Projective Invariants for Geometric Calibration in Flat-Panel Computed Tomography

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Related Work

- “offline” calibration
  → Dedicated calibration scan before acquisition
  → Employs known (marker based) 3D phantom
    → Accurately manufactured
    → Designed for specific reproducible trajectories
Related Work

• “offline” calibration
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• “online” motion/calibration correction
  → Assumes existing projection data
  → Optimizes geometry parameters “after the fact”
    (e.g. consistency conditions)
Offline Calibration

- Parametrization
  - Either: Model parameters for trajectory (e.g. circular)
  - Or: Parameters of each linear projection (projection matrix)
- Detection of calibration objects in images
- Model fitting
Estimation of Projection Matrices from Point Correspondences

**Multiple View Geometry in Computer Vision**
Richard Hartley and Andrew Zisserman
Calibration with DLT

- Direct linear transformation
  → Linear least squares estimate for projection matrix
  → Efficient, fast, simple, *but*…
Calibration with DLT

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  - Linear least squares estimate for projection matrix
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- Unstable in presence of outliers
- Assumes perfect point matches
Calibration with DLT

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Standard Pipeline

1. 2D Images of Phantom
2. Bead Detection
3. Descriptor-based matching
4. Estimation of Projection Matrices
5. 3D Bead Locations
Standard Pipeline

Bead Detection

Descriptor-based matching

Estimation of Projection Matrices

2D Images of Phantom

RANSAC & DLT

Non-linear Refinement

Projection Matrices

“Gold Standard”

3D Bead Locations

Estimation of Projection Matrices

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Projection Matrices
Computer Vision Answer

Find Analytically special image points which are
→ **Locally** defined for a certain scale
→ **Invariant** w.r.t. scale, lighting etc.
→ **Salient** compared to other points
Computer Vision Answer

Find Analytically special image points which are

→ **Locally** defined for a certain scale
→ **Invariant** w.r.t. scale, lighting etc.
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Matching Problem: PDS2 solution

→ Every sequence of 7 beads is unambiguous!
Problem Solved?

→ Accurately manufactured
→ Designed for specific reproducible trajectories
The Cross-ratio of Collinear Points
A Projective Invariant Property

Multiple View Geometry in Computer Vision
Richard Hartley and Andrew Zisserman
## Invariant Theory

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid</td>
<td>(Translation and Rotation)</td>
<td><img src="image1" alt="Diagram" /></td>
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<tr>
<td></td>
<td>...length, area</td>
<td></td>
</tr>
<tr>
<td>Similarity</td>
<td>(+Scale)</td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>...angles</td>
<td></td>
</tr>
<tr>
<td>Affinity</td>
<td>(+Shear)</td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>...parallelism</td>
<td></td>
</tr>
<tr>
<td>Projective</td>
<td>(+Perspectivity)</td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>...?</td>
<td></td>
</tr>
</tbody>
</table>
The Cross-ratio

\[
\frac{(a-c)(b-d)}{(a-d)(b-c)} \qquad \text{def} \quad cr\ (a, b; c, d)
\]

- Definition in line coordinates
- Projective: Determinant expression
- Cross-ratio is invariant to dot-product
The Cross-ratio: a Projective Invariant

\[
\frac{(a-c) \cdot (b-d)}{(a-d) \cdot (b-c)} \quad \text{def} \quad \text{cr} \ (a, b; c, d)
\]

- Definition in line coordinates
- Projective: Determinant expression
- Cross-ratio is invariant to dot-product

\[
\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} \lambda \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

\[
\text{cr} \ (\infty, 0; 1, \lambda) = \lim_{a \to \infty} \frac{(a - 1) \cdot \lambda}{(a - \lambda) \cdot 1} = \lambda
\]
Projective Scales

\[
x = 2
\]
Projective Scales

\[ p_0, p_1, x, p_\infty \]

\[ x = 5 \]
Projective Scales

$x = 10$
Projective Scales

\[ x = 20 \]
Projective Scales
Projective Scales
Phantom Design
Possible Point Configurations
Cross-ratio as a Descriptor
Cross-ratio as a Descriptor

Beads placed at regular distance $n=9$

0 2
1 4
2 7
3 11
4 21
5 49
6 1
7 3
8 5
9 9
10 21
11 1
12 2
13 5
14 11
15 1
16 3
17 7
18 1
19 4
20 2
Cross-ratio as a Descriptor

Beads placed at regular distance n=9
Cross-ratio as a Descriptor

Beads placed at regular distance n=9
Cross-ratio as a Descriptor

Beads placed at regular distance n=9

Example set of 18 unambiguous point configurations.
Cross-ratio as a Descriptor (cr=2)
Cross-ratio as a Descriptor (cr=3)
Cross-ratio as a Descriptor (cr=4)
Cross-ratio as a Descriptor (cr=10)
Cross-ratio as a Descriptor

Beads placed at increasing distance from center
Phantom Design
Flexibility in Spacial Arrangement
What could phantoms look like?

For Reference: PDS2

n=108
What could phantoms look like?

Randomized phantoms with varying number of points.

n=28
What could phantoms look like?

Randomized phantoms with varying number of points.

n=60
What could phantoms look like?

Randomized phantoms with varying number of points.

$n=90$
What could phantoms look like?

Randomized phantoms with varying number of points.

n=120
Example Matching

n=60
Validation using Numerical Simulations
Trajectories

Evaluation on Circular and “Spherical” trajectories

120 projections

1449 projections
Results

Residuals

Target Projection Error

Source Position Error

Correct Matches
Results – comparison to PDS2
Conclusion
Conclusion & Future Work

- A “building block” for geometry calibration phantoms
  - Flexible w.r.t. truncation, scale etc.
  - Cheap manufacturing
  - Treats all directions equally
Conclusion & Future Work

● A “building block” for geometry calibration phantoms
  → Flexible w.r.t. truncation, scale etc.
  → Cheap manufacturing
  → Treats all directions equally

● Other computer vision algorithms
  → Factorization, stratification and bundle adjustment

● Motion tracking individual blocks
Preview: factorization
Questions.

Publication partially covered by this talk

Projective Invariants for Geometric Calibration in Flat Panel Computed Tomography
A. Aichert, B. Bier, L. Rist and A. K. Maier
Accepted for CT Meeting 2018
The Cross Product and the Plücker Matrix

\[ l = a \times b = \begin{pmatrix} a_1 b_2 - b_1 a_2 \\ b_0 a_2 - a_0 b_2 \\ a_0 b_1 - a_1 b_0 \end{pmatrix} = \begin{pmatrix} l_0 \\ l_1 \\ l_2 \end{pmatrix}, \]

\[ b^\top a - a^\top b = \begin{pmatrix} a_0 b_0 - b_0 a_0 & a_0 b_1 - b_0 a_1 & a_0 b_2 - b_0 a_2 \\ a_1 b_0 - b_1 a_0 & a_1 b_1 - b_1 a_1 & a_1 b_2 - b_1 a_2 \\ a_2 b_0 - b_2 a_0 & a_2 b_1 - b_2 a_1 & a_2 b_2 - b_2 a_2 \end{pmatrix} = \begin{pmatrix} 0 & l_2 & -l_1 \\ -l_2 & 0 & l_0 \\ l_1 & -l_0 & 0 \end{pmatrix}. \]
Rest of this talk: how to figure out $P$?

$x \approx PX$

- $x$ is up to scale identical to $PX$
  $\rightarrow$ Cross-product is zero

$$x \times PX = 0$$

- Can be written in matrix form

$$\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
0^\top & x_3X^\top & -x_2X^\top \\
-x_3X^\top & 0^\top & x_1X^\top \\
x_2X^\top & -x_1X^\top & 0^\top
\end{pmatrix} \cdot \begin{pmatrix}
p^1 \\
p^2 \\
p^3
\end{pmatrix}$$