Epipolar Consistency Guided Beam Hardening Reduction - ECC^2

ECC^2

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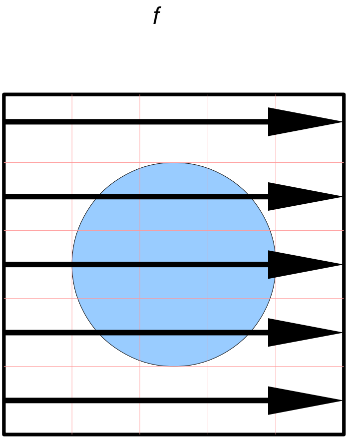
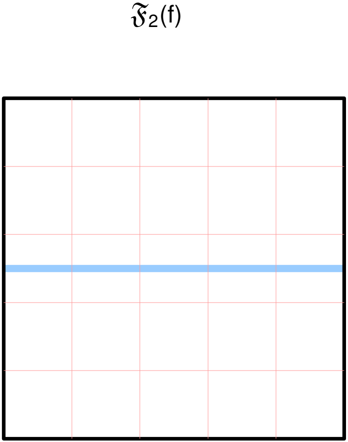
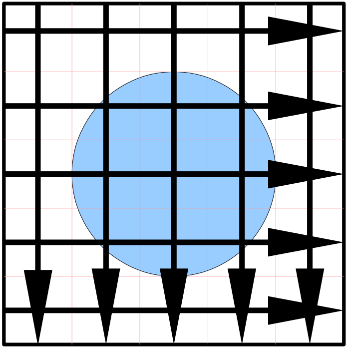
Redundancy in CT provides intrinsic measurements



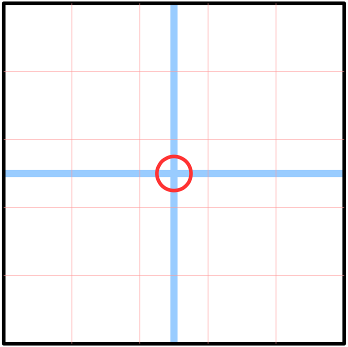


Redundancy in CT provides intrinsic measurements

f

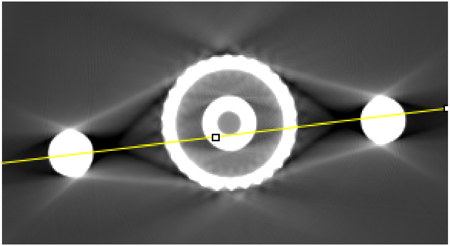


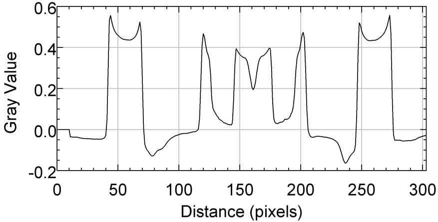
$\mathfrak{F}_2(\mathrm{f})$



Visual Effect of Beam Hardening

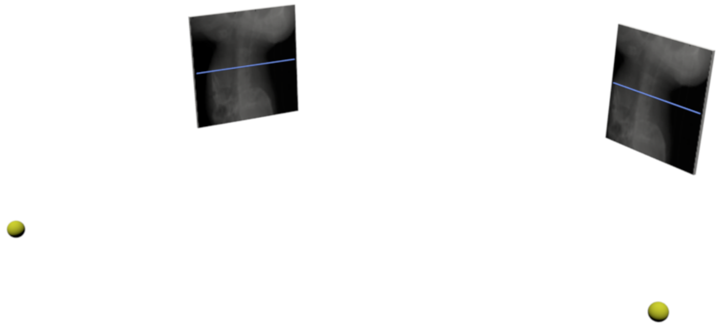
Stopwatch



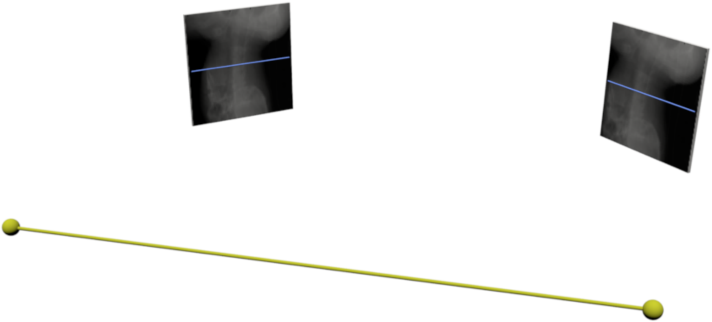


Exemplary stopwatch

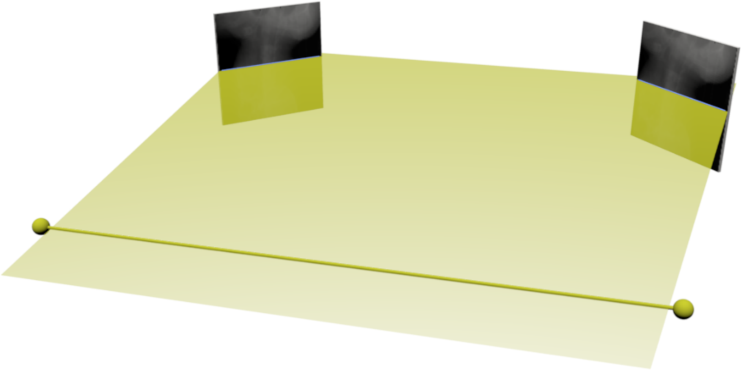
Epipolar Geometry



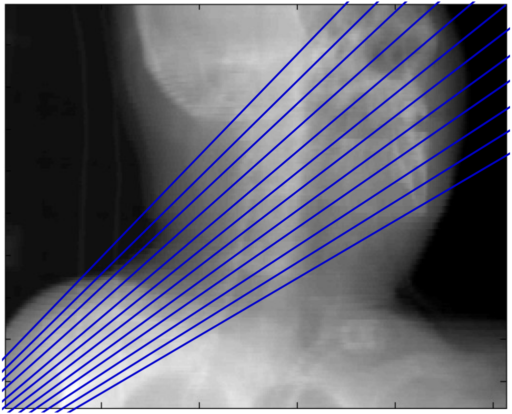
Epipolar Geometry



Epipolar Geometry



Efficient integration by sampling an intermediate function



$$\frac{d}{dn}\int_a^b \mathbf{I}(\ell)ds \approx \frac{\partial}{\partial t}\rho_{\mathbf{I}}(\alpha, t)$$

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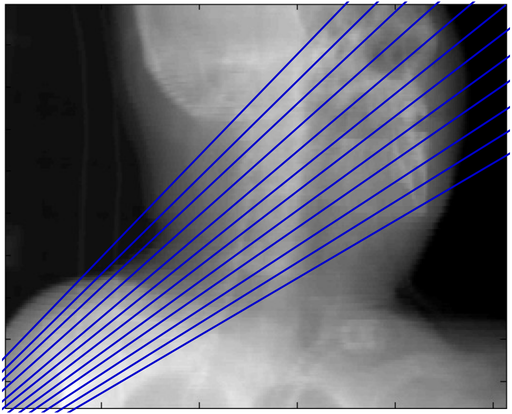

$$\frac{d}{dn}\int_a^b \mathbf{I}(\ell)ds \approx \frac{\partial}{\partial t}\rho_{\mathbf{I}}(\alpha, t)$$

$$\frac{\partial}{\partial t}\rho_{\mathbf{I}_0}(\ell_0) = \frac{\partial}{\partial t}\rho_{\mathbf{I}_1}(\ell_1)$$

Two key problems

$$\frac{\partial}{\partial t}\rho_{f(\mathbf{I}_0)}(\ell_0) = \frac{\partial}{\partial t}\rho_{f(\mathbf{I}_1)}(\ell_1) \tag{1}$$

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Two key problems

$$\frac{\partial}{\partial t}\rho_{f(\mathbf{I}_0)}(\ell_0) = \frac{\partial}{\partial t}\rho_{f(\mathbf{I}_1)}(\ell_1) \tag{1}$$

- Intermediate **depends** on f
- Solution is **up to scale**

Model

We model $f(\mathbf{I})$ parametric:

$$\mathbf{I}_{corr} = \sum_{n=1}^{N} w_n \cdot \mathbf{I}^n$$

No constant term

- $\frac{\partial}{\partial t} const = 0$
- Fits physics

Transferring the problem to intermediate domain

$$\frac{\partial}{\partial t}\rho_{\left(\sum_{n=1}^{N} w_n \mathbf{I}^n\right)}^{(\ell)}$$

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$$\sum_{n=1}^{N} w_n \left(\frac{\partial}{\partial t}\rho_{\mathbf{I}_0^n}(\ell_0)\right) = \sum_{n=1}^{N} w_n \left(\frac{\partial}{\partial t}\rho_{\mathbf{I}_1^n}(\ell_1)\right)$$

Objective function

M equations:

$$a_n = \left(\frac{\partial}{\partial t} \rho_{\mathbf{I}_0^n}(\ell_0) - \frac{\partial}{\partial t} \rho_{\mathbf{I}_1^n}(\ell_1) \right) = 0$$

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For $M >> N$:

$$\min_{\mathbf{w}} \left(\|\mathbf{Aw}\|_2^2 \right)$$

Regularization

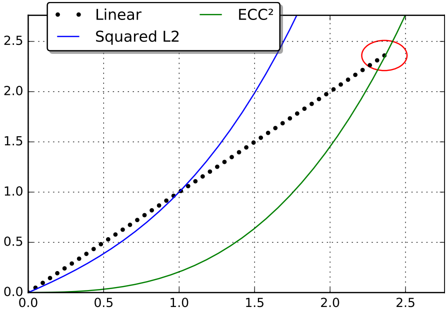

Figure: Ideal vs. measured attenuation

$$\beta = \sum_{n=1}^{N} w_n b^n = \mathbf{w}^T \mathbf{b}$$

$$\beta = b = \mathbf{max}(\mathbf{I})$$

ECC^2 optimization problem and solution

$$\min\left(\|\mathbf{A}\mathbf{w}\|_2^2\right) \quad s.t.: \mathbf{w}^T\mathbf{b} = \beta$$

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$$\hat{\mathbf{w}} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{b}$$

$$\mathbf{w} = \frac{\beta}{\mathbf{b}^T\hat{\mathbf{w}}}\hat{\mathbf{w}}$$

Robust estimation

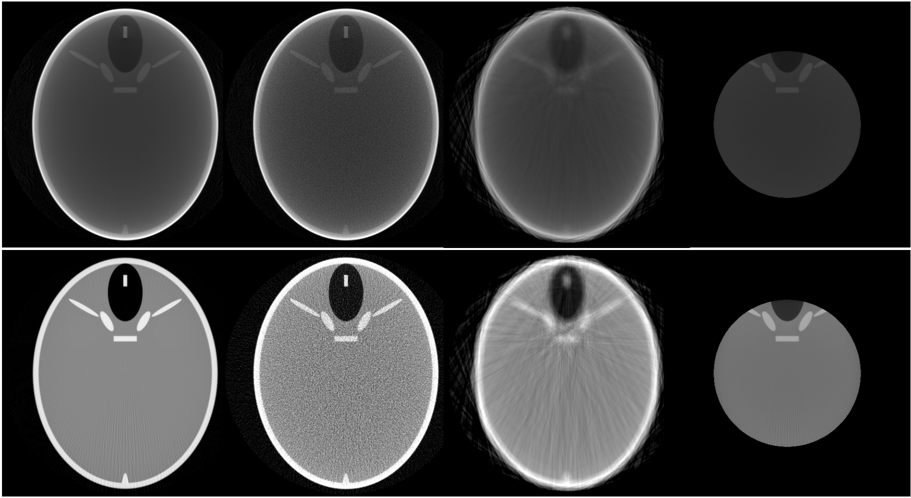
$$min(\|\mathbf{Aw}\|_2^2) \quad s.t.: \mathbf{w}^T\mathbf{b} = \beta; \quad w \geq 0 \quad \forall w \in \mathbf{w}$$

- **Not** a **necessary**
- but a **sufficient**

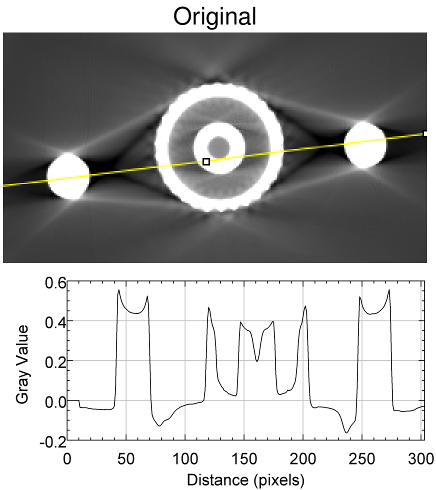
condition for **convexity**.

Overview of the simulation experiments

Beam hardening + Image Noise + Geometric noise + Truncation



ECC^2 works on real data



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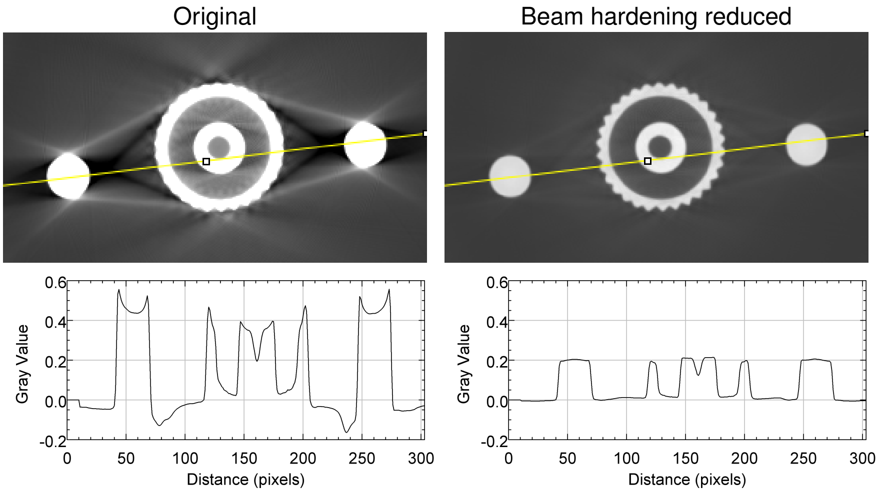

Conclusion and Outlook

Properties of ECC^2

- **Can't** recover absolute HU
- Calibration **free**
- **Efficient**
- **Robust**

Outlook

- **Multi**-material
- **Simultaneous** reduction
- **Scatter** reduction
- Different **Trajectories**

Thanks for listening.

Any questions?