## Parametric LV Model Fitting to Coronary Arteries

Tobias Geimer ${ }^{1,2,3}$, Johannes Höhn ${ }^{1}$, Mathias Unberath ${ }^{1,2}$, Andreas Maier¹,2
${ }_{1}$ Pattern Recognition Lab, Department of Computer Science, Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany
2 Erlangen Graduate School in Advanced Optical Technologies (SAOT), Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany
${ }^{3}$ Department of Radiation Oncology, Universitätsklinikum Erlangen, Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany

## Abstract

In the context of rotational coronary angiography, research focus is shifting towards 3D+t applications. Here, heart models allow for the extraction of functional parameters from the heart motion receive increasing attention. We present an approach to fit a parametric left ventricular heart model to centerlines of coronary arteries that accommodates the sparse point set conditional to the underlying angiography data. Using a coarse-to-fine optimization based on simulated annealing and ellipsoid pseudo-distances, we achieve a reprojection error of 0.794 mm compared to 0.422 mm of the 3D centerline ground truth.

## Introduction

## Rotational X-ray Angiography

- 3D+t reconstruction of arteries [1]
- Structure and movement of myocardium [2]


## Left Ventricular (LV) Parametric Heart Model

- Originally developed for tagged MRI [3]
- Ellipsoid shape described by parameter functions


## Adaption to Coronary Artery Centerlines

- No uniform distribution over the surface renders fitting complicated


## Materials and Methods

## Parameter Function Ellipsoid (PFE)

$$
\begin{aligned}
& f_{t, a_{x}, a_{y}, a_{z}, e_{x} \cdot e_{y}(u, v)}^{\left(\begin{array}{ccc}
\cos \tau(u) & -\sin \tau(u) & 0 \\
\sin \tau(u) & \cos \tau(u) & 0 \\
0 & 0 & 1
\end{array}\right)} \underbrace{\left(\begin{array}{c}
a_{x}(u) \cos u \cos v \\
a_{y}(u) \cos u \sin v \\
a_{z}(u) \sin v
\end{array}\right)}_{\text {twisting }}+\underbrace{\left(\begin{array}{c}
e_{x}(u) \\
e_{y}(u) \\
0
\end{array}\right)}_{\text {ellipsoid \& scaling }} \\
& =\underbrace{\left(\begin{array}{c}
0
\end{array}\right.}_{\text {axis offset }})
\end{aligned}
$$

with $u \in\left[-\frac{\pi}{2}, \frac{\pi}{4}\right]$ and $v \in[-\pi, \pi]$.
Equidistant piecewise linear functions (Fig. 1):

- $a_{x}(u), a_{y}(u)$ : width of minor ellipsoid axes
- $e_{x}(u), e_{y}(u)$ : offset from the principal axis
- $\tau(u) \quad:$ rotation around long axis (twist)


## Pseudo-closest Point

For a segmented 3D point $\boldsymbol{p} \in P$ on the coronary tree:

$$
\tilde{c}(\boldsymbol{p})=\binom{\operatorname{atan} 2\left(p_{z}^{\prime}, a_{z} \sqrt{\left(\frac{p_{x}^{\prime}}{a_{x}}\right)^{2}+\left(\frac{p_{y}^{\prime}}{a_{y}}\right)^{2}}\right)}{\operatorname{atan} 2\left(a_{x} p_{y}^{\prime}, a_{y} p_{x}^{\prime}\right)}
$$

with $\boldsymbol{p}^{\prime}=\boldsymbol{R}^{-1}(\boldsymbol{p}-\boldsymbol{c})$ in the ellipsoid's reference system
(a) Initial Post Estimation

- Semi-manual selection of LV base and apex in two views (supported by epipolar geometry)
- Triangulation yields center and orientation of cut-off ellipsoid
- Initial optimization of $a_{x}=a_{y}$ (further refined in (b))


## (b) Parameter Function Fitting

Coarse-to-fine optimization based on simulated annealing [4]:

$$
\underset{a_{x}, a_{y}, e_{x}, e_{y}}{\arg \min } \sum_{\boldsymbol{p} \in P}\left\|\boldsymbol{p}-f_{t, a_{x}, a_{y}, e_{x}, e_{y}}(\tilde{c}(\boldsymbol{p}))\right\|_{2}
$$

## Contact

© tobias.geimer@fau.de


The authors gratefully acknowledge funding of the Erlangen Graduate School in Advanced Optical Technologies (SAOT) by the German Research Foundation (DFG) in the framework of the German excellence initiative.

a) $a_{x}(u), a_{y}(u)$
b) $e_{x}(u), e_{y}(u)$
c) $a_{z}(u)$
d) $\tau(u)$

Figure 1: Effects of changing parameters in the ellipsoid.


Figure 2: 3D PFE for patient 1 (top) and patient 2 (bottom).
Table 1: Average fitting and reprojection error in mm for points on the LV model surface and the centerline reconstruction.

|  | patient 1 | patient 2 |
| :--- | ---: | ---: |
| 3D fitting error | 1.020 | 0.810 |
| reconstruction reprojection error | 0.532 | 0.422 |
| model surface reprojection error | 0.815 | 0.794 |

## Results and Discussion

## Data \& Evaluation

- X-ray angiography, two patients, 133 projections
- 3D left artery tree reconstruction [5], at cardiac time $t=0.1$


## Results

- Qualitative fitting results in Fig. 2
- 3D fitting error, 2D reprojection error in Table 1


## Discussion

- Initial 3D reconstruction: lower bound on the reprojection error
- Further compromised by erroneous segmentation
- Comparably small error increase for surface points


## Conclusions

- Approach to fitting a parametric LV model to coronary artery centerlines
- Direct involvement of functional heart parameters


## Future Work

$>$ Extension to 3D+t
$>$ Extraction of twist from the cardiac cycle
[1] Çimen, S. et al., Med Image Anal, 32:46-68, (2016)
[2] Frangi, A. F. et al., IEEE Trans Med Imaging, 20(1):2-5, (2001)
[3] Park, J. et al., IEEE Trans Med Imaging, 15(3):278-289, (1996)
[4] Ledersma, S. et al., Simulated Annealing, 20:401-420, (2008)
[5] Unberath, M. et al., Proc IEEE ISBI, 1143-1146, (2016)

