Parametric LV Model Fitting to Coronary Arteries

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Abstract

X-ray angiography is the gold standard in assessing coronary artery diseases. With research focus shifting towards 3D+t applications, heart models that allow for the extraction of functional parameters from the heart motion receive increasing attention. We present an approach to fit a parametric left ventricular heart model originally developed for tagged MRI to the centerlines of coronary arteries reconstructed from rotational coronary angiography. This 3D model fitting process must accommodate the sparse point set conditional to the underlying angiography data. Using a coarse-to-fine optimization based on simulated annealing and ellipsoid pseudo-distances, we achieve a reprojection error of 0.794 mm for the surface points compared to 0.422 mm of the 3D centerline ground truth. Results are promising and form the basis to extend the model to 3D+t in order to monitor radial and longitudinal contraction as well as left ventricular twist over the cardiac cycle.

I. Introduction

X-ray angiography is the diagnostic gold standard for assessing coronary artery disease. Rotational acquisition of X-ray projections of contrast-enhanced arteries on a circular source trajectory provides both high spatial and temporal resolution [1]. 3D and 3D+t reconstructions of the arteries offer further diagnostic value over direct assessment of the 2D projection images [2]. One possible application is the estimation of structure and movement of the myocardium to recover functional heart parameters [3], [4]. To this end, surface models can be fitted to the imaging data. So far, bicubic hermite splines [5] and superquadrics [6] have been used. Among these, only a few approaches explicitly model functional parameters, such as radial and longitudinal contraction and left ventricular twist [7], and often in a purely global manner. Thus, these parameters need to be derived rather than being part of the model representation.

In contrast, Park et al. [8] established a left ventricular (LV) heart model for the analysis of tagged MRI. They refine global ellipsoid parameters along the apical-basal axis, enabling them to model the twist regionally and express the shape in terms of parameter functions. We present the first step in adapting said model from MRI to 3D coronary artery centerlines reconstructed from X-ray angiography. Unlike previous work in the context of angiography, 3D+t fitting of this model is performed by optimizing physiologically meaningful parameters.

II. Material & Methods

In the following, the existing LV heart model by Park et al. is introduced [8]. Our contribution is the fitting process to the coronary artery centerlines at a particular heart phase. This fitting is complicated, as the coronary arteries are not equally distributed over the complete model surface. A list of experiments concludes the section.

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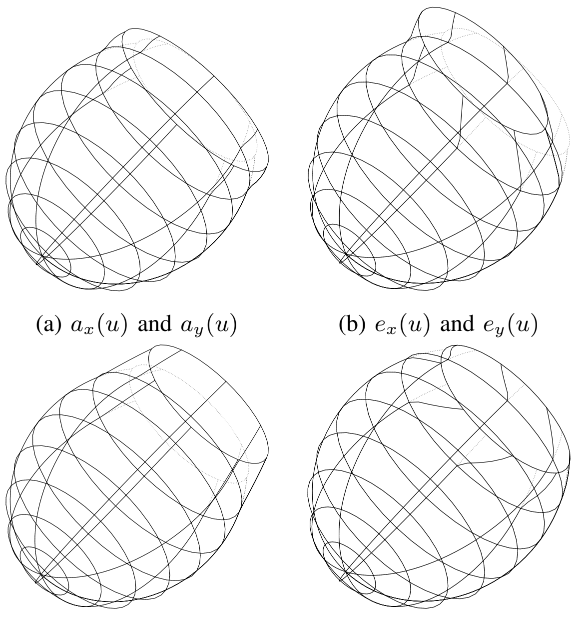
Parameter Function Ellipsoid: The parameter function ellipsoid (PFE) by Park et al. is based on an ellipsoid cut off at $u = \frac{\pi}{4}$ to represent the basis of the left ventricle [8]. Some ellipsoid parameters are replaced by parameter functions that vary along the apical-basal u-axis. Without global rigid motion the parametric representation is given by $f_{\mathbf{t},\mathbf{a}_x,\mathbf{a}_y,\mathbf{a}_z,\mathbf{e}_x,\mathbf{e}_y}(u,v) =$

$$\underbrace{\begin{pmatrix}\cos\tau(u) & -\sin\tau(u) & 0\\ \sin\tau(u) & \cos\tau(u) & 0\\ 0 & 0 & 1\end{pmatrix}}_{\text{twisting}}\underbrace{\begin{pmatrix}a_x(u)\cos u\cos v\\ a_y(u)\cos u\sin v\\ a_z(u)\sin u\end{pmatrix}}_{\text{ellipsoid and scaling}}+\underbrace{\begin{pmatrix}e_x(u)\\ e_y(u)\\ 0\end{pmatrix}}_{\text{axis offset}}$$

with $u \in [\frac{-\pi}{2}; \frac{\pi}{4}]$ and $v \in [-\pi; \pi]$. Thus, the z-axis in the PFE's reference system coincides with the apical-basal axis and the u-axis of the surface reference system.

$\tau(u)$, $a_x(u)$, $a_y(u)$, $a_z(u)$, $e_x(u)$, $e_y(u)$: $[u_{\min}; u_{\max}] \to \mathbb{R}$ are equidistant piecewise linear functions with $d_u = \frac{u_{\max}-u_{\min}}{n-1}$ being the distance between sampling points $u_i = u_{\min} + (i-1)\cdot d_u$. $a_x(u)$ and $a_y(u)$ control the length of the two minor ellipsoid axes to reflect the width of real ventricles not following a perfect sinus curve. $e_x(u)$ and $e_y(u)$ control the offset from the principal axis following the bent shape of the left ventricle. Finally, $\tau(u)$ rotates the model around the long axis, representing a twist of the ventricle. In the static case, $a_x(u)$, $a_y(u)$, $e_x(u)$, and $e_y(u)$ describe the three-dimensional shape of the left ventricle without motion as the twist can be set to zero for the initial heart phase. Fig. 1 visualizes the effect of the different parameters on the ellipsoid's shape.
To accommodate the angiography data, the model was slightly modified. Instead of u, the parameter functions interpolate over $\sin u$ such that $d_s = \frac{\sin(u_{\max})-\sin(u_{\min})}{n}$ is the distance between the sampling points $s_i = \sin(u_{\min}) + i\cdot d_s$, which are equidistant along the z-axis before transformation by $a_z(u)$. This results in a piecewise definition in z by a quadratic equation over $\sin(u)$. Unlike the original definition, this formulation can be inverted very efficiently for the fitting procedure.

The fitting process is two-fold: (a) position and orientation of a cut-off ellipsoid are determined semi-automatically using epipolar geometry and, from this initial guess, (b) the parameter functions are optimized using simulated annealing.



(a) $a_x(u)$ and $a_y(u)$ (b) $e_x(u)$ and $e_y(u)$

(c) $a_z(u)$ (d) $\tau(u)$

Fig. 1: Effects of changing parameters of the ellipsoid.

(a) Initial Pose Estimation: Given a point $\mathbf{p}$, there is usually no closed-form solution for the closest point $c(\mathbf{p})$ on the ellipsoidal surface. The pseudo-closest point $\tilde{c}(\mathbf{p})$ is a good approximation, given by:

$$\tilde{c}(\mathbf{p}) = \begin{pmatrix} \operatorname{atan2}\left(p'_z, a_z\sqrt{\left(\frac{p'_x}{a_x}\right)^2 + (\frac{p'_y}{a_y})^2}\right) \\ \operatorname{atan2}\left(a_x p'_y, a_y p'_x\right) \end{pmatrix},$$

with $\mathbf{p}' = \mathbf{R}^{-1}(\mathbf{p} - \mathbf{c})$ being $\mathbf{p}$ transformed into the ellipsoid reference system.

For initialization, an expert is asked to mark the base and apex of the LV in two views. The selection process is visually supported based on epipolar geometry, i.e. the corresponding point in the second view must be positioned on the epipolar line determined by the first point. After triangulation, the line segment between the two 3D points yields the center and orientation of the initial cut-off ellipsoid as well as the parameter a_z. Then, the only parameter left is $a_x = a_y$ which we optimise using simulated annealing and the above mentioned pseudo-distance. This initial assumption is valid, as the parameter functions will be refined in the next step.

(b) Parameter Function Fitting: With the PFE reversible in z a pseudo-distance function can be constructed. Given a point $\mathbf{p}$, there is a unique line connecting said point to the principal axis, such that the line is perpendicular to the principal axis. The intersection of this line and the PFE is the pseudo-closest point. The line and pseudo-closest point are within the plane $z = p_z$, which yields the u-parametrization. Updating the parameter functions with $\sin u$, v of the pseudo-closest point is calculated in the same way as for the normal ellipsoid.

For initialization, the parameters $\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$ are set to the values of the semi-automatic cut-off ellipsoid, $\mathbf{t}, \mathbf{e}_x, \mathbf{e}_y$ are set to zero. The PFE is fit against a set of points P by minimizing

$$\underset{\mathbf{a}_x,\mathbf{a}_y,\mathbf{e}_x,\mathbf{e}_y}{\arg\min} \sum_{\mathbf{p}}^{P} \|\mathbf{p} - f_{\mathbf{t},\mathbf{a}_x,\mathbf{a}_y,\mathbf{a}_z,\mathbf{e}_x,\mathbf{e}_y}\left(\tilde{c}\left(\mathbf{p}\right)\right)\|_2.$$

For robustness, an iterative coarse-to-fine scheme is used, increasing the parameter dimension in each step. In order to prevent overfitting to the arteries, e_x and e_y are further restricted to 10 mm in each direction.

Experiments & Evaluation: Rotational X-ray angiography scans of two patients having 133 projections each were used to reconstruct the 3D left coronary artery tree centerlines at cardiac time $t = 0.1$ [9]. For evaluation, PFE fitting (b) was applied on the semi-automatic initial guess as described in (a). Both 3D fitting error and 2D reprojection error were assessed with regards to the reconstructed centerlines and their segmentation in 2D, respectively.

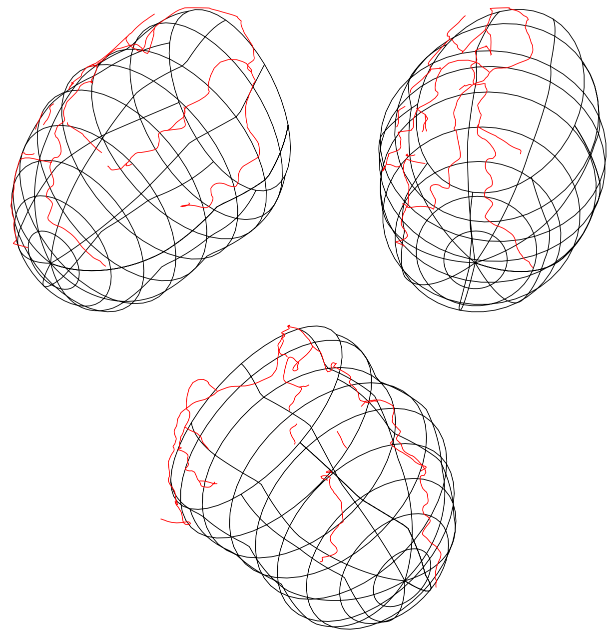


Fig. 2: 3D parameter function ellipsoid for patient 1.

TABLE I: Average fitting and reprojection error for points on the LV model surface and the centerline reconstruction.

[mm]	patient 1	patient 2
3D fitting error	1.02	0.81
reconstruction reprojection error	0.532	0.422
model surface reprojection error	0.815	0.794

III. Results

Figure 2 and 3 show the fitted PFE, in the same view as the projections that were used to reconstruct the artery tree. According to Table I, the 3D error is $1.02\,$mm for patient 1 and $0.81\,$mm for patient 2. The average reprojection errors for both the reconstructed centerline points and the corresponding surface points after fitting are also stated by Table I.

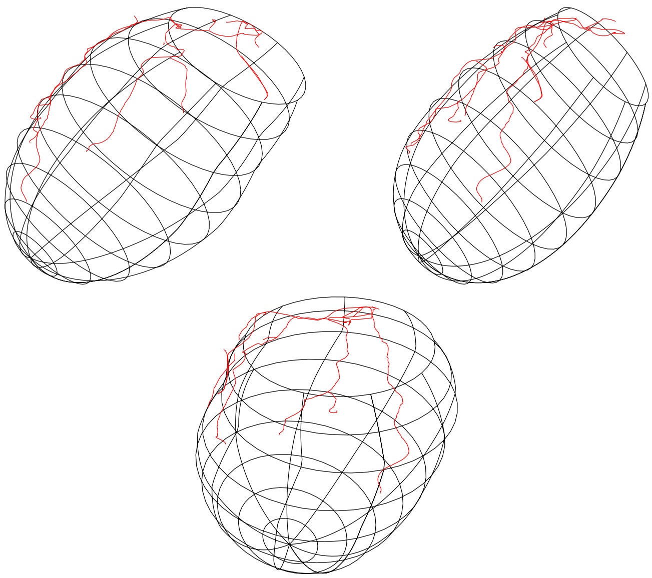


Fig. 3: 3D parameter function ellipsoid for patient 2.

IV. Discussion & Conclusion

The lower bound for the reprojection error is given by the initial 3D reconstruction, which is imperfect due to residual motion. In addition, an erroneous segmentation further compromises the optimization result [10]. With that in mind, the comparably small increase in error for the corresponding surface points of the PFE is promising.

In conclusion, we presented an approach to fit a parametric LV heart model to coronary artery centerlines, that directly carries functional heart parameters. For future work an extension of the model to 3D+t is necessary in order to extract twist and further functional parameters from the heart motion.

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