm. The second iteration accurately reflects the heart motion.

We observed that our method is more stable than grid-search with respect to the selection of the right parameters and sequence of optimization. Grid-search tends to run into local minima, which leads to jumps in the motion estimation.  $\alpha$ -expansion moves are resistant to jumps due to the smoothness regularizer and searching strategy. In various experiments we found the solution to be insensitive to the choice of  $\lambda$  up to a value which is unfortunately dataset dependent.

## IV. CONCLUSION

We propose a new method for motion estimation of rotational angiography. We show the reliability of the method and discussed the advantages compared to grid-search. Using a graph based method we also gain some flexibility in how we choose a neighborhood term and how we solve the problem. For example,  $\alpha$ -expansion allows us to represent relative shifts. This reduces the number of labels and makes it considerably faster while retaining the reliability of the results. Future work can further accelerate the framework by using a coarse-to-fine approach. It may be possible to incorporate gating information by defining local neighborhood in terms of heart phase.

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