Overexposure Correction by Mixed One-bit Compressive Sensing for C-Arm CT

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Abstract. This paper proposes a novel method to deal with overexposure for C-arm CT reconstruction. The proposed method is based on recent progress of one-bit compressive sensing (1bit-CS), which is to recover sparse signals from sign measurements. Overexposure could be regarded as a kind of sign information, thus the application of 1bit-CS to overexposure correction in CT reconstruction is expected. This method is evaluated on a phantom and its promising performance implies potential application on clinical data.

1 Introduction

In the Angiographic C-arm Computed Tomography (C-arm CT), due to the limited dynamic range of C-arm flat detectors and the high contrast variation of different imaged object components, the problem of overexposure arises in the acquired projections during a 3D acquisition. Consequently, the reconstructed image, especially the low contrast structures, will be severely degraded by streak artifacts and capping artifacts due to the overexposed projection values. Thus, it is important to establish overexposure correction methods to reduce these artifacts.

The overexposure problem is similar to the truncation problem [1] in the sense of the resulting discontinuity between measured and unmeasured data and hence the truncation correction methods [2,3,4,5] are potentially feasible for overexposure correction. Generally, these methods heavily rely on the prior-knowledge about the object structure. As a specific example, [6] is to correct the overexposure for knee images based on cylinder shapes that are fitted in the sinogram domain. But such methods are no longer accurate if there is little prior-knowledge or the shapes are too complicated to be modeled.

The essence of overexposure artifacts is the lack of measurements, which inspires us to think about compressive sensing (CS). Based on sparsity, CS can recover signals/images with a relatively small number of observations. The related theory and algorithms can be found in, e.g., [7,8,9]. When overexposure occurs, the observed projection value is zero. The value itself is useless, but it implies that the true projection is less than the threshold. Thus, in overexposure correction, it is still possible to acquire some information from those projections, which is closely linked with so-called one-bit compressive sensing (1bit-CS) [10,11,12]. Our task is between CS and 1bit-CS: we have both analogy observations (un-overexposed part) and one-bit information (overexposed part). Therefore, we call our correction method as mixed one-bit compressive sensing (M1bit-CS).

In the rest of this paper, we first mathematically formulate the overexposure correction problem and give M1bit-CS method for this problem. Then the proposed method is evaluated on the Shepp-Logan phantom and the paper is concluded with some discussions.

2 Materials and Methods

2.1 Overexposure on CT projection

The X-ray transform of an object $\mathbf{f}$ is denoted by $\mathbf{R}$. Then the ideal acquired projection is

$$\mathbf{p} = \mathbf{R}\mathbf{f}. \tag{1}$$

However, due to the dynamic range of the detector, projections could be overexposed such that our observations $\mathbf{y}$ is a truncation of $\mathbf{p}$. Mathematically,

$$y_i = \begin{cases} p_i, & \text{if } p_i > s, \\ 0, & \text{if } p_i \leq s, \end{cases}$$

where s is the threshold of overexposure determined by the highest X-ray intensity that can be measured by the detector. In this paper we assume that we know which projection is overexposured, which could be modeled as a boolean indicator vector Φ:

$$\Phi_i = 1 \;\Leftrightarrow\; p_i > 0 \text{ and } y_i = 0.$$

Our aim in this paper is to reconstruct $\mathbf{f}$ from the truncated projection $\mathbf{y}$ with the above prior assumption.

2.2 (Mixed) one-bit compressive sensing

Compared to the regular CT construction, the major problem of overexposure is that we do not have the exact values for the overexposed projections. Instead, we only know that $(\mathbf{R}\mathbf{f})_i < s$ if $\Phi_i = 1$. This inequality inspires us to consider 1bit-CS, which is to recover sparse signals from sign measurements. As aforementioned, we have both un-overexposed projections and one-bit information. Therefore the following M1bit-CS model is proposed,

$$\min_{\mathbf{f}} \; \mu\|\mathbf{f}\|_{\mathrm{TV}} + \frac{1}{2}\sum_{i:\Phi_i=0} \left((\mathbf{R}\mathbf{f})_i - y_i\right)^2 + \lambda \sum_{i:\Phi_i=1} \max\left\{0, (\mathbf{R}\mathbf{f})_i - s\right\}, \tag{2}$$

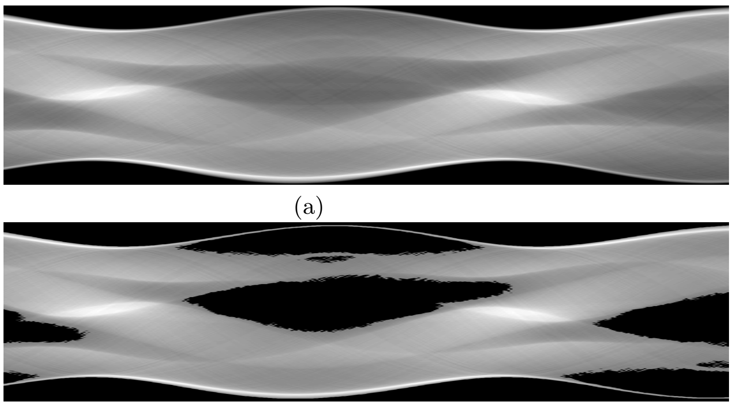
where μ and λ are relaxation parameters, $\|\cdot\|_{\mathrm{TV}}$ is the total variation term that pursues sparsity, $(\cdot)^2$ is the least squares loss to penalize the inconsistency on analogy measurers, and $\max\{0,\cdot\}$ is the hinge loss for the inequality consistency. Obviously, (2) is a convex model and can be solved by standard convex optimization methods, such as interior-point method, coordinate descent algorithm, alternating direction method of multipliers, and so on.

3 Results

3.1 Simulated Phantom Design

To demonstrate the performance of M1bit-CS method on overexposure correction, the standard high contrast Shepp-Logan phantom is employed; see, Fig. 2(a). The image size is 256×256 with an isotropic pixel length of 1 mm. We simulate a fan-beam scan to acquire the overexposed sinogram. The source-to-isocenter distance is 750 mm and isocenter-to-detector distance is 450 mm. The angular step is 1° and the total scan range is 360°. The equal-spaced detector length $s_{\max}$ is 620 mm with pixel element length $\Delta s = 1$ mm.

The ideal sinogram of the Shepp-Logan phantom is shown in Fig.1(a). Without overexposure, the classical reconstruction algorithms such as FBP [13] and SART [14] can be applied for reconstruction. However, overexposure that leads to severe information loss makes these reconstruction algorithms not applicable. To simulate the overexposure, we take the threshold $s = 0.55p_{\max}$, where $p_{\max}$ is the maximum value in the projection domain. The sinogram with overexposure is shown in Fig.1(b) and our task is to accurately reconstruct the image from this overexposed sinogram.



(a)

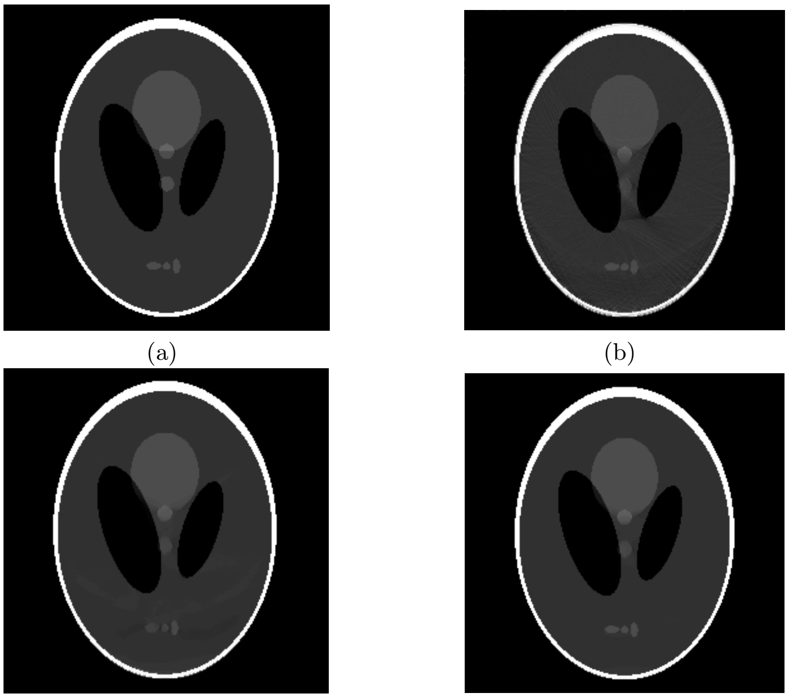
(b)

Fig. 1. (a) Projection of the Shepp-Logan; (b) Projection with overexposure.

3.2 Reconstructed image

We first consider the reconstruction performance of FBP and SART. FBP utilizes all the "fake" zeros, i.e., the overexposure projections that correspond to non-zero real values but zero observations. For SART, if we use those fake projection data, the performance will be similar to FBP. If we simply drop away them and only use the remaining un-overexposed projections, the reconstructed result is much better than FBP, as illustrated in Fig.2(b).

Recall the projections in Fig.1(b), where the overexposed part is not in the boundary. Thus, SART yields good image quality on the boundaries of the Shepp-Logan phantom. However, in the center, the performance is not satisfactory: there are streaks inside and detailed structures are blurred.



(a) (b) (c) (d)

Fig. 2. Image reconstructed from the overexposed sinogram with different algorithms: (a) Shepp-Logan phantom; (b) SART; (c) SART with TV; (d) M1bit-CS.

The result of M1bit-CS is shown in Fig.2(d). Intuitively, the reconstructed image is very close to the ground truth and the streaks are significantly reduced. The reconstruction performance can also be quantitatively measured by the the root-mean-square error, which are presented in Table 1. The gray value of the Shepp-Logan is between 0 and 1. One can see that the reconstructed image of

M1bit-CS is quite accurate, although there is serious overexposure in projections. To further highlight the effect of one-bit measurements, we also display the reconstruction result of applying SART together with TV minimization in Fig.2(c). The improvement obtained by using one-bit measurements can also be observed clearly from Table 1.

Table 1. Root of Mean Square Error (RMSE) of Reconstruction Results

method	FBP	SART	SART-TV	M1bit-CS
RMSE	0.3148	0.0242	0.0147	0.0098

4 Discussion

From the evaluation results, we can conclude that the proposed M1bit-CS method is beneficial for overexposure correction. The corresponding theory is closely linked with (one-bit) compressive sensing and is interesting to be investigated in the future. Establishing an efficient algorithm to solve (2), together with evaluation on clinical data, are both necessary before applying the method to clinical trials. In the above experiment, we assume that the overexposure positions is known. But in clinical applications, this may not be available. Thus, a good overexposure detection method is required for practical use.

References

1. Sidky EY, Kraemer DN, Roth EG, Ullberg C, Reiser IS, Pan X. Analysis of iterative region-of-interest image reconstruction for X-ray computed tomography. Journal of Medical Imaging. 2014;1(3):031007–031007.
2. Ohnesorge B, Flohr T, Schwarz K, Heiken J, Bae K. Efficient correction for CT image artifacts caused by objects extending outside the scan field of view. Medical Physics. 2000;27(1):39–46.
3. Hsieh J, Chao E, Thibault J, Grekowicz B, Horst A, McOlash S, et al. A novel reconstruction algorithm to extend the CT scan field-of-view. Medical Physics. 2004;31(9):2385–2391.
4. Maier A, Scholz B, Dennerlein F. Optimization-based extrapolation for truncation correction. In: 2nd CT Meeting; 2012. p. 390–394.
5. Xia Y, Hofmann H, Dennerlein F, Mueller K, Schwemmer C, Bauer S, et al. Towards clinical application of a Laplace operator-based region of interest reconstruction algorithm in C-arm CT. IEEE Transactions on Medical Imaging. 2014;33(3):593–606.
6. Preuhs A, Berger M, Xia Y, Maier A, Hornegger J, Fahrig R. Over-exposure correction in CT using optimization-based multiple cylinder fitting. In: Bildverarbeitung für die Medizin 2015. Springer; 2015. p. 35–40.

7. Donoho DL. Compressed sensing. IEEE Transactions on Information Theory. 2006;52(4):1289–1306.
8. Candes EJ. The restricted isometry property and its implications for compressed sensing. Comptes Rendus Mathematique. 2008;346(9):589–592.
9. Eldar YC, Kutyniok G. Compressed sensing: Theory and applications. Cambridge University Press; 2012.
10. Boufounos PT, Baraniuk RG. 1-bit compressive sensing. In: 42nd Annual Conference on Information Sciences and Systems, 2008. IEEE; 2008. p. 16–21.
11. Yan M, Yang Y, Osher S. Robust 1-bit compressive sensing using adaptive outlier pursuit. IEEE Transactions on Signal Processing. 2012;60(7):3868–3875.
12. Jacques L, Laska JN, Boufounos PT, Baraniuk RG. Robust 1-bit compressive sensing via binary stable embeddings of sparse vectors. IEEE Transactions on Information Theory. 2013;59(4):2082–2102.
13. Feldkamp L, Davis L, Kress J. Practical cone-beam algorithm. Journal of the Optical Society of America A. 1984;1(6):612–619.
14. Andersen AH, Kak AC. Simultaneous algebraic reconstruction technique (SART): A superior implementation of the ART algorithm. Ultrasonic Imaging. 1984;6(1):81–94.