Ray-Density Weighted Algebraic Reconstruction for Volume-of-Interest CT

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Introduction and Method

ABSTRACT

In computed tomography (CT), it is desirable to scan only a limited field of view in order to reduce the overall patient dose, when there is only a small region inside the patient body that is of interest. The CT reconstruction problem gets more challenging in that circumstance and typical truncation artifacts occur in the reconstructed volume, even for iterative reconstruction techniques.
Given such a volume-of-interest (VOI) problem, we achieve an improvement of image quality compared to the standard algebraic reconstruction technique (Kaczmarz method), when applying Kaczmarz's algorithm to seek the optimal truncated forward projection for the normal equation instead of the whole volume vector x for the system $Ax = b$ directly. This leads to a weighting matrix that is also known in forms of SIRT and SART algorithms. In this work, we motivate and derive this method from a new perspective and show the benefits for VOI imaging on simulated data.

CLASSICAL KACZMARZ METHOD (ART)

- basic CT problem:

  $Ax = b$ with $A \in \mathbb{R}^{M \times N}$

- ART update for each ray m (equation m):

  $$x^{k+1} = x^k - \omega A_m^{\mathrm{T}} R_m^{-1} \left(A_m x^k - b_m\right) \ , \ \omega \leq 1^{[6]}$$

- R_m can be interpreted roughly as a **ray length**:

  $$R_m = A_m A_m^{\mathrm{T}} = \| A_m \|_2^2 = \sum_{n=1}^{N} (A_{mn})^2$$

- this form implicitly minimizes $\| x \|_2^2$
  → solution avoids attenuation in outer region

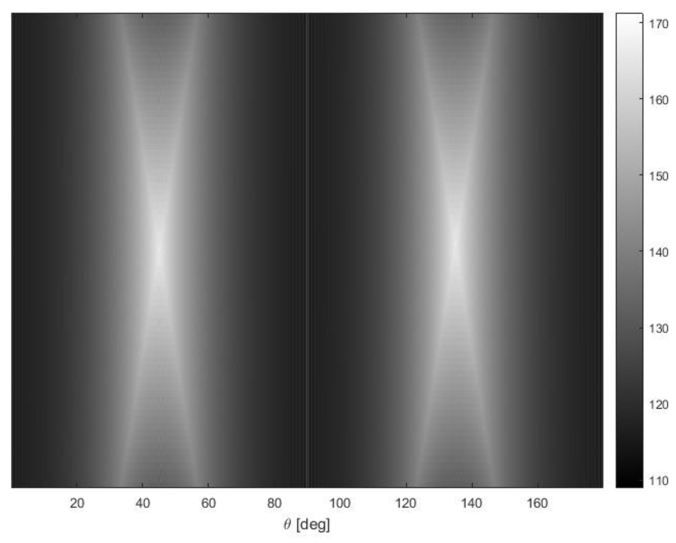

Fig. 1: Classical ray weighting R_m

DENSITY WEIGHTED ART (DWART)

- consider normal equation:

  $$A^{\mathrm{T}} A x - A^{\mathrm{T}} b = 0$$

  $$\Longleftrightarrow \boxed{B\beta = \gamma}$$

  → read as new equation system for the back projection in the whole volume x with truncated forward projection β as the unknown.

  with $B := A^{\mathrm{T}}$, $\beta := Ax$ and $\gamma := A^{\mathrm{T}} b$

- application of Kaczmarz method leads to following algorithm:

  $$x^{k+1} = x^k - \omega D^{-1} A^{\mathrm{T}} \left(A x^k - b\right)$$

  with $D = \mathrm{diag}(A^{\mathrm{T}} A) \ , \ D_{nn} = \sum_{m=1}^{M} (A_{mn})^2$,

  which can be interpreted roughly as a **ray density** within a voxel.

- This form tends to minimize the contribution of a voxel to the set of truncated projections. Solution prefers attenuation in outer region, since only a few views are contributing.

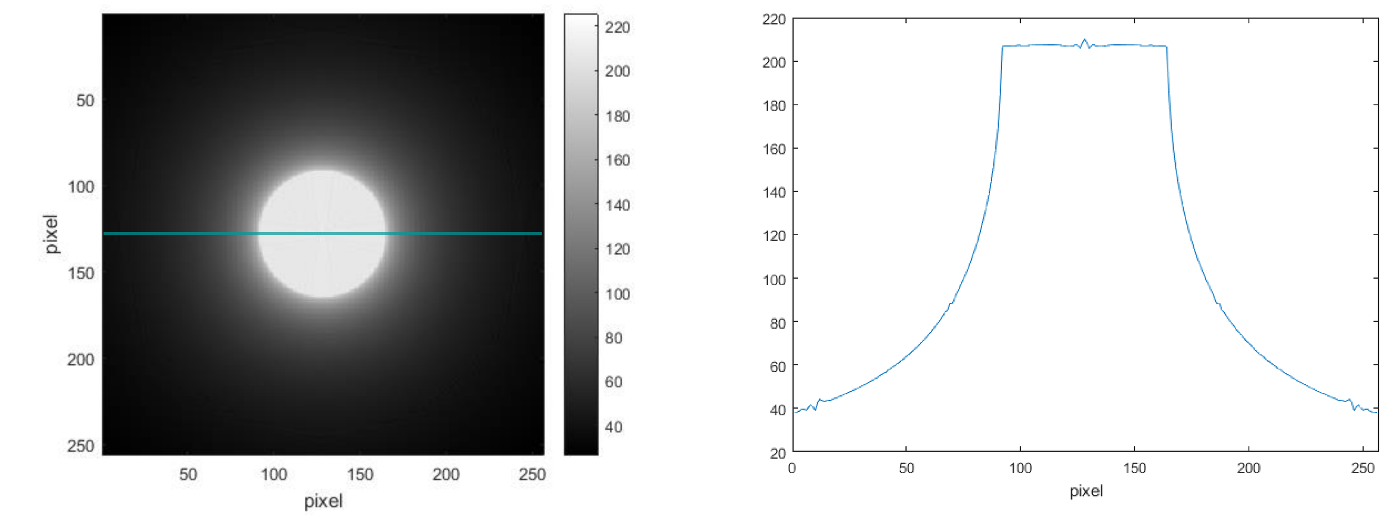

Fig. 2: Density weighting D_{nn} for a truncated projection data set

Results

INTERIOR SZENARIO

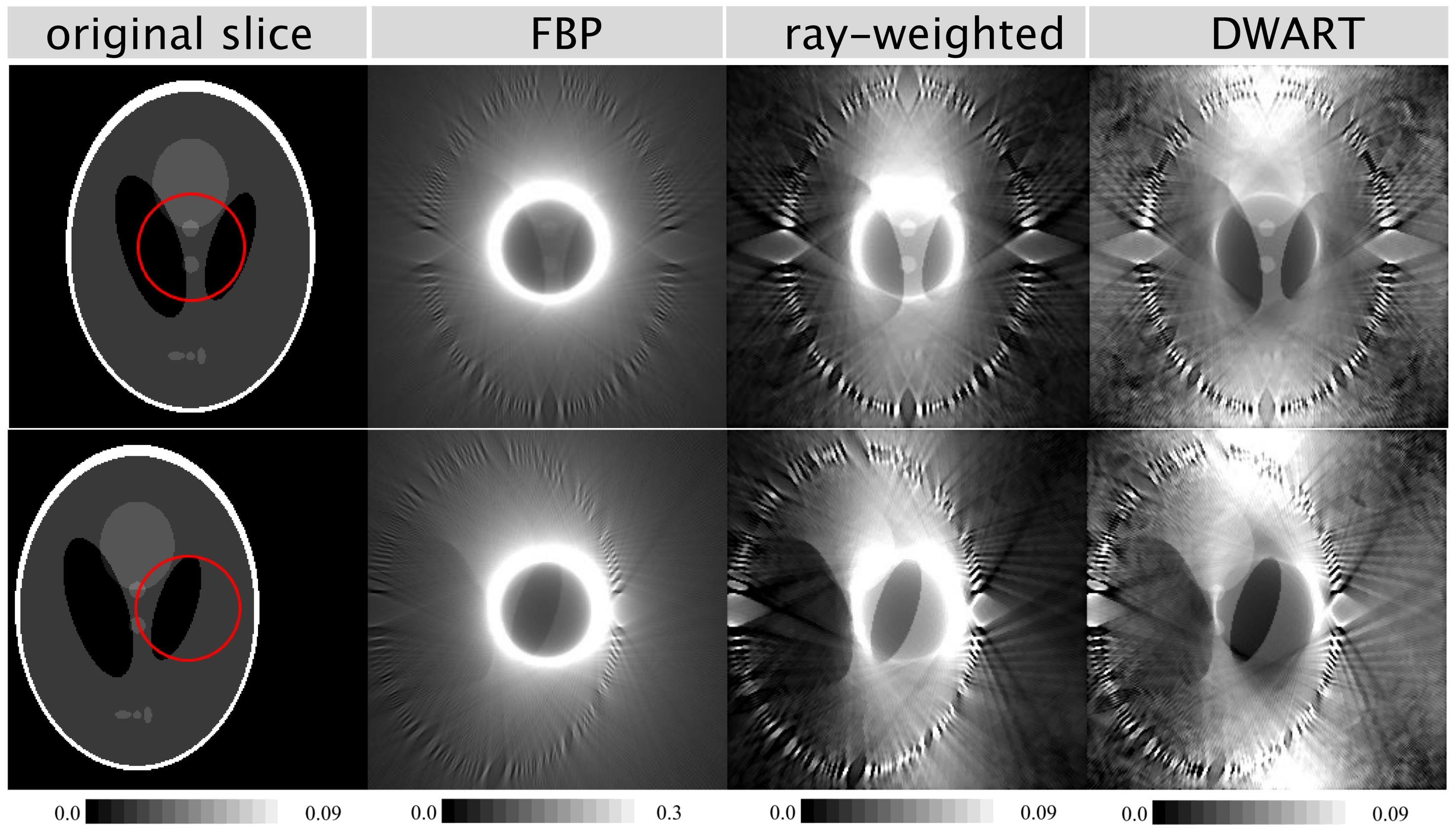

Fig. 3: Comparison of the two ART reconstructions and FBP of a Shepp-Logan phantom, which is shifted slightly to the left in the second row.

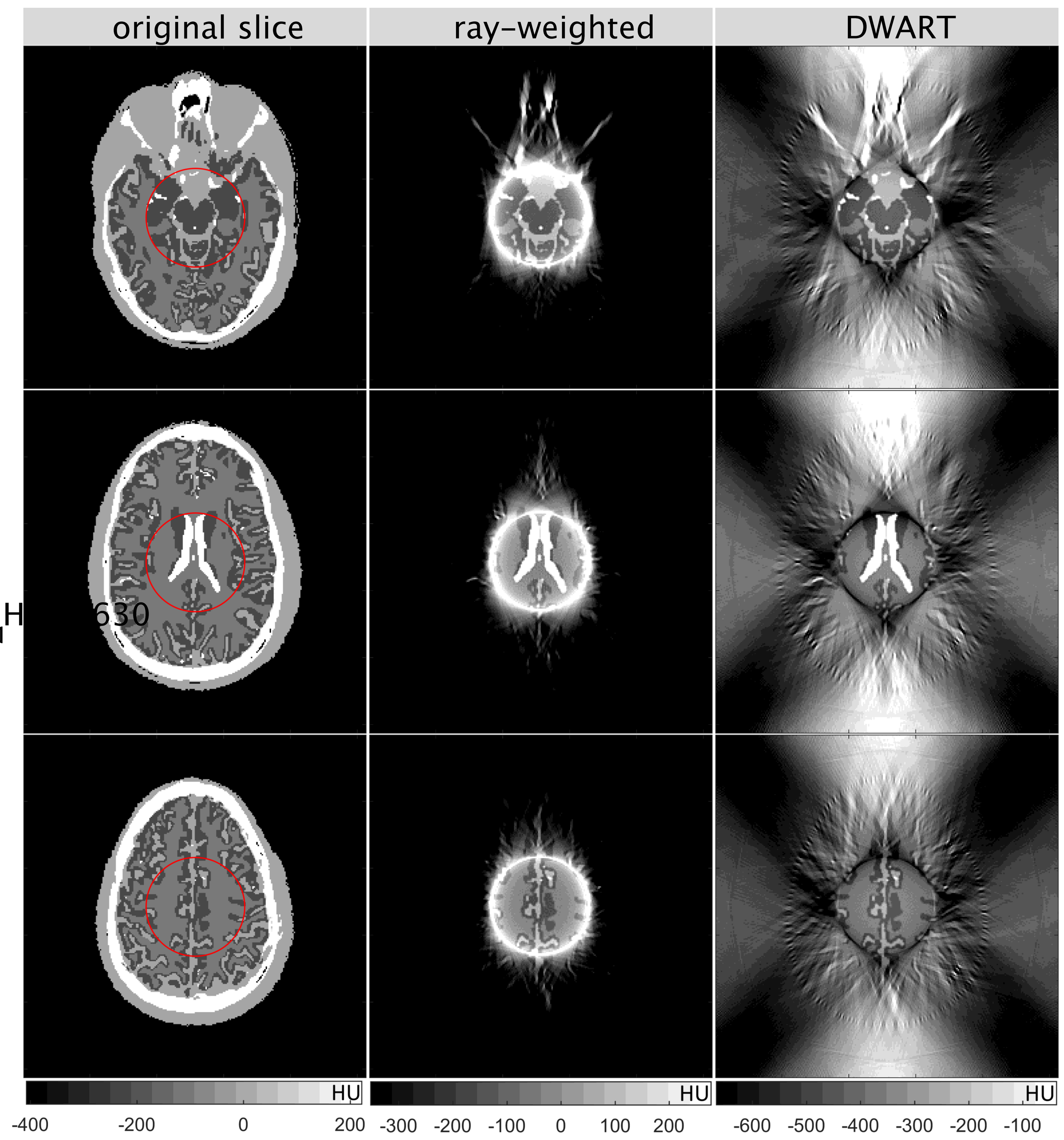

Fig. 4: Comparison of the ART recons. of an anatomical software phantom [2].

NON-INTERIOR SZENARIO

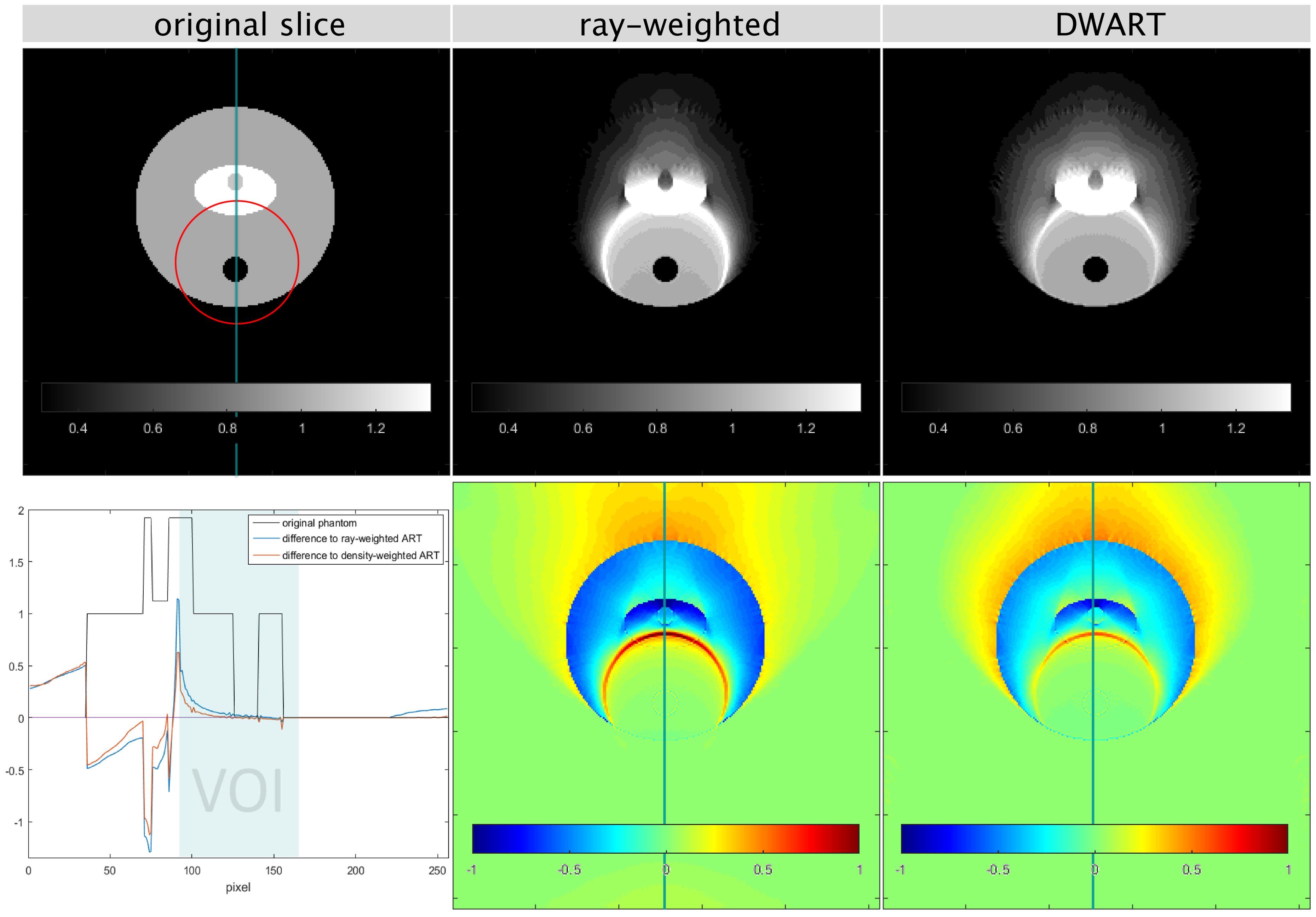

Fig. 5: ART reconstructions of a partially truncated scan (top) and differences to the ground truth (bottom) which is a slice of a jaw phantom[1]. The differences between ray-weighted ART and density-weighted ART (DWART) are smaller than in case of the interior problems (Fig. 3,4). However, the DWART is always closer to the ground truth than the ray-weighted ART even in the region outside the VOI. The direct comparison with the ground truth is possible, since the scan was not entirely truncated (no offset/scaling drift).

Conclusion

We derive an intuitive and compact solution for strong truncated VOI imaging in the algebraic domain of iterative algorithms. We came out with a weighting technique that appeared only in general terms of SIRT/SART[4,5] algorithms so far. We show its suitability to VOI imaging by using software phantoms.
The DWART is extendable to more sophisticated applications, e.g. regularization or statistical weightings. Further investigations will be performed regarding the comparison or combination with specialized analytical reconstruction methods (e.g. ATRACT[3]). Moreover, we consider to apply the DWART to 3D real cone-beam flat-panel CT or C-arm data as future work.

References

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