Projective Invariants for Geometric Calibration in Flat-Panel Computed Tomography

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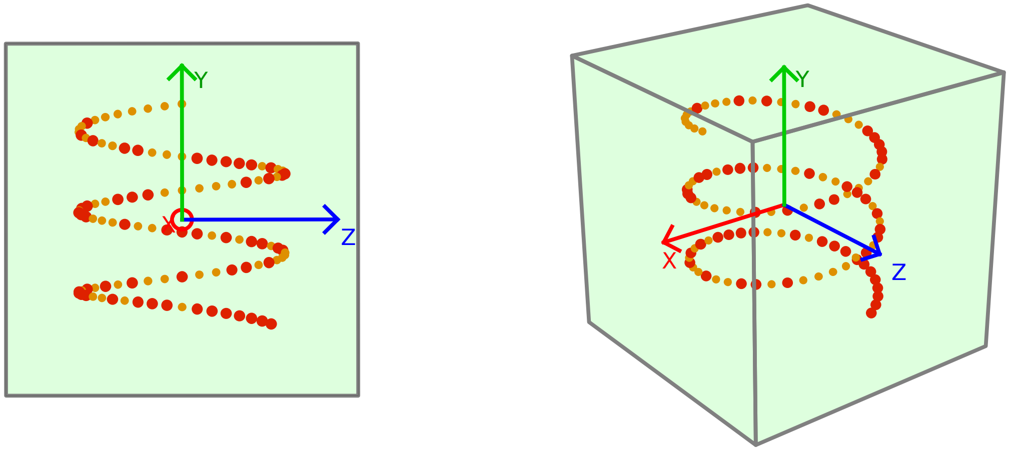
5th CT-Meeting, May 21st 2018, Salt Lake City, UT

CT Calibration

Related Work

- “offline” calibration
  → Dedicated calibration scan before acquisition
  → Employs known (marker based) 3D phantom
    → Accurately manufactured
    → Designed for specific reproducible trajectories



Related Work

- "offline" calibration

  → Dedicated calibration scan before acquisition

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- "online" motion/calibration correction

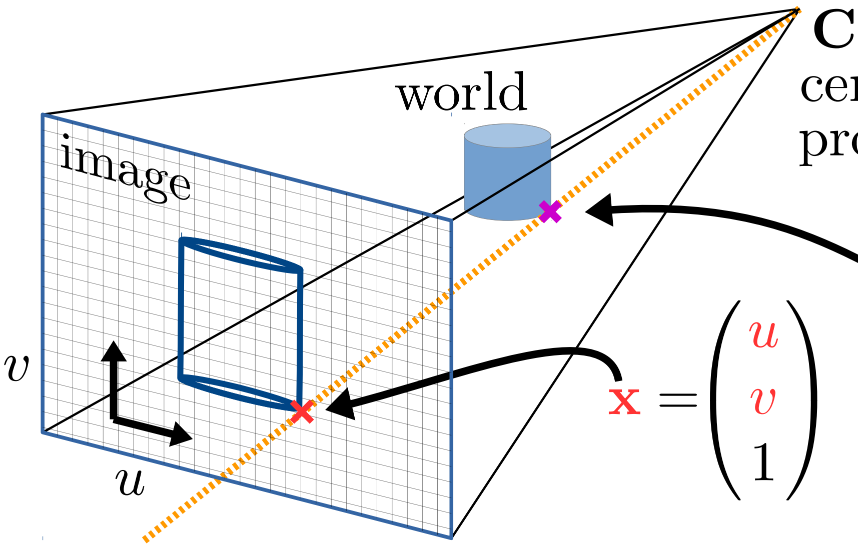
  → Assumes existing projection data

  → Optimizes geometry parameters "after the fact"

    (e.g. consistency conditions)

Offline Calibration

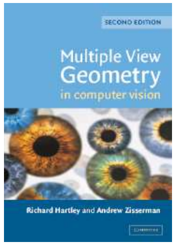
- Parametrization
  → Either: Model parameters for trajectory (e.g. circular)
  → Or: Parameters of *each* linear projection (projection matrix)
- Detection of calibration objects in images
- Model fitting



$$\mathbf{x} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \qquad \mathbf{X} \cong \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Estimation of Projection Matrices

from Point Correspondences



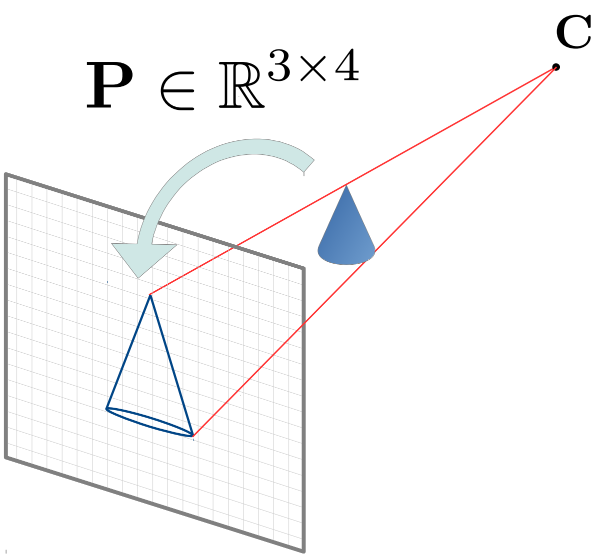
Multiple View Geometry in Computer Vision
Richard Hartley and Andrew Zisserman
Cambridge University Press, March 2004.

Calibration with DLT

- Direct linear transformation

  → Linear least squares estimate for projection matrix

  → Efficient, fast, simple, *but*…



Calibration with DLT

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  $\longrightarrow$ Linear least squares estimate for projection matrix

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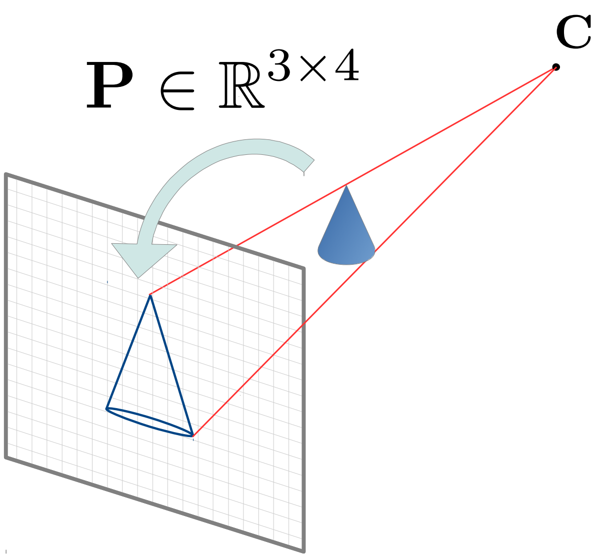
- Unstable in presence of outliers
- Assumes perfect point matches



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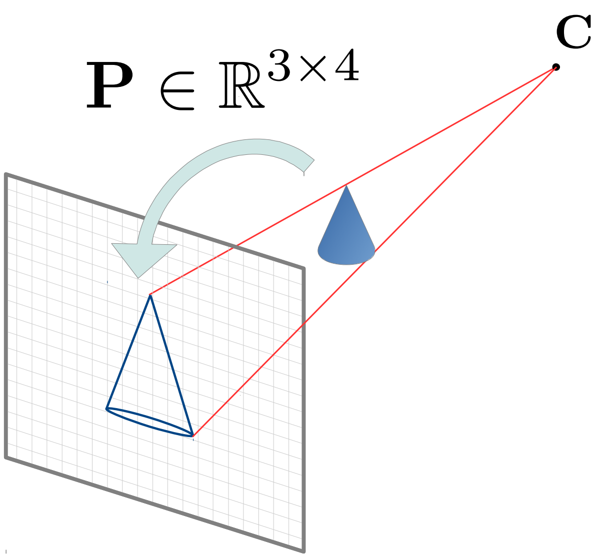
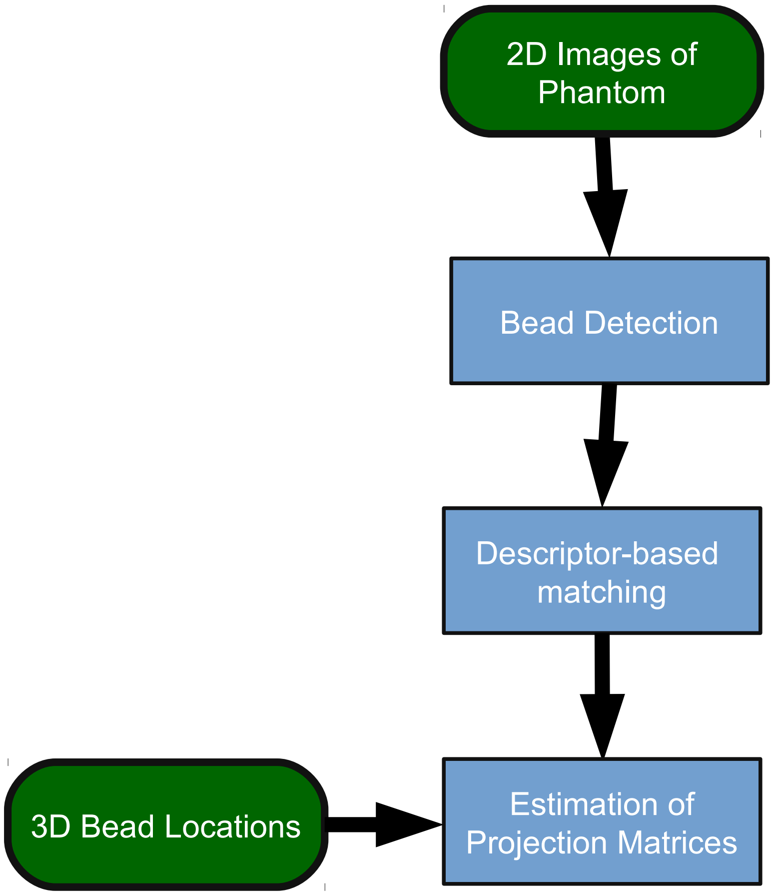
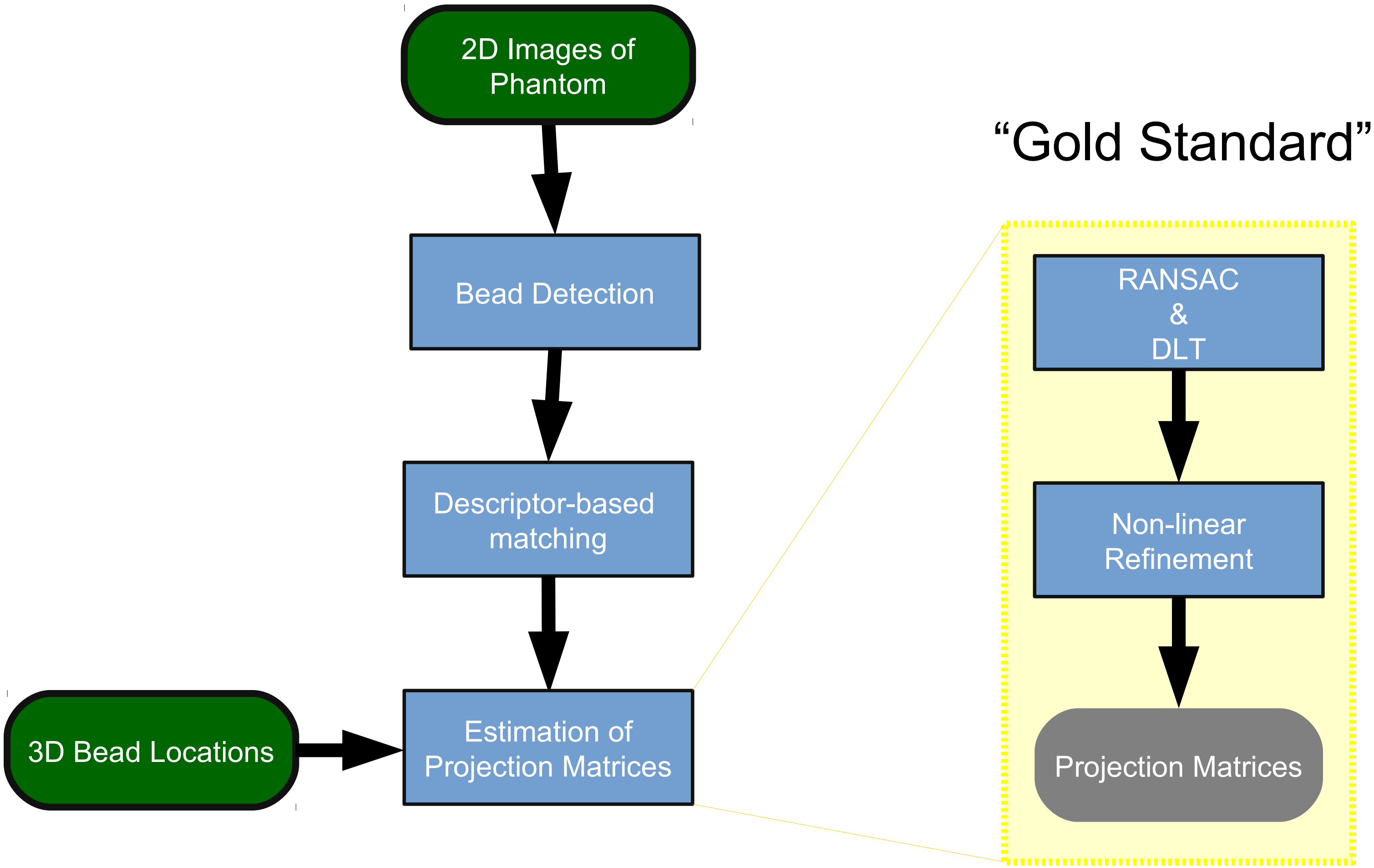


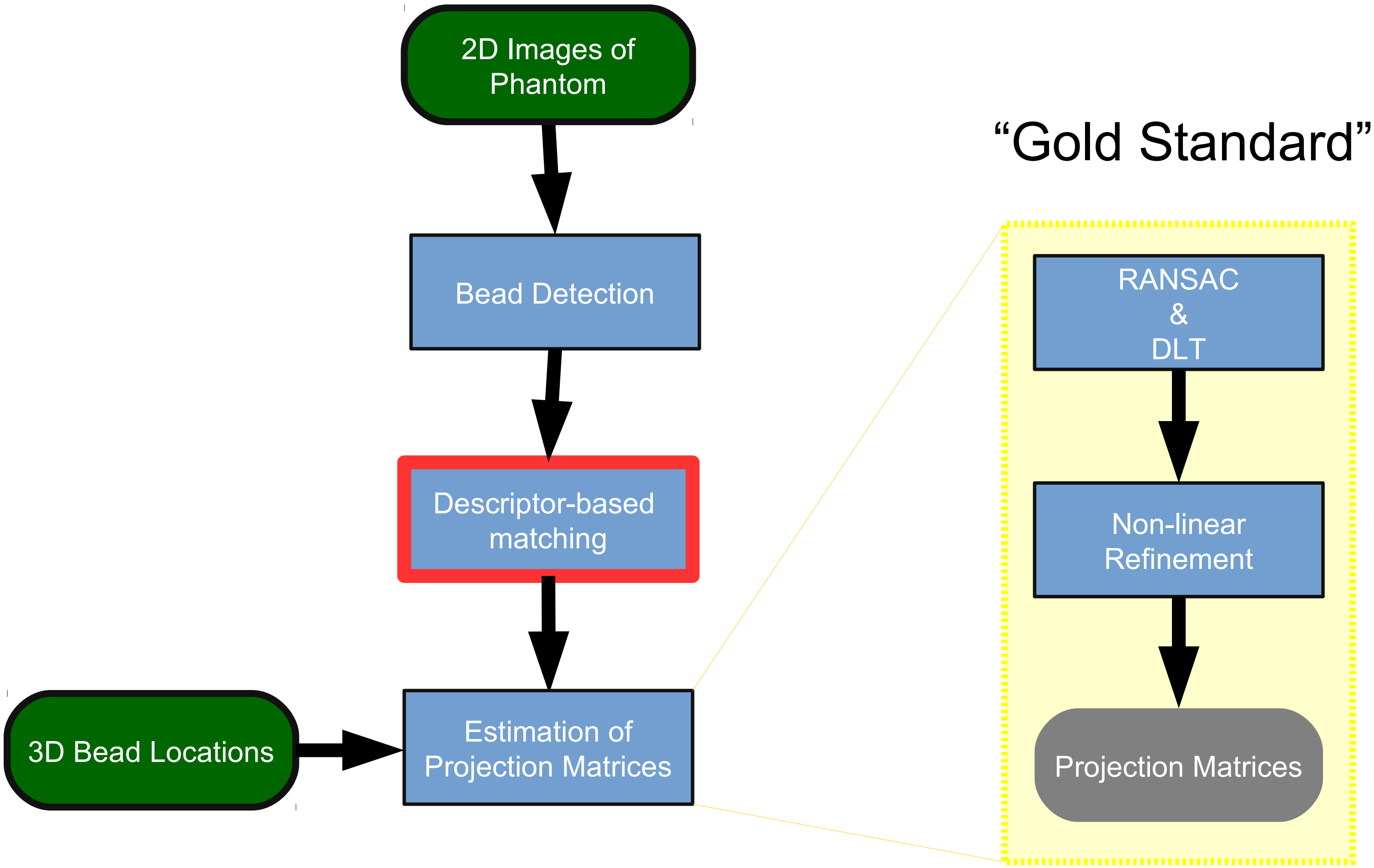
Standard Pipeline



Standard Pipeline



Standard Pipeline



Computer Vision Answer

Find Analytically special image points which are

→ **Locally** defined for a certain scale

→ **Invariant** w.r.t. scale, lighting etc.

→ **Salient** compared to other points

Feature Detection → Feature Description → Feature Matching

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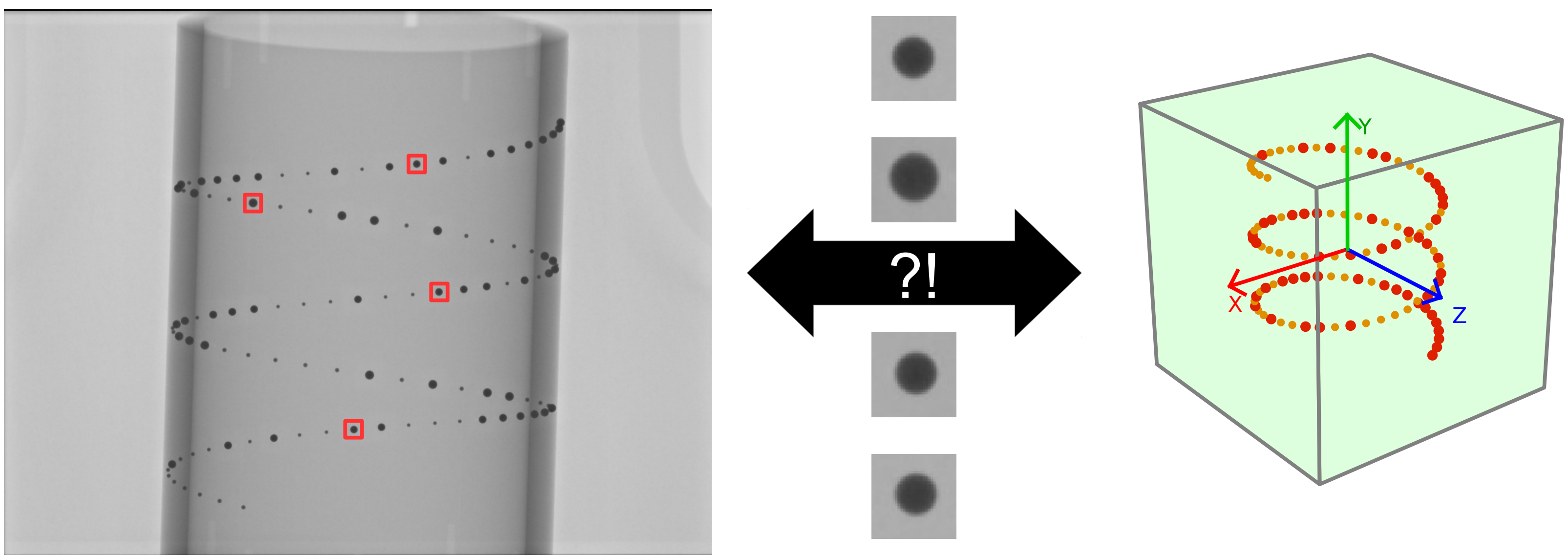
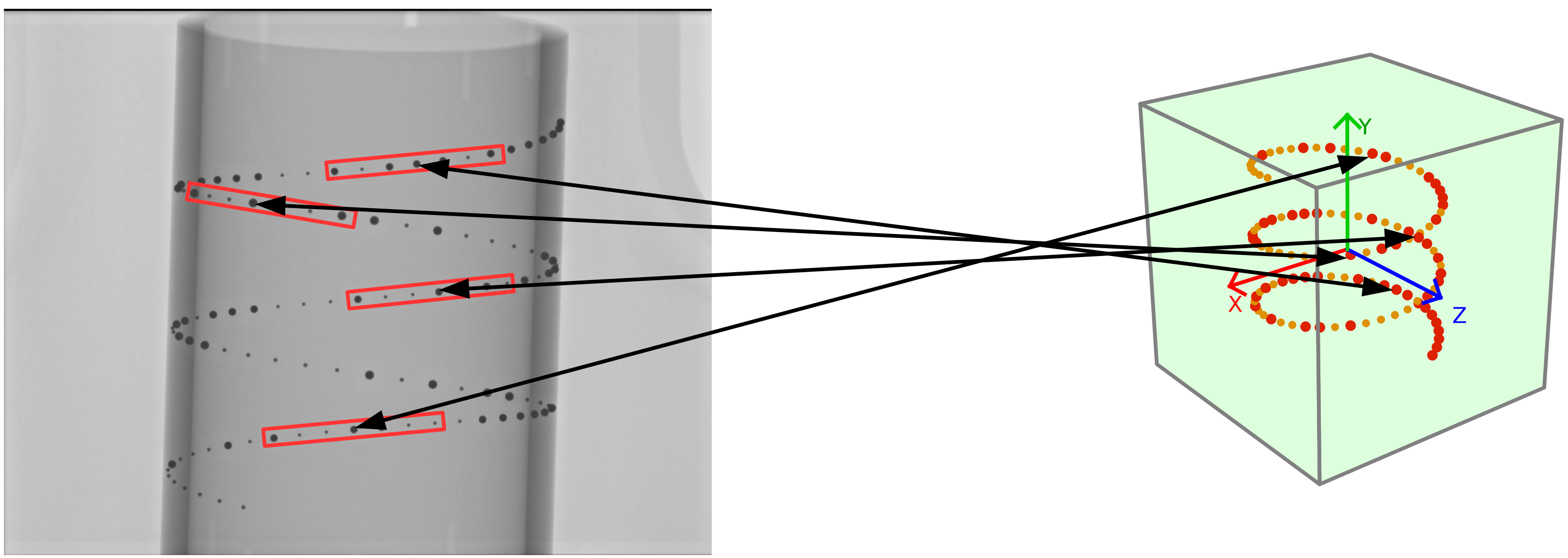


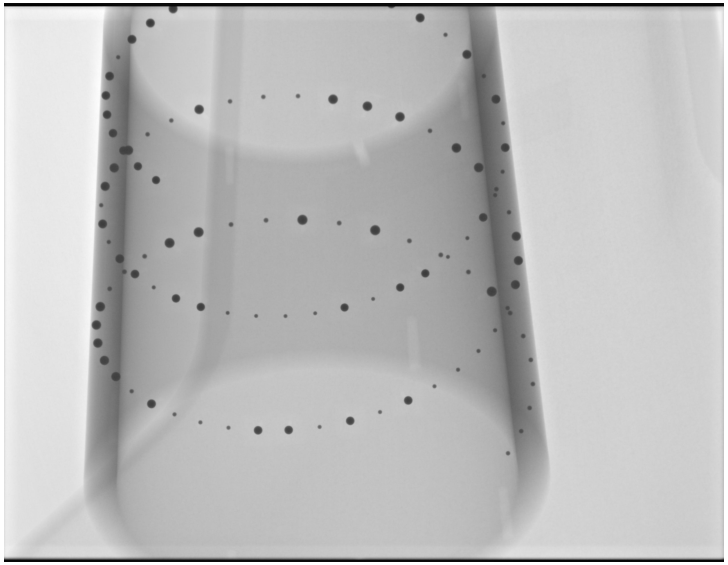
Matching Problem: PDS2 solution

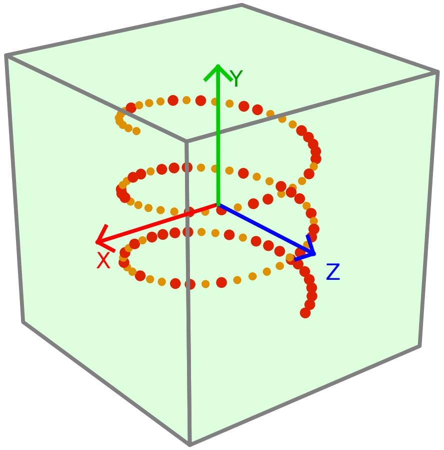
→ Every sequence of 7 beads is unambiguous!



Problem Solved?

→ Accurately manufactured
→ Designed for specific reproducible trajectories





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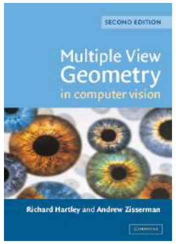
The Cross-ratio of Collinear Points

A Projective Invariant Property



Multiple View Geometry in Computer Vision
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Cambridge University Press, March 2004.

Invariant Theory

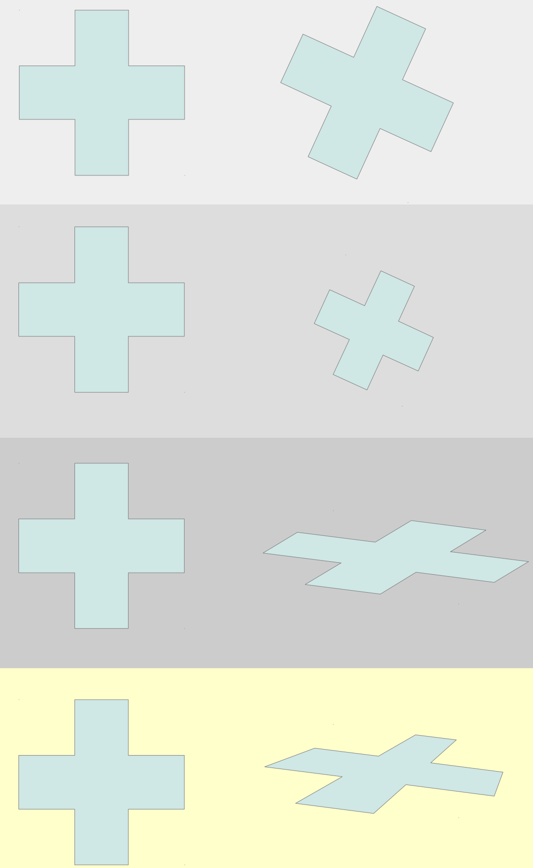
- Rigid (Translation and Rotation)

  ...length, area
- Similarity (+Scale)

  ...angles
- Affinity (+Shear)

  ...parallelism
- Projective (+Perspectivity)

  … ?



The Cross-ratio

$$\frac{(a-c)\cdot(b-d)}{(a-d)\cdot(b-c)} \stackrel{\text{def}}{=} \operatorname{cr}(a,b;c,d)$$

- Definition in line coordinates
- Projective: Determinant expression
- Cross-ratio is invariant to dot-product

The Cross-ratio: a Projective Invariant

$$\frac{(a-c)\cdot(b-d)}{(a-d)\cdot(b-c)} \stackrel{\text{def}}{=} \operatorname{cr}(a, b; c, d)$$

- Definition in line coordinates
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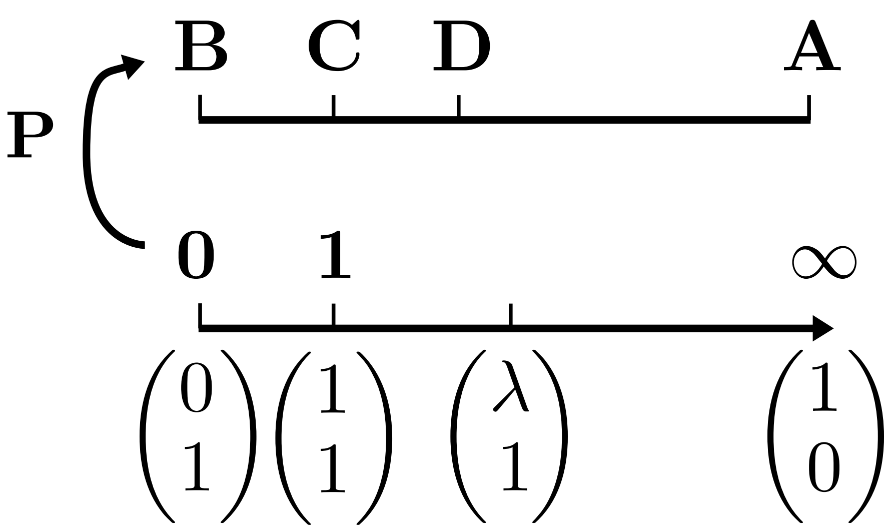
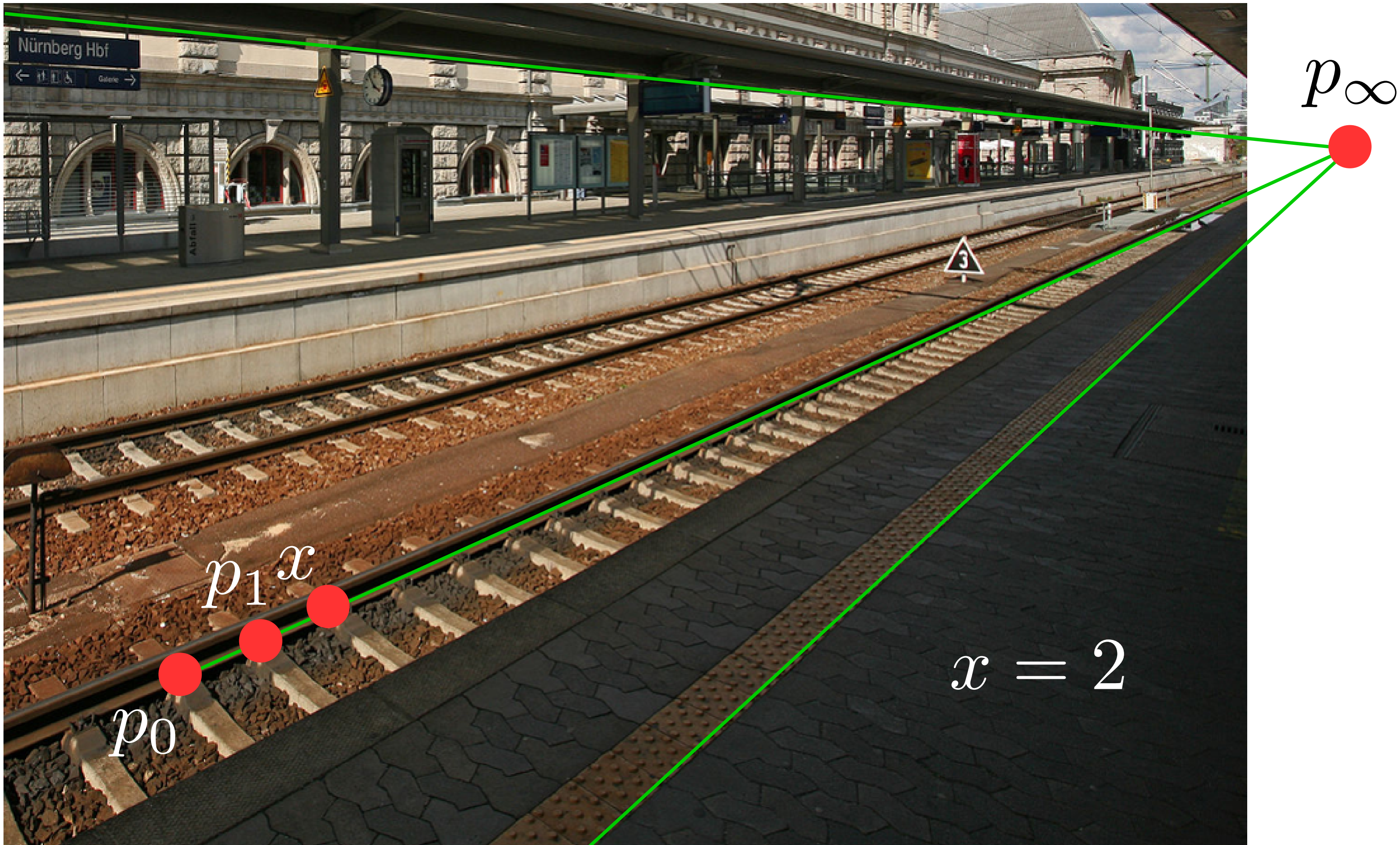


$$\operatorname{cr}(\infty,\ 0;\ 1,\ \lambda)$$

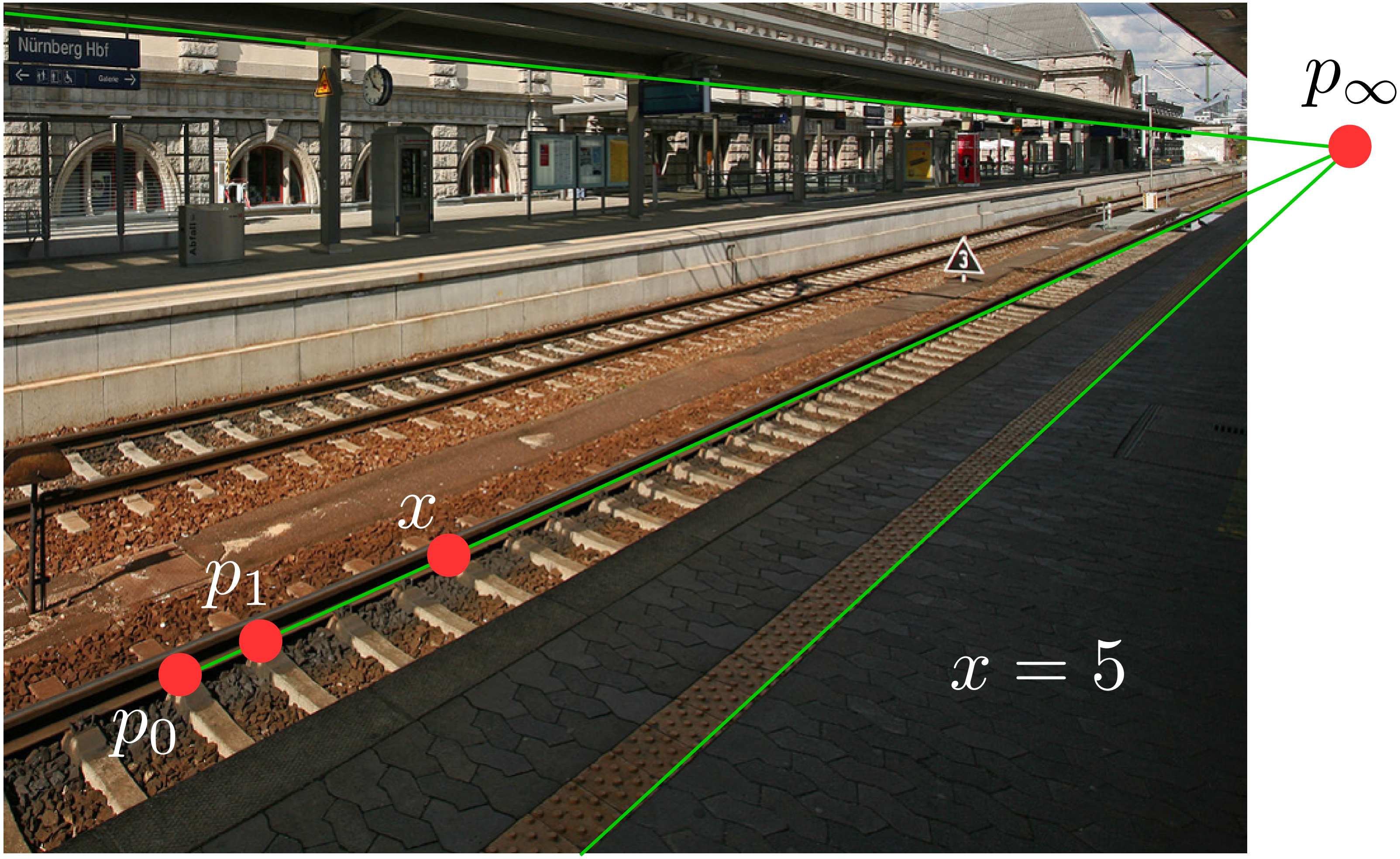
$$= \lim_{a\to\infty} \frac{(a-1)\cdot\lambda}{(a-\lambda)\cdot 1}$$

$$= \lambda$$

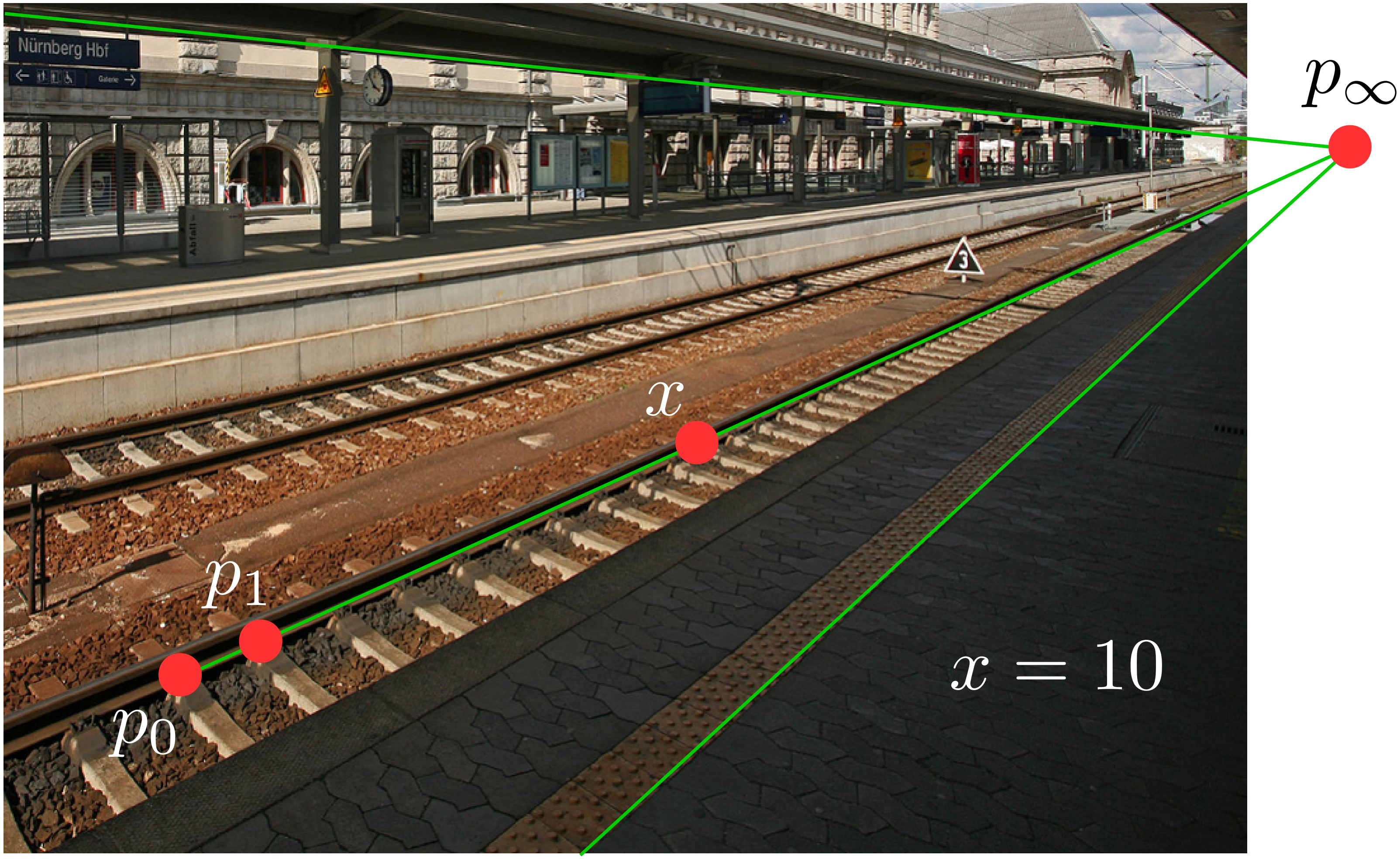
Projective Scales



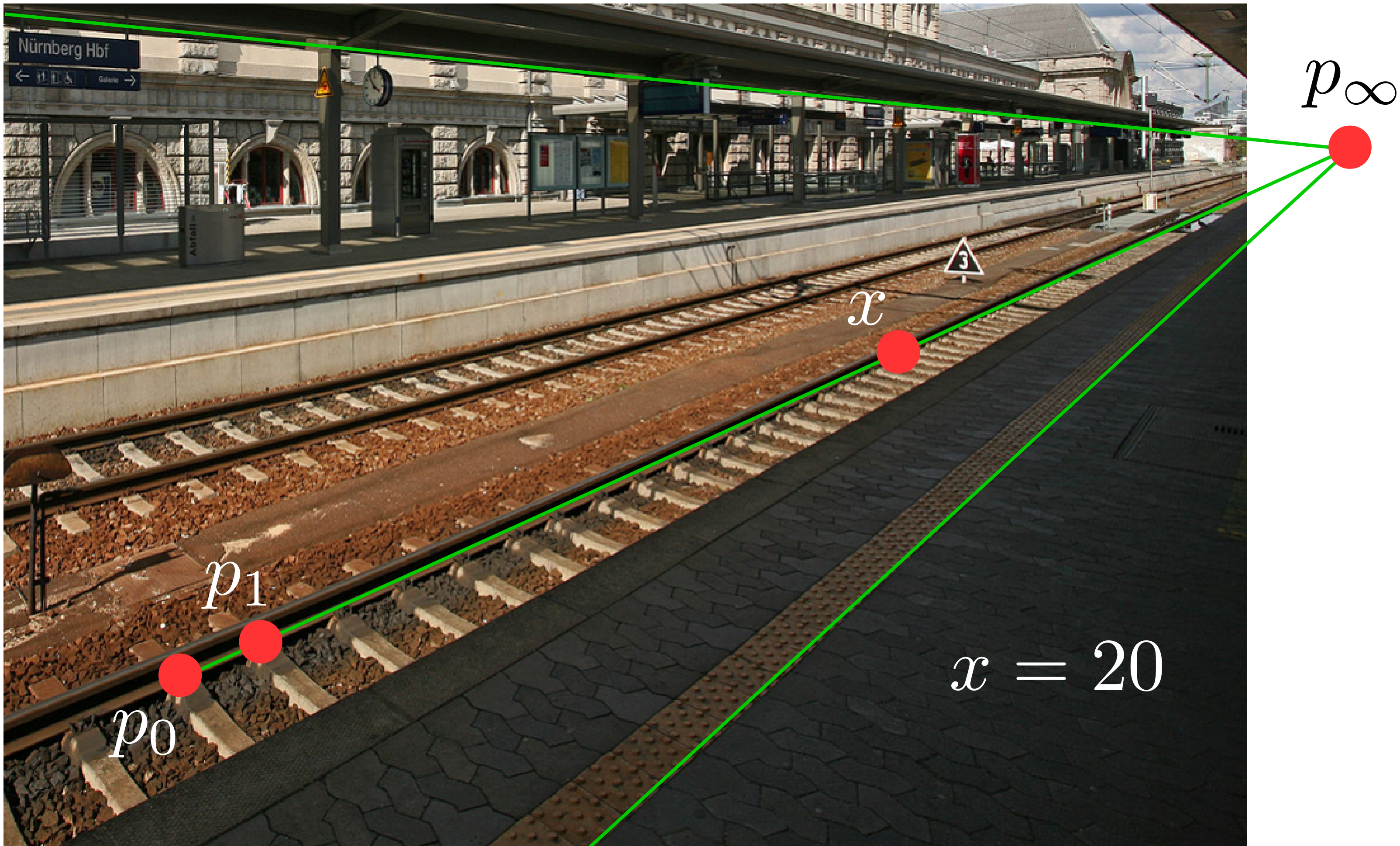
Projective Scales



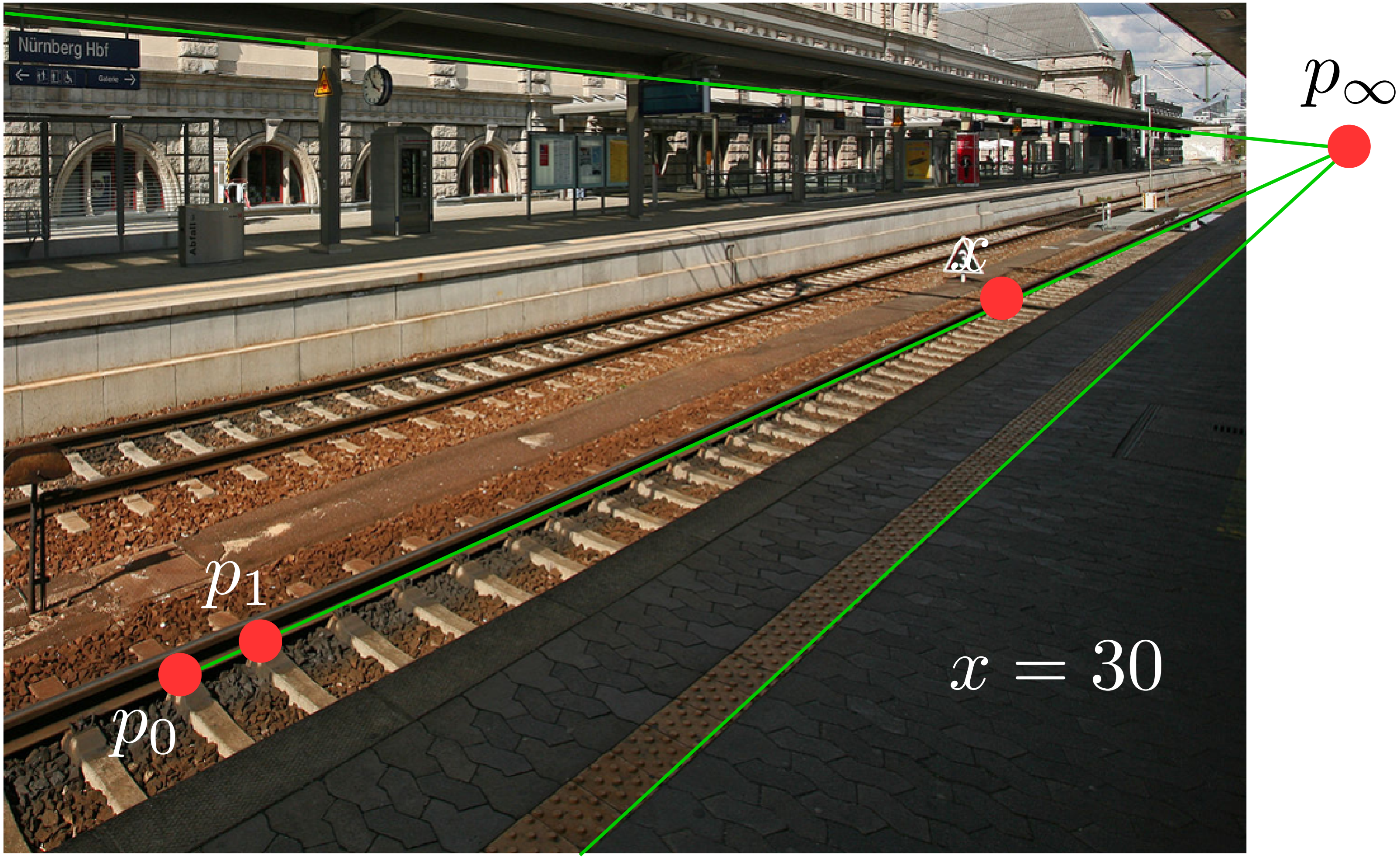
Projective Scales



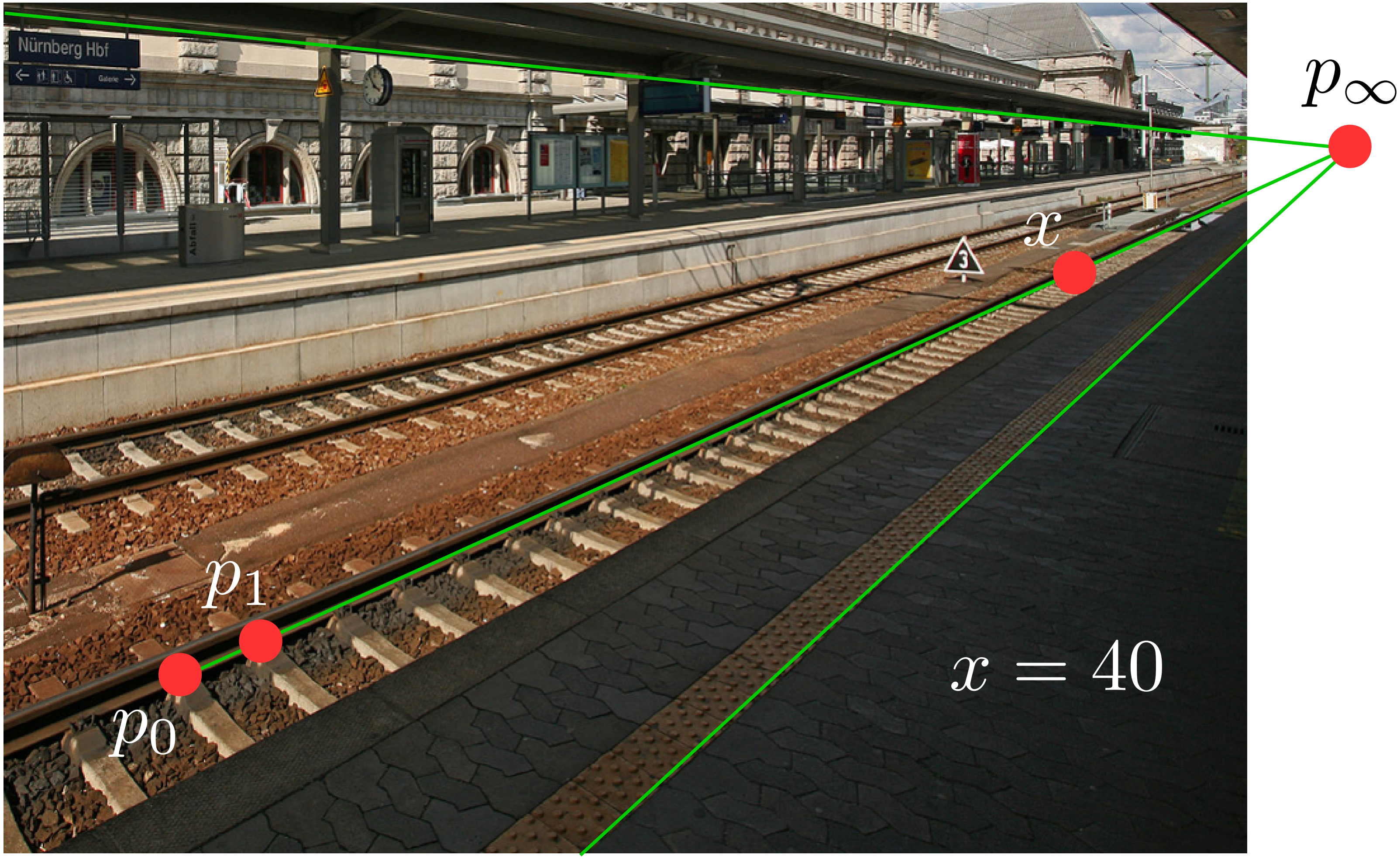
Projective Scales



Projective Scales



Projective Scales



Phantom Design

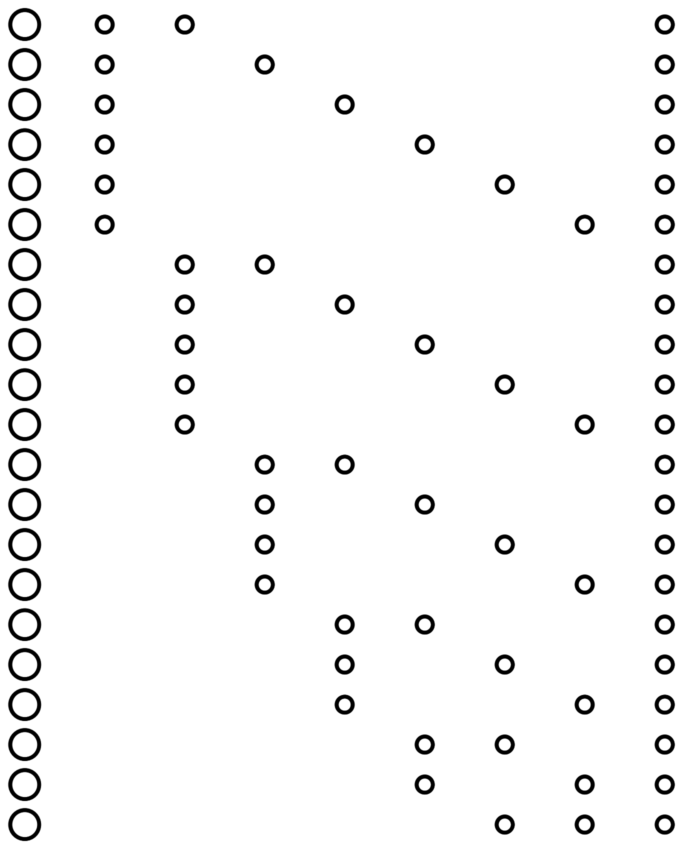
Possible Point Configurations

Cross-ratio as a Descriptor

Cross-ratio as a Descriptor

Beads placed at regular distance n=9

0	2
1	4
2	7
3	11
4	21
5	49
6	1
7	3
8	5
9	9
10	21
11	1
12	2
13	5
14	11
15	1
16	3
17	7
18	1
19	4
20	2



Cross-ratio as a Descriptor

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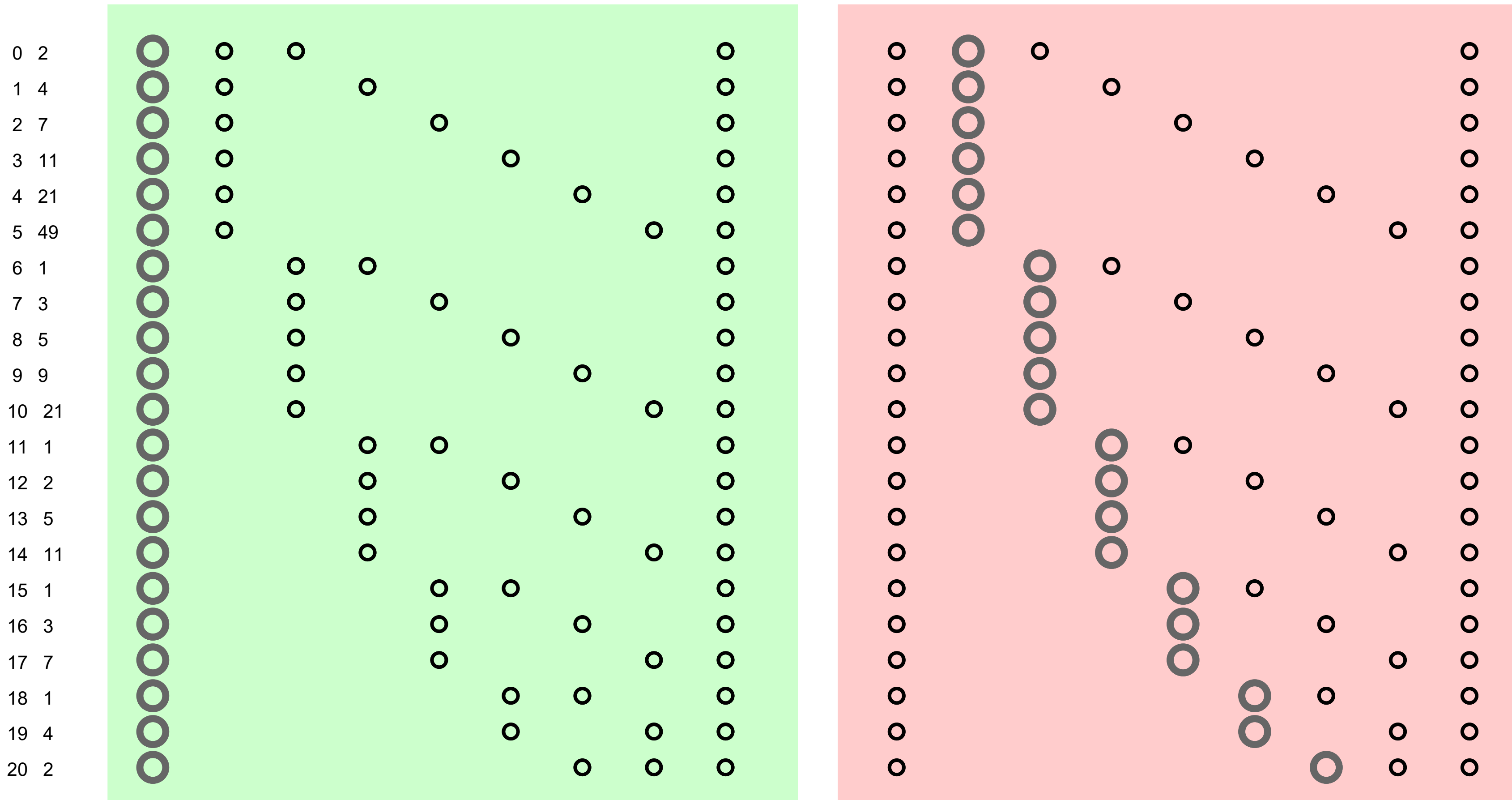

Cross-ratio as a Descriptor

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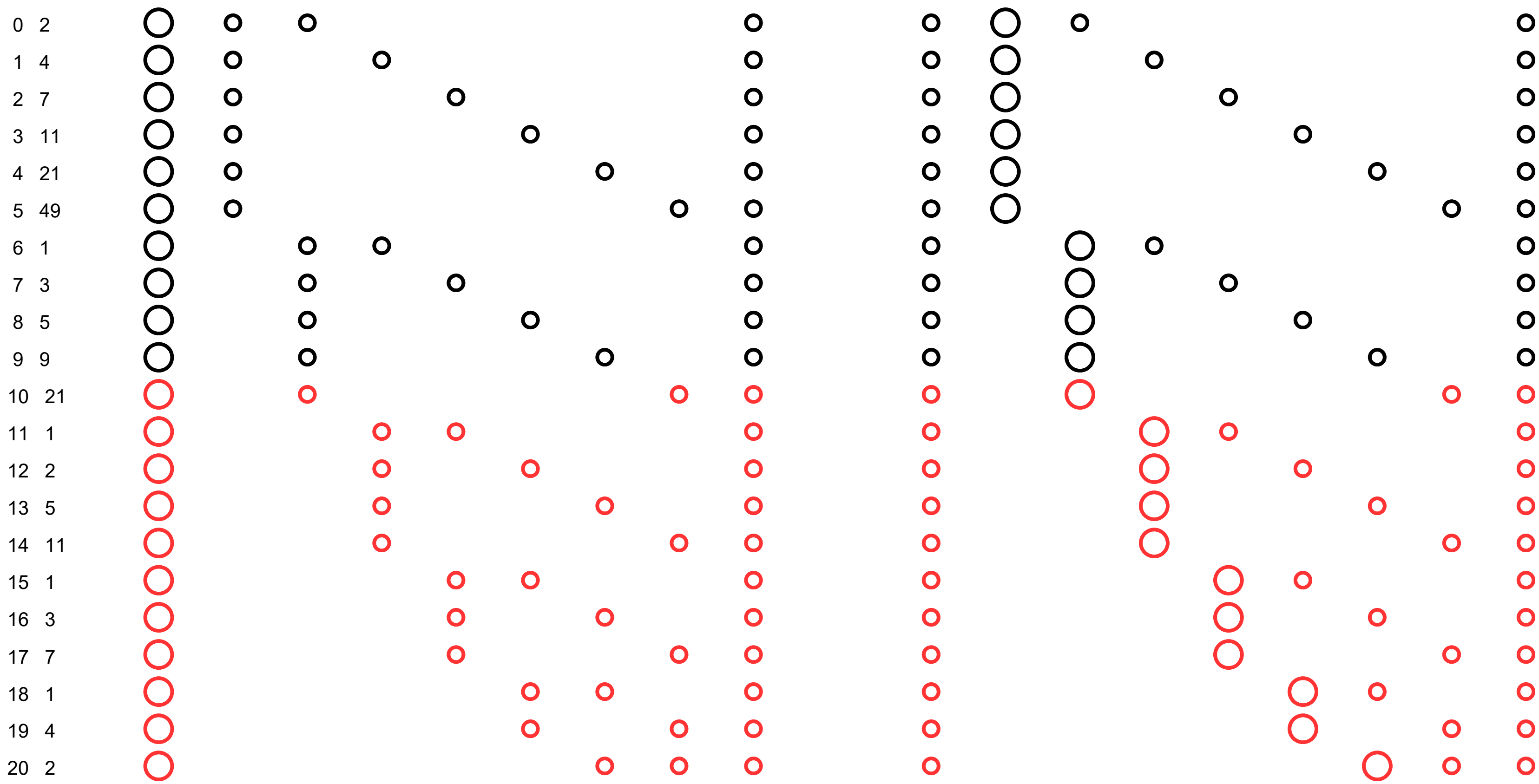

Cross-ratio as a Descriptor

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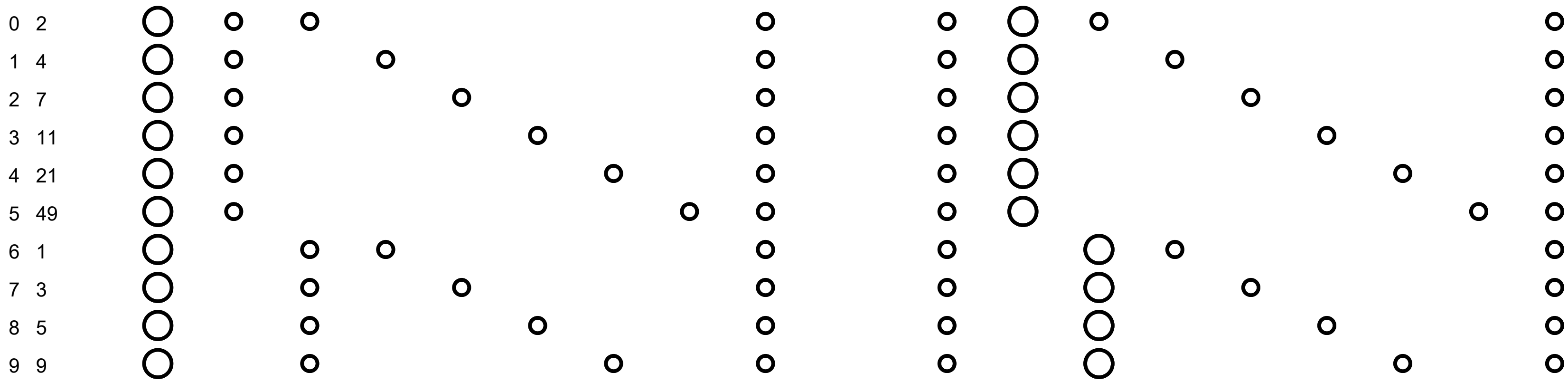
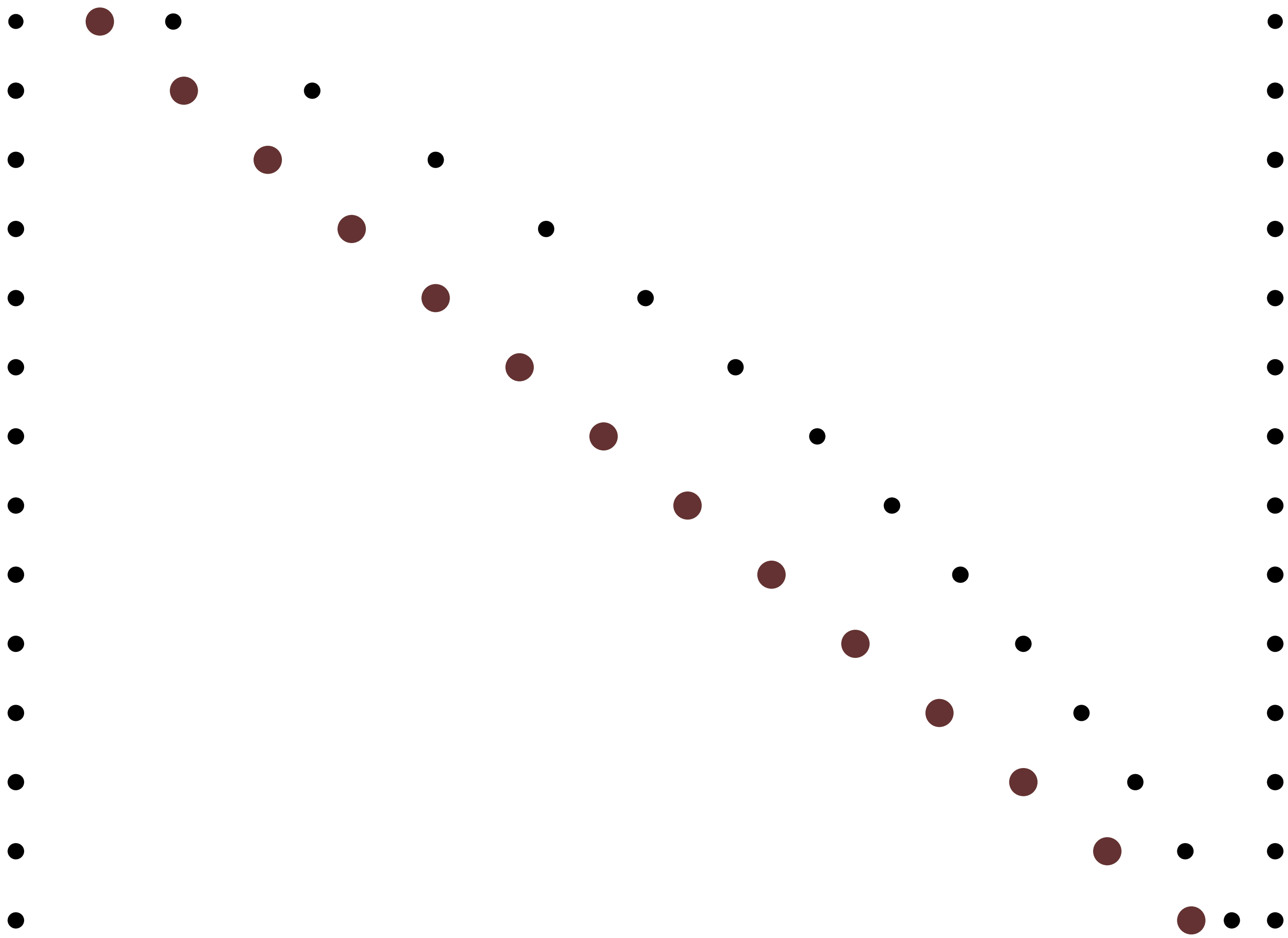
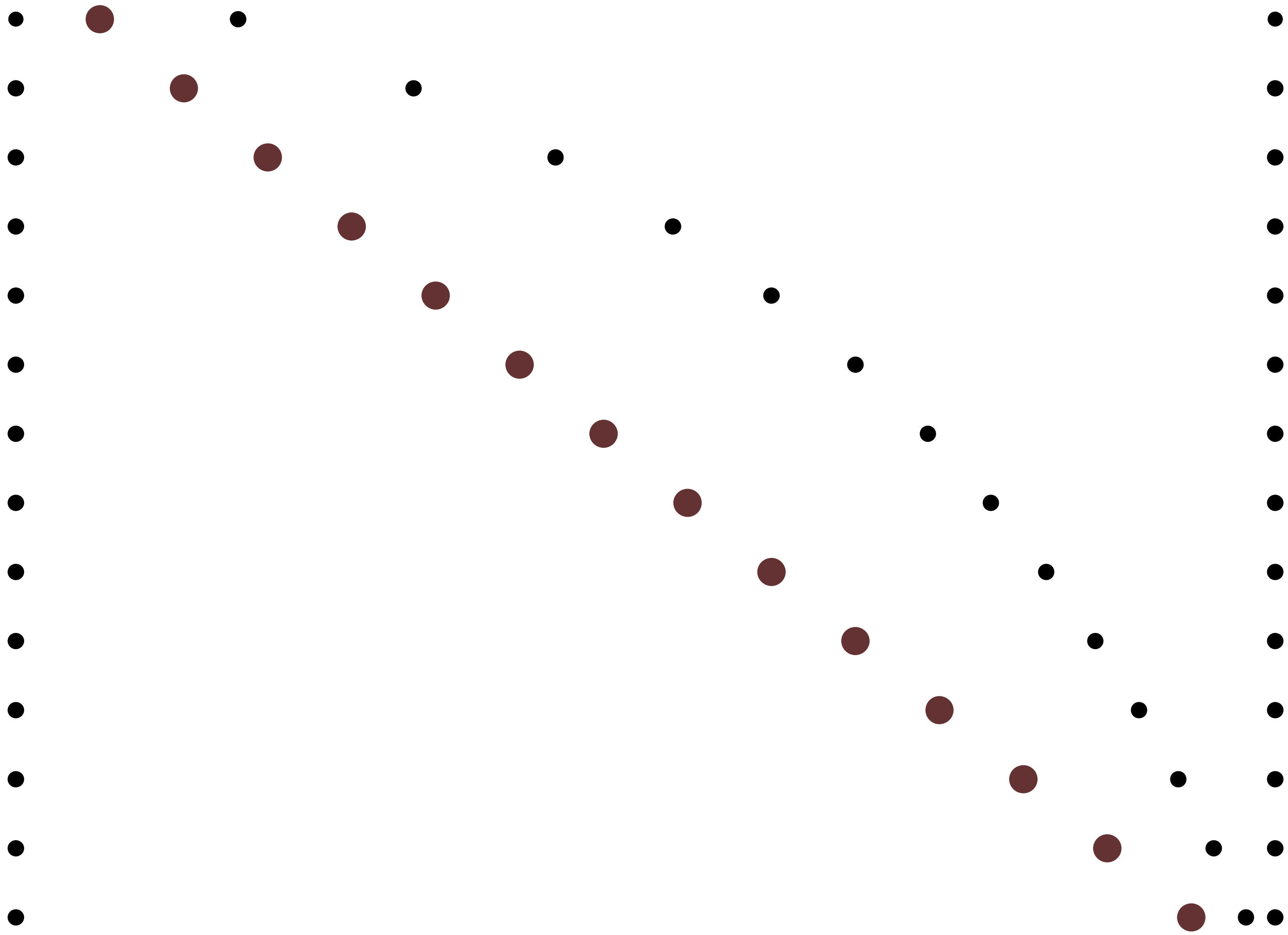


Example set of 18 unambiguous point configurations.

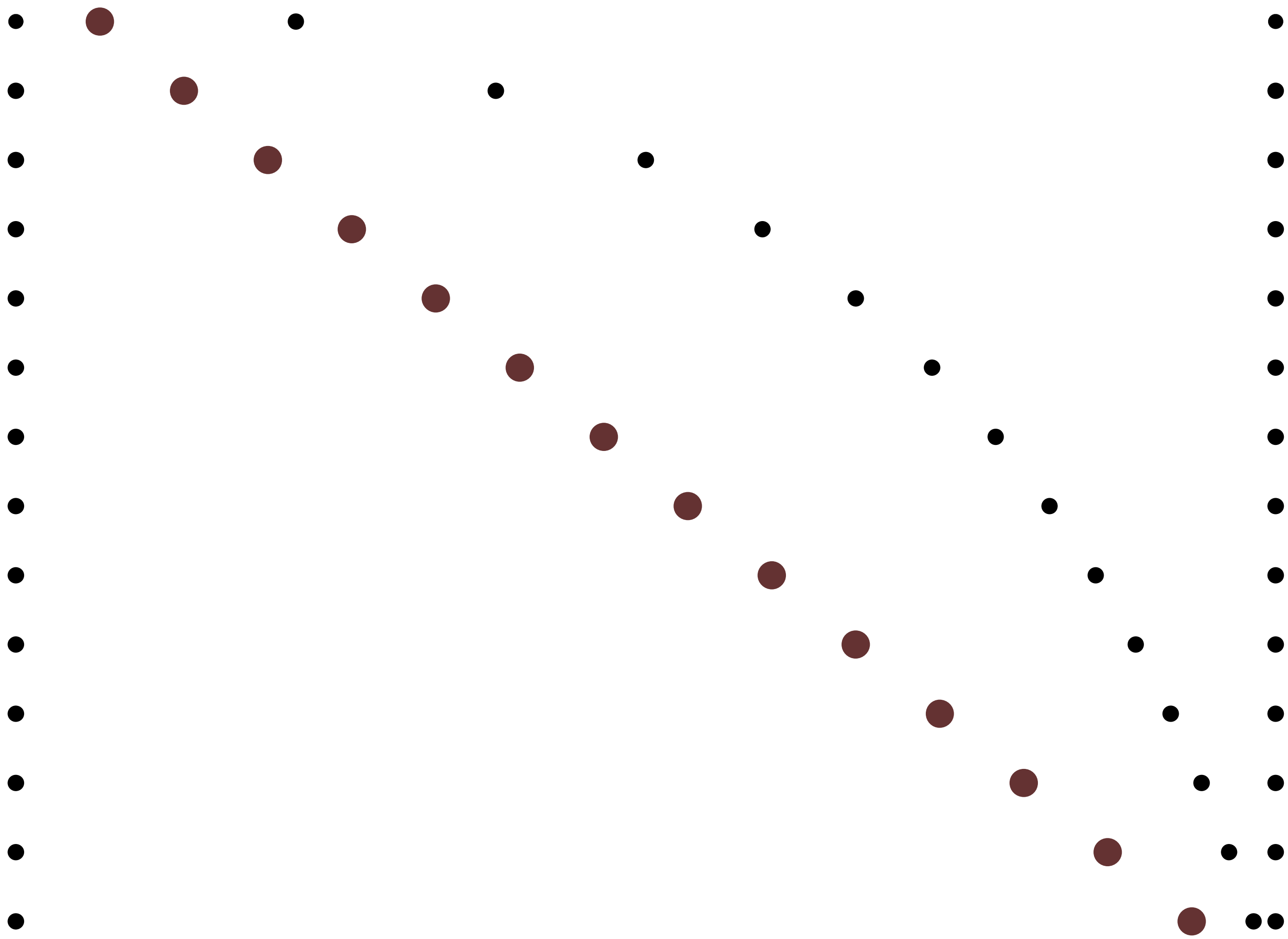
Cross-ratio as a Descriptor (cr=2)



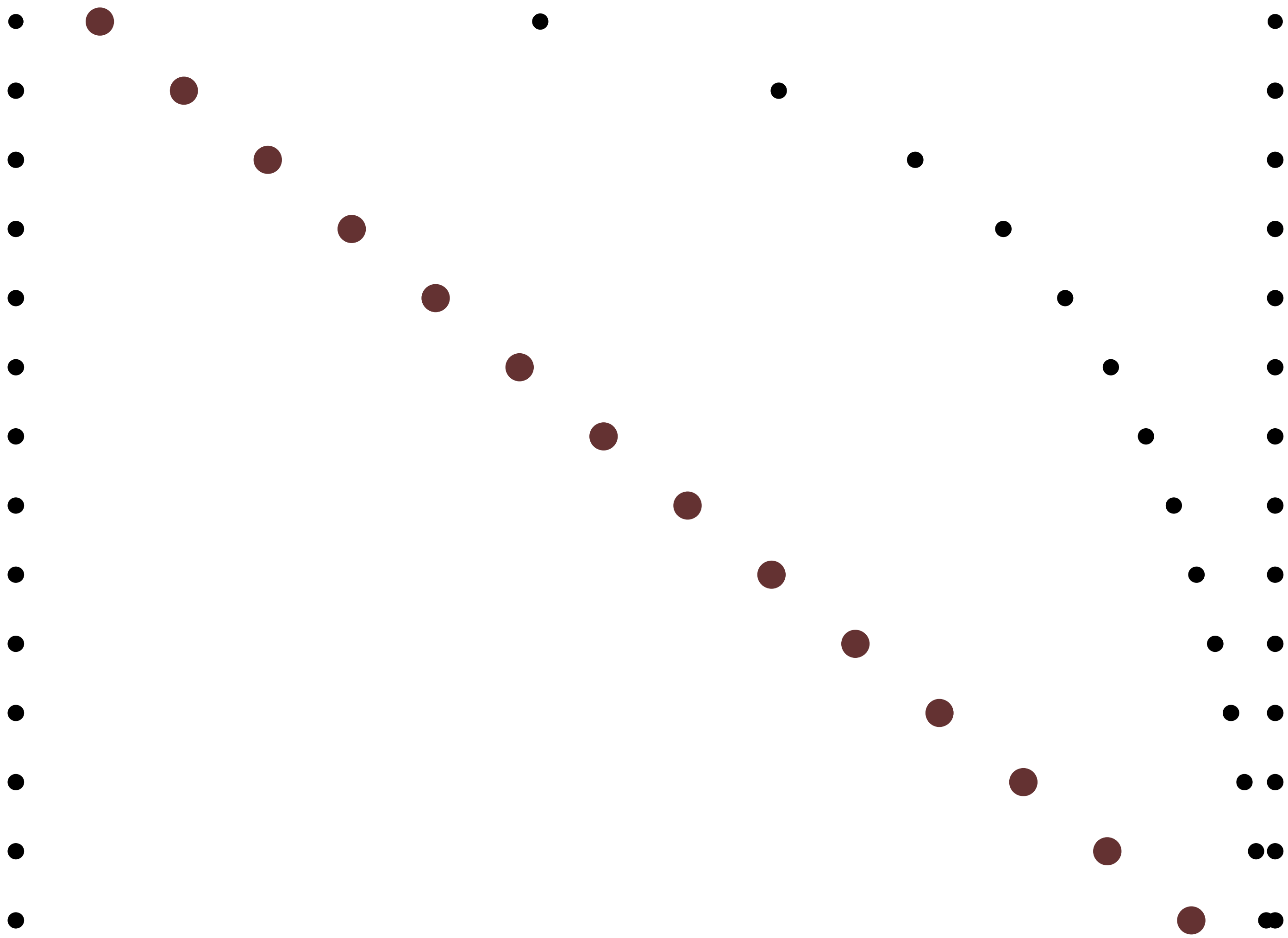
Cross-ratio as a Descriptor (cr=3)



Cross-ratio as a Descriptor (cr=4)

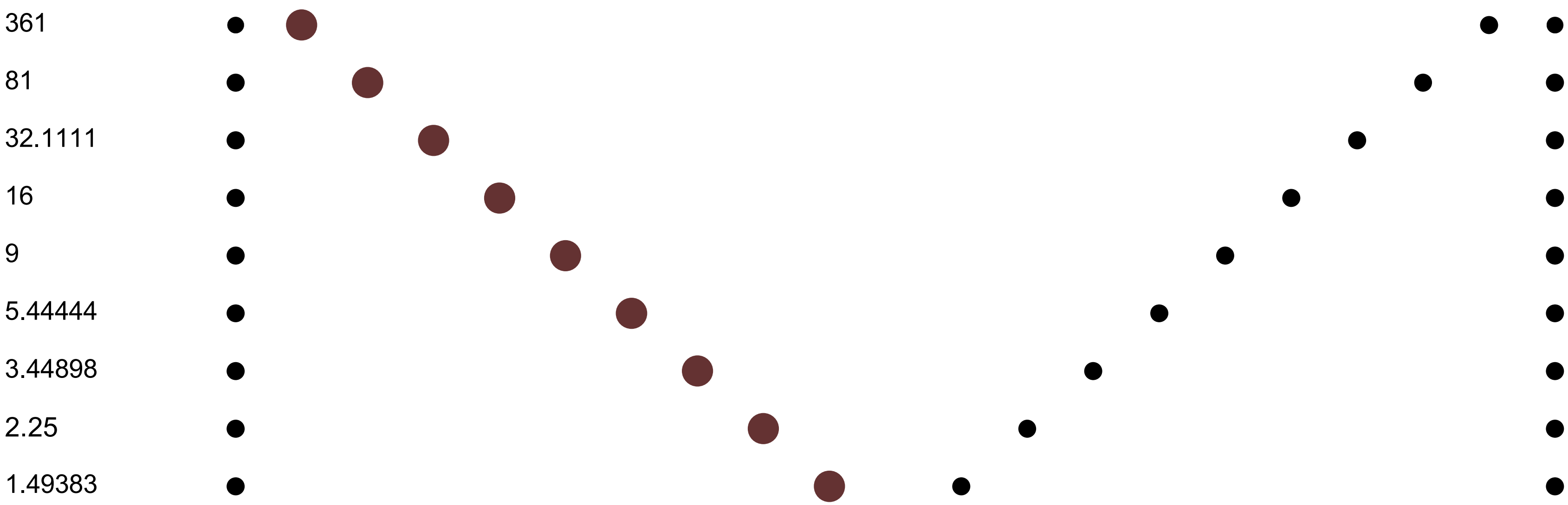


Cross-ratio as a Descriptor (cr=10)



Cross-ratio as a Descriptor

Beads placed at increasing distance from center



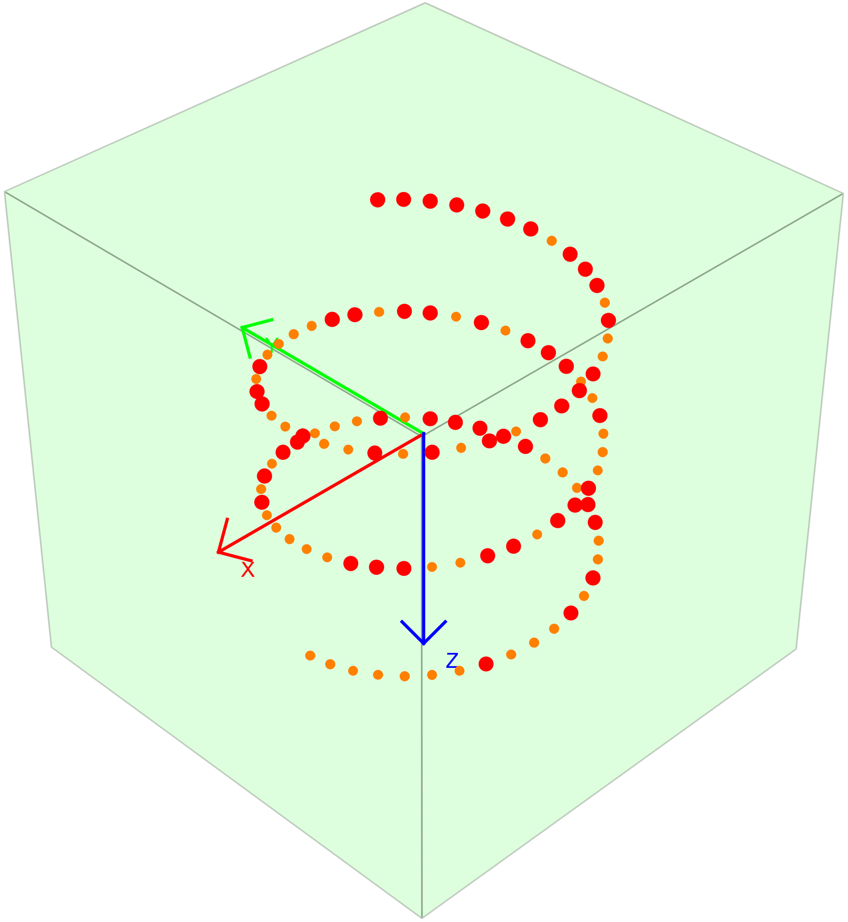
Phantom Design

Flexibility in Spacial Arrangement

What could phantoms look like?

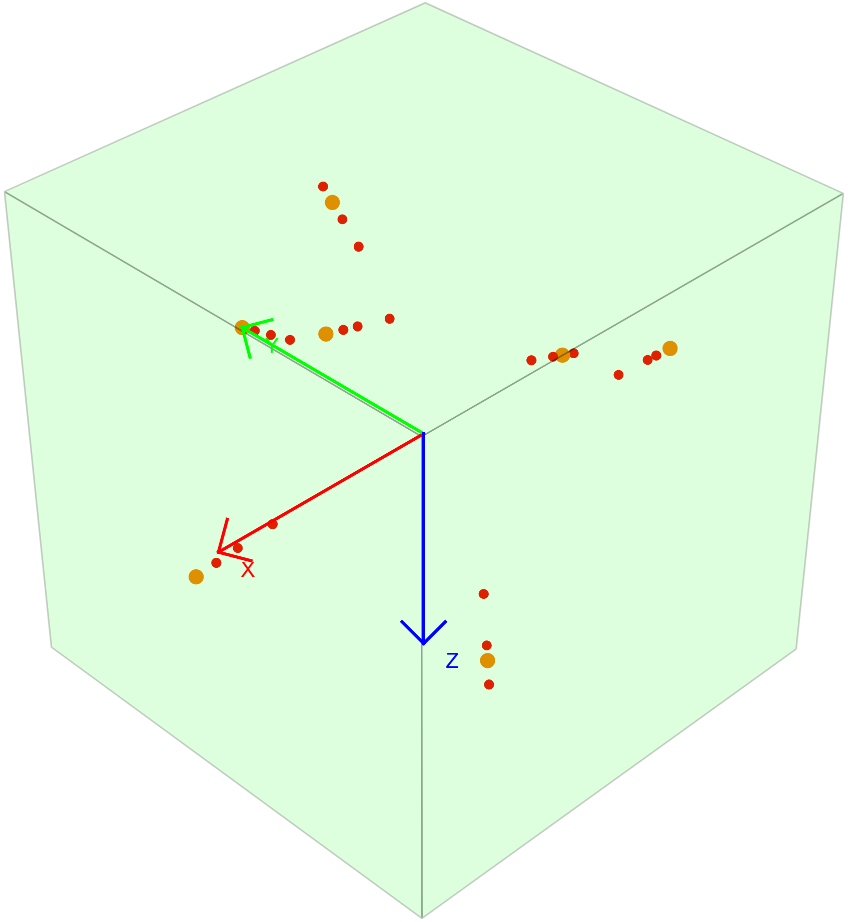
For Reference: PDS2

n=108



What could phantoms look like?

Randomized phantoms with varying number of points.

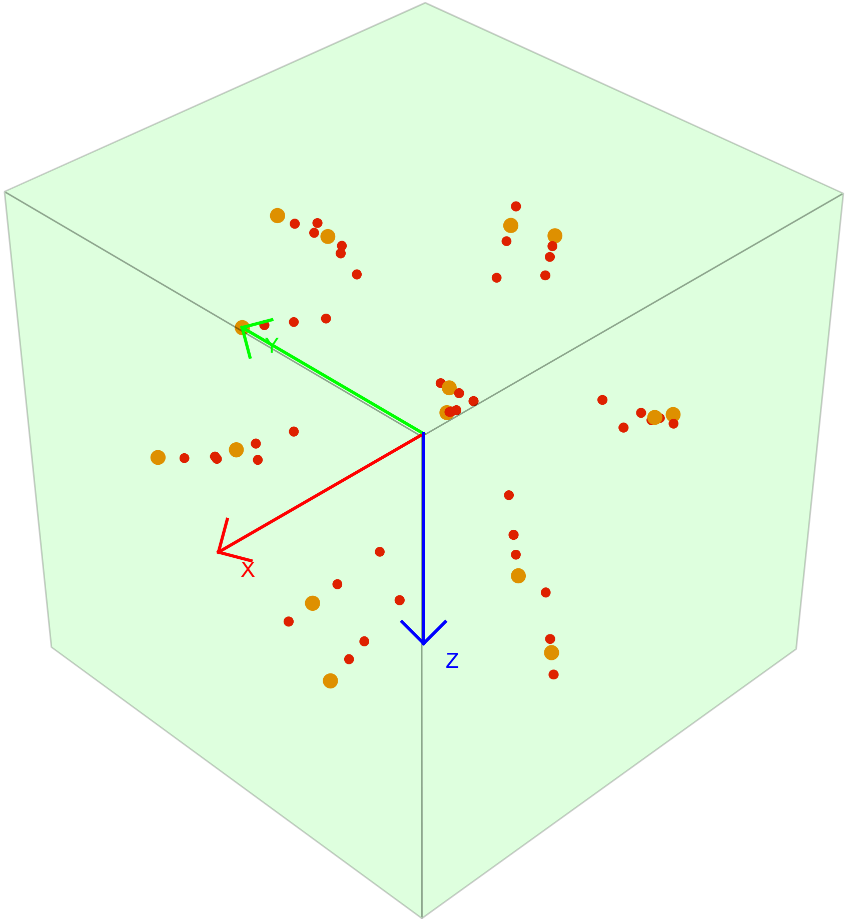


n=28

What could phantoms look like?

Randomized phantoms with varying number of points.

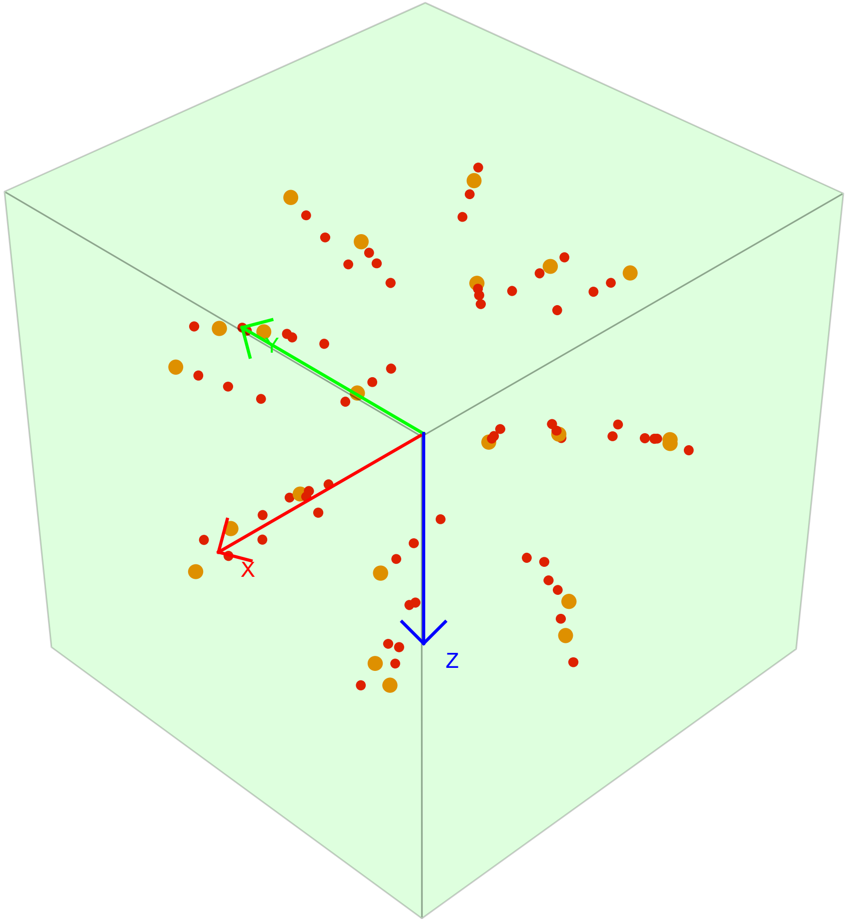
n=60



What could phantoms look like?

Randomized phantoms with varying number of points.

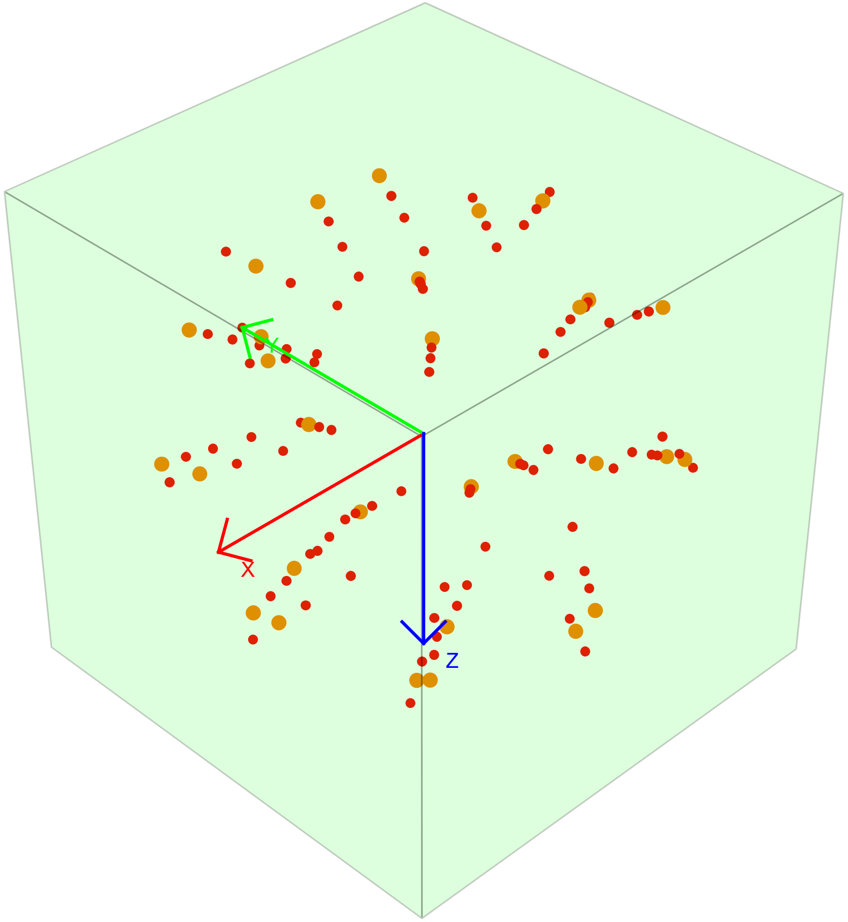
n=90



What could phantoms look like?

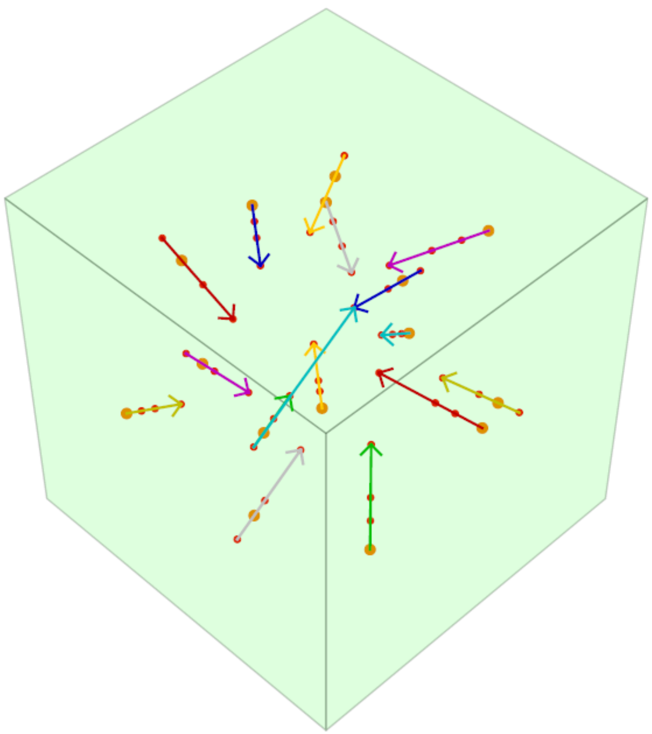
Randomized phantoms with varying number of points.

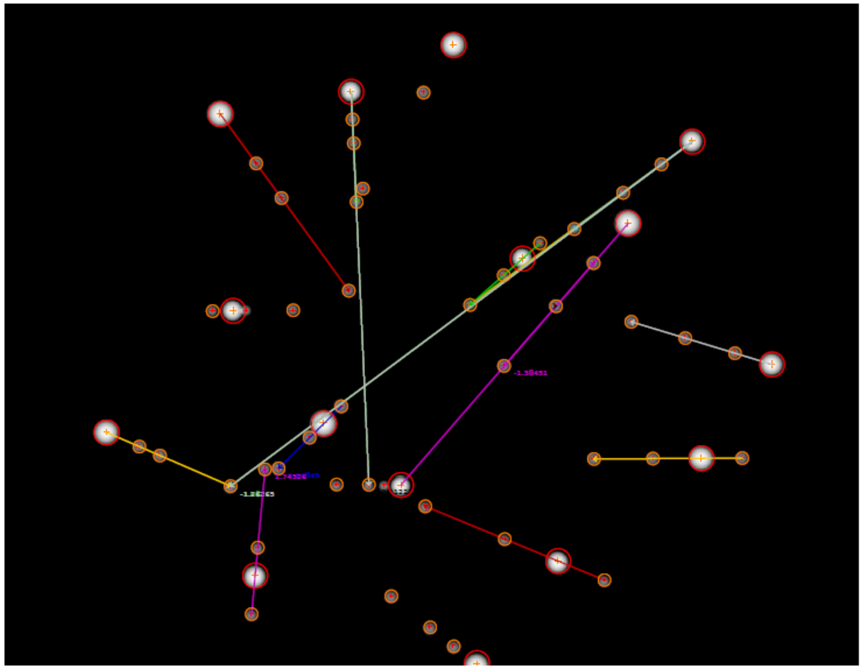
n=120



Example Matching

n=60



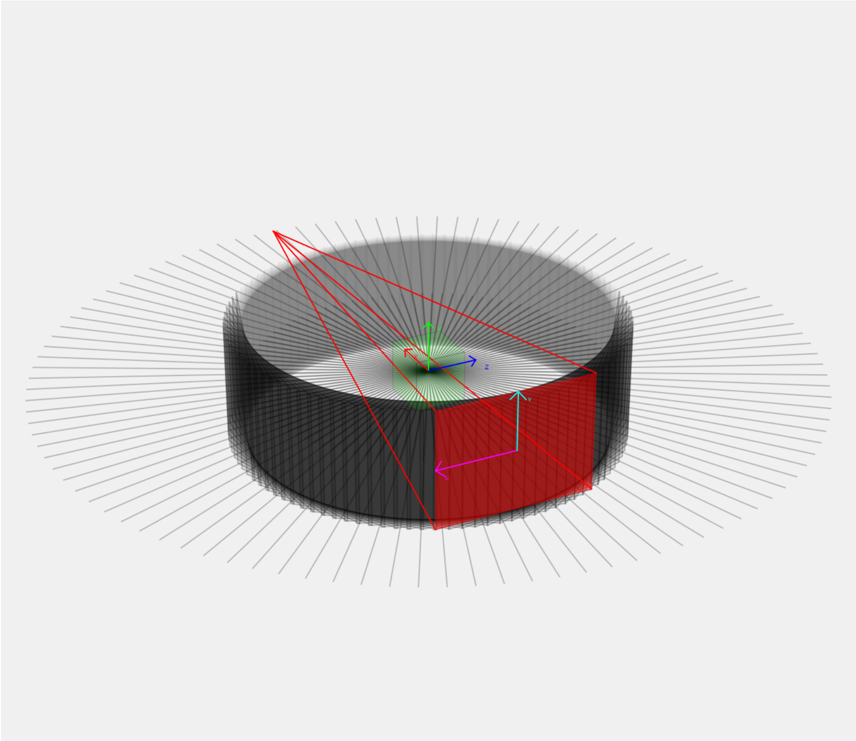


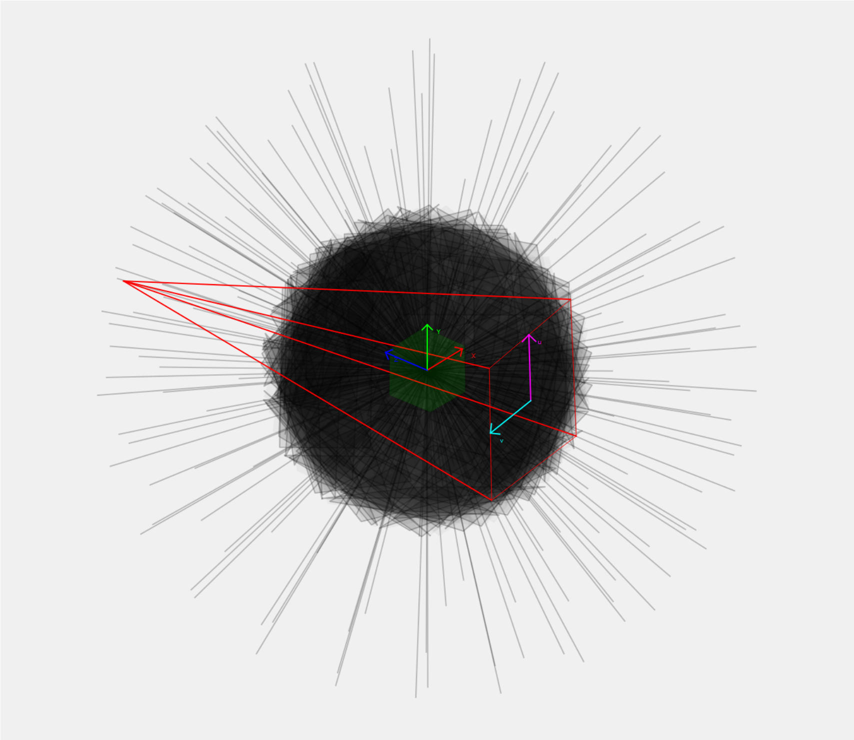
Validation

using Numerical Simulations

Trajectories

Evaluation on Circular and “Spherical” trajectories

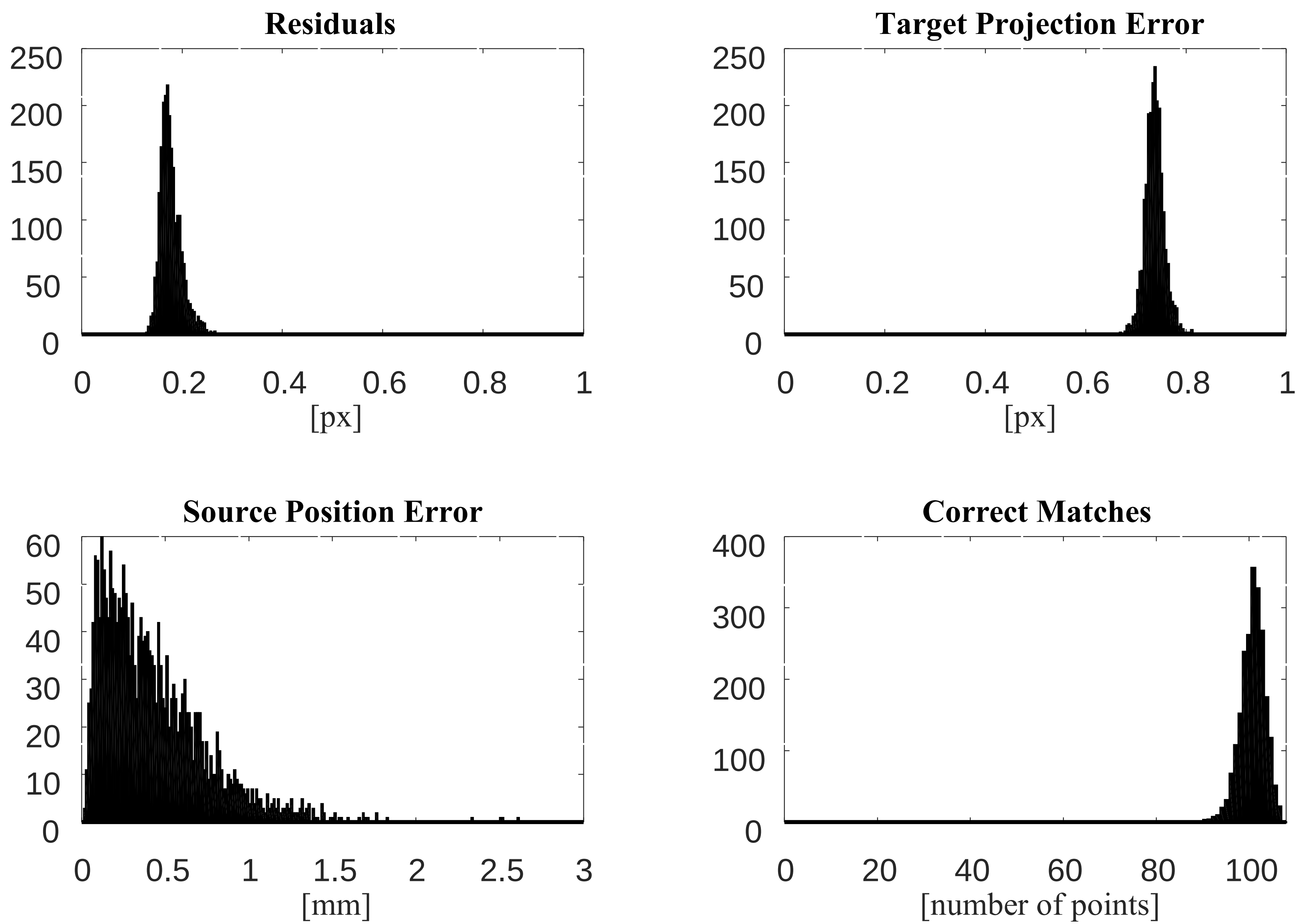




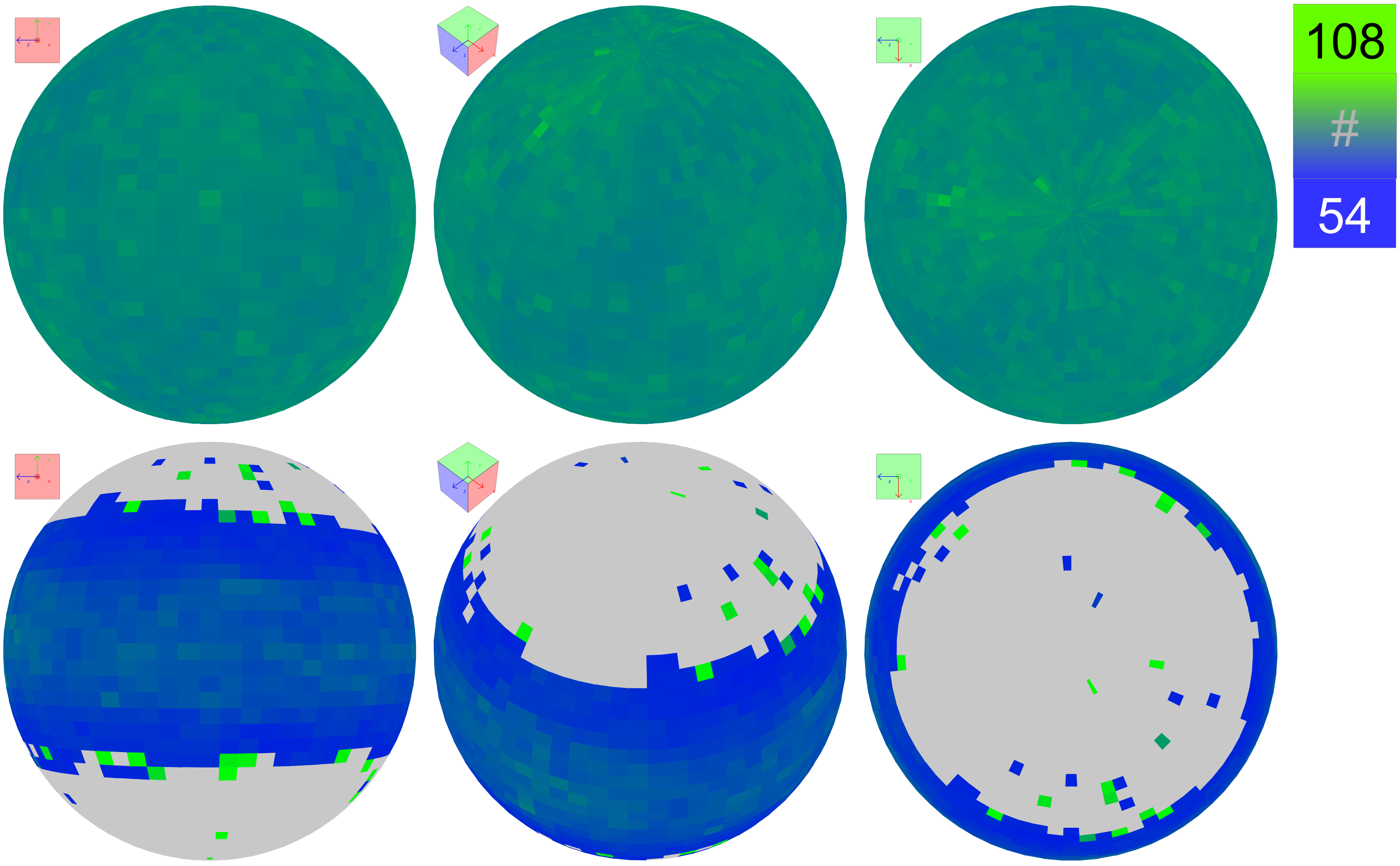
120 projections

1449 projections

Results


Residuals
250
200
150
100
50
0
0 0.2 0.4 0.6 0.8 1
[px]
Target Projection Error
250
200
150
100
50
0
0 0.2 0.4 0.6 0.8 1
[px]
Source Position Error
60
50
40
30
20
10
0
0 0.5 1 1.5 2 2.5 3
[mm]
Correct Matches
400
300
200
100
0
0 20 40 60 80 100
[number of points]

Results – comparison to PDS2



Conclusion

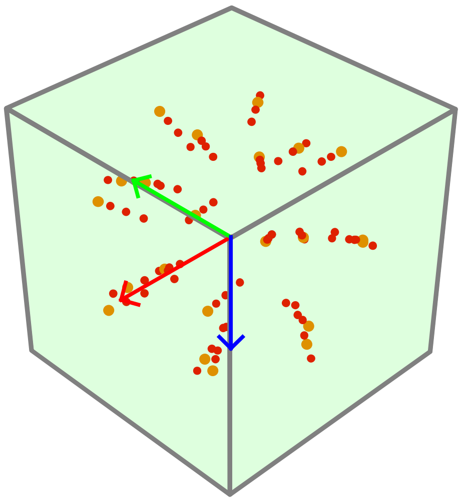
Conclusion & Future Work

- A “building block” for geometry calibration phantoms

  → Flexible w.r.t. truncation, scale etc.

  → Cheap manufacturing

  → Treats all directions equally



Conclusion & Future Work

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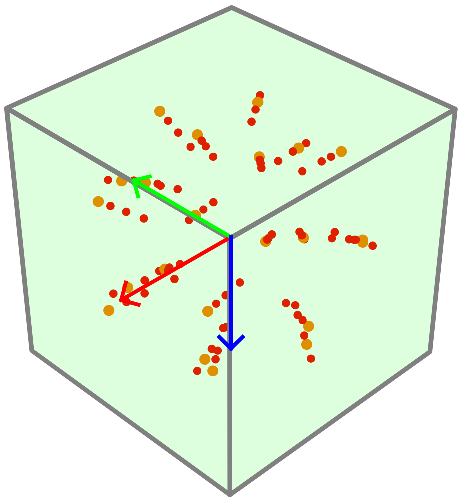
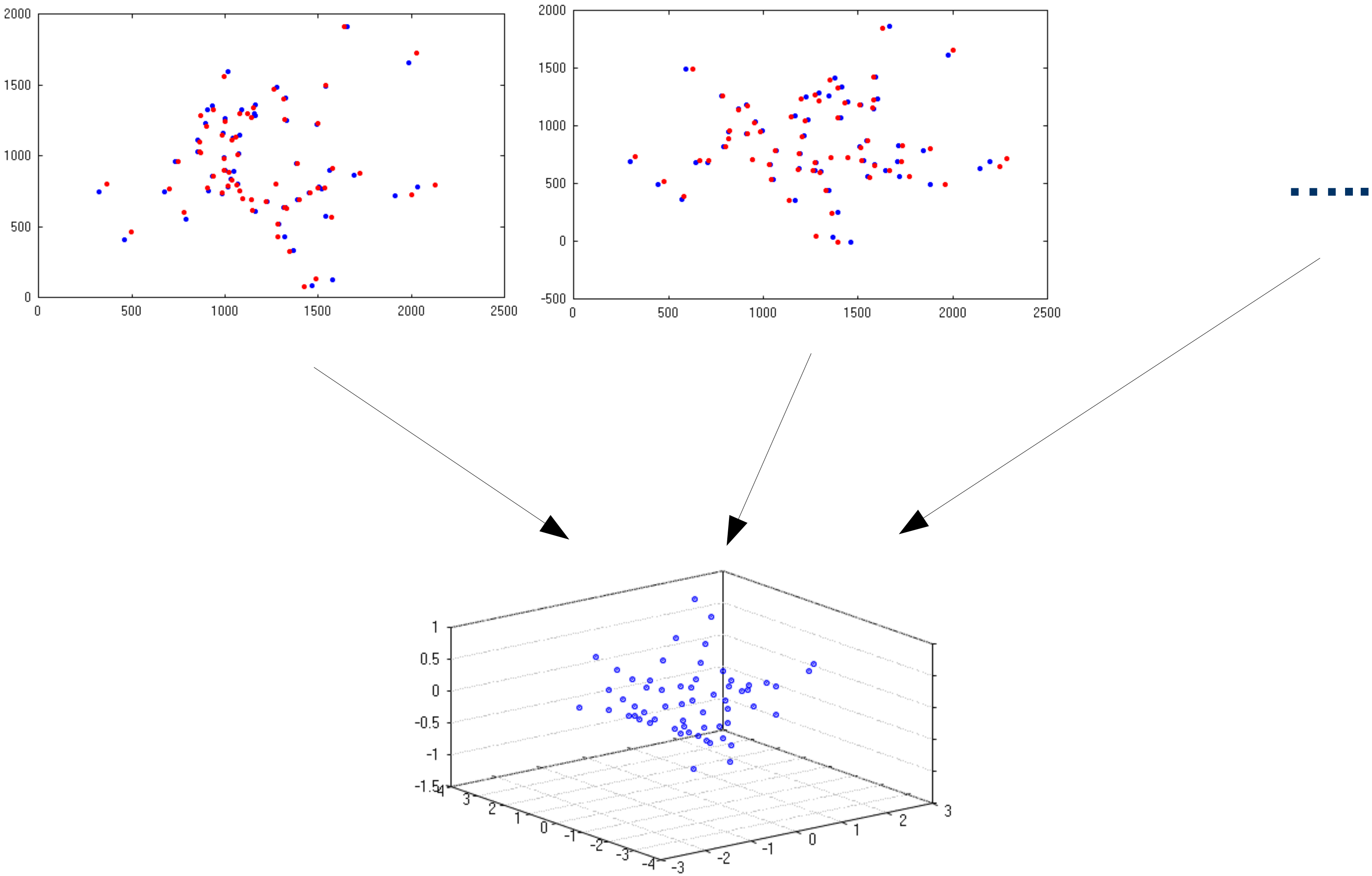
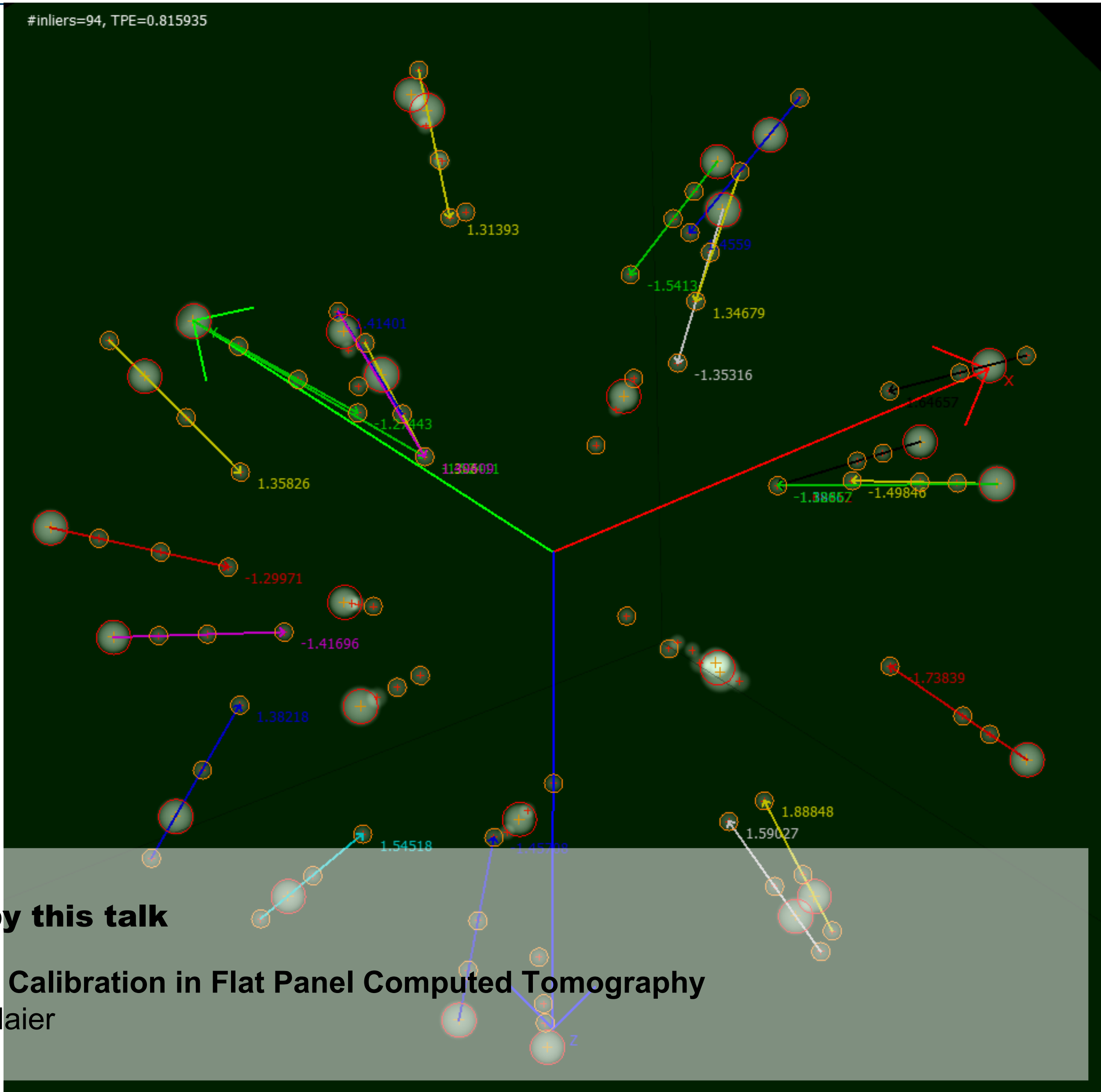


- Other computer vision algorithms
  - → Factorization, stratification and bundle adjustment
- Motion tracking individual blocks

Preview: factorization



Questions.



Publication partially covered by this talk

Projective Invariants for Geometric Calibration in Flat Panel Computed Tomography
A. Aichert, B. Bier, L. Rist and A. K. Maier
Accepted for CT Meeting 2018

The Cross Product and the Plücker Matrix

$$\mathbf{l} = \mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 b_2 - b_1 a_2 \\ b_0 a_2 - a_0 b_2 \\ a_0 b_1 - a_1 b_0 \end{pmatrix} = \begin{pmatrix} l_0 \\ l_1 \\ l_2 \end{pmatrix},$$

$$\mathbf{b}^\top \mathbf{a} - \mathbf{a}^\top \mathbf{b} = \begin{pmatrix} a_0 b_0 - b_0 a_0 & a_0 b_1 - b_0 a_1 & a_0 b_2 - b_0 a_2 \\ a_1 b_0 - b_1 a_0 & a_1 b_1 - b_1 a_1 & a_1 b_2 - b_1 a_2 \\ a_2 b_0 - b_2 a_0 & a_2 b_1 - b_2 a_1 & a_2 b_2 - b_2 a_2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & l_2 & -l_1 \\ -l_2 & 0 & l_0 \\ l_1 & -l_0 & 0 \end{pmatrix}.$$

Rest of this talk: how to figure out $\mathbf{P}$?

$$\mathbf{x} \cong \mathbf{P}\mathbf{X}$$

- $\mathbf{x}$ is up to scale identical to $\mathbf{P}\mathbf{X}$

  $\rightarrow$ Cross-product is zero

$$\mathbf{x} \times \mathbf{P}\mathbf{X} = \mathbf{0}$$

- Can be written in matrix form

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \boxed{\begin{matrix} \mathbf{0}^\top & x_3\mathbf{X}^\top & -x_2\mathbf{X}^\top \\ -x_3\mathbf{X}^\top & \mathbf{0}^\top & x_1\mathbf{X}^\top \end{matrix}} \\ \begin{matrix} x_2\mathbf{X}^\top & -x_1\mathbf{X}^\top & \mathbf{0}^\top \end{matrix} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{p}^1 \\ \mathbf{p}^2 \\ \mathbf{p}^3 \end{pmatrix}$$