Epipolar Consistency Guided Beam Hardening Reduction - ECC$^2$

ECC$^2$
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Redundancy in CT provides intrinsic measurements

\[ f \]

\[ \mathcal{F}_2(f) \]
Redundancy in CT provides intrinsic measurements

\[ f \quad \hat{\mathcal{F}}_2(f) \]
Visual Effect of Beam Hardening

Stopwatch

Exemplary stopwatch

![Graph showing gray value vs distance in pixels](attachment:image.png)
Epipolar Geometry
Epipolar Geometry
Epipolar Geometry
Efficient integration by sampling an intermediate function

\[
\frac{d}{dn} \int_a^b I(\ell) ds \approx \frac{\partial}{\partial t} \rho_1(\alpha, t)
\]
Efficient integration by sampling an intermediate function

\[
\frac{d}{dn} \int_a^b I(\ell) \, ds \approx \frac{\partial}{\partial t} \rho_1(\alpha, t)
\]

\[
\frac{\partial}{\partial t} \rho_0(\ell_0) = \frac{\partial}{\partial t} \rho_1(\ell_1)
\]
Two key problems

\[
\frac{\partial}{\partial t} \rho_{f(1)}(\ell_0) = \frac{\partial}{\partial t} \rho_{f(1)}(\ell_1)
\]  

(1)
Two key problems

\[
\frac{\partial}{\partial t} \rho_{f(I_0)}(\ell_0) = \frac{\partial}{\partial t} \rho_{f(I_1)}(\ell_1)
\]  

(1)

• Intermediate depends on \( f \)
Two key problems

\[ \frac{\partial}{\partial t} \rho_f(l_0)(l_0) = \frac{\partial}{\partial t} \rho_f(l_1)(l_1) \]  

• Intermediate depends on f
• Solution is up to scale
Model

We model $f(I)$ parametric:

$$I_{corr} = \sum_{n=1}^{N} w_n \cdot I^n$$

**No constant term**

- $\frac{\partial}{\partial t} const = 0$
- Fits physics
Transferring the problem to intermediate domain

\[ \frac{\partial}{\partial t} \rho \left( \sum_{n=1}^{N} w_n I^n(\ell) \right) \]
Transferring the problem to intermediate domain

\[
\frac{\partial}{\partial t} \rho \left( \sum_{n=1}^{N} w_n l^n \right)(\ell) = \sum_{n=1}^{N} w_n \left( \frac{\partial}{\partial t} \rho_{l^n}(\ell) \right)
\]
Transferring the problem to intermediate domain

\[
\frac{\partial}{\partial t} \rho \left( \sum_{n=1}^{N} w_n I^n \right) (\ell) = \sum_{n=1}^{N} w_n \left( \frac{\partial}{\partial t} \rho_{I^n}(\ell) \right)
\]

\[
\sum_{n=1}^{N} w_n \left( \frac{\partial}{\partial t} \rho_{I^n}(\ell_0) \right) = \sum_{n=1}^{N} w_n \left( \frac{\partial}{\partial t} \rho_{I^n}(\ell_1) \right)
\]
Objective function

$M$ equations:

$$a_n = \left( \frac{\partial}{\partial t} \rho^n_0 (\ell_0) - \frac{\partial}{\partial t} \rho^n_1 (\ell_1) \right) = 0$$
Objective function

\[ M \text{ equations:} \]

\[ a_n = \left( \frac{\partial}{\partial t} \rho_{l_0}^n(\ell_0) - \frac{\partial}{\partial t} \rho_{l_1}^n(\ell_1) \right) = 0 \]

For \( M \gg N \):

\[ \min_w (\|Aw\|_2^2) \]
Regularization

\[ \beta = \sum_{n=1}^{N} w_n b^n = w^T b \]

\[ \beta = b = \max(I) \]

Figure: Ideal vs. measured attenuation
ECC$^2$ optimization problem and solution

$$\min(\|Aw\|_2^2) \quad s.t. : w^T b = \beta$$
$\text{ECC}^2$ optimization problem and solution

$$\min(\|Aw\|_2^2) \quad s.t.: \quad w^Tb = \beta$$

$$\hat{w} = (A^TA)^{-1}b$$
ECC$^2$ optimization problem and solution

$$\min(\|A\mathbf{w}\|_2^2) \quad s.t. : \mathbf{w}^T\mathbf{b} = \beta$$

$$\hat{\mathbf{w}} = (A^T A)^{-1} \mathbf{b}$$

$$\mathbf{w} = \frac{\beta}{\mathbf{b}^T \hat{\mathbf{w}}} \hat{\mathbf{w}}$$
Robust estimation

\[ \min(\|Aw\|_2^2) \quad s.t.: \quad w^T b = \beta; \quad w \geq 0 \quad \forall w \in w \]

- Not a necessary condition for convexity.
- but a sufficient condition for convexity.
Overview of the simulation experiments

Beam hardening + Image Noise + Geometric noise + Truncation
ECC\textsuperscript{2} works on real data

Original

![Graph showing gray value against distance in pixels](image)

- Gray Value
- Distance (pixels)
ECC$^2$ works on real data

Original

Beam hardening reduced
Conclusion and Outlook
Properties of ECC$^2$

- Can’t recover absolute HU
- Calibration free
- Efficient
- Robust
Outlook

- Multi-material
- Simultaneous reduction
- Scatter reduction
- Different Trajectories
Thanks for listening.

Any questions?