Deriving Neural Network Architectures using Precision Learning: Parallel-to-fan beam Conversion

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Introduction

- Up to now the concept of *precision learning* [1] was only used to augment networks with prior knowledge
- We want to demonstrate that we can use *precision learning* to formulate different hypothesis on **efficient solution schemes** that are then found as the point of optimality of a deep learning traning process
- We show that on the parallel-to-fan beam conversion problem in the context of Hybrid MR-/X-ray imaging device and compare with an geometrical rebinning method (Fig. 1)

Aim:

- Show that neural networks can be used to find efficient solution schemes for unknown operators in a mathematical model
- Find a convolution-based algorithm for the conversion task

Material and Methods

Finding a convolution-based algorithm:

- parallel-to-fan conversion problem using the discrete reconstruction formulas:

$$\boldsymbol{A}_f\boldsymbol{A}_p^\top\boxed{(\boldsymbol{A}_p\boldsymbol{A}_p^\top)^{-1}}\boldsymbol{p}_p = \boldsymbol{p}_f$$

- **formulate a hypothesis** that the inverse bracket can be approximated by an **projection-independent or –dependent filter**:

$$\boldsymbol{A}_f\boldsymbol{A}_p^\top\boxed{\boldsymbol{F}^H\boldsymbol{K}\boldsymbol{F}}\boldsymbol{p}_p = \hat{\boldsymbol{p}}_f$$

- This formula directly gives us the network topology (Fig. 3) to find the unknown operator as in [2] .

Training:

- Additional scaling layer **S** compensates for the normalization between the forward- / back-projector. **S** gets pre-trained.
- Only numerical phantoms are used (Fig. 2**)**
- Weights of filter layer **K** are smoothed with a small Gaussian kernel after each epoch to obtain continuous weights

Results

- Projection-**indepented** filter can be found but is instable (Fig. 4)
- Projection-**depented** filter can be found (Fig. 5)
- Sharper results than the interpolation-based method (Fig. 6)

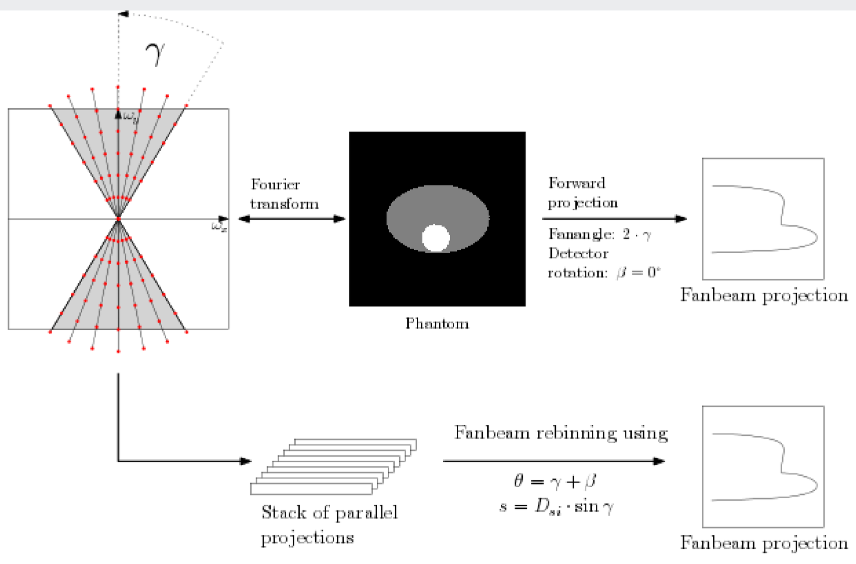

Figure 1: Geometrical rebinning approach [3].

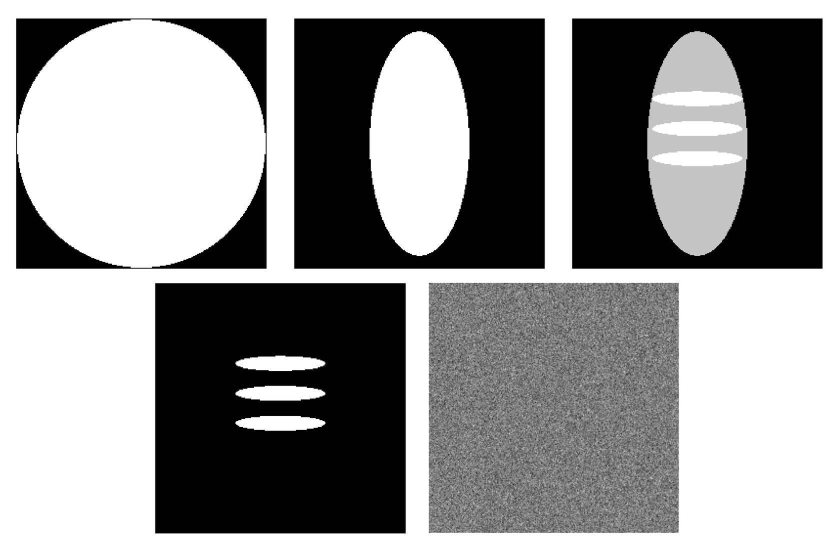

Figure 2: Numerical training data.

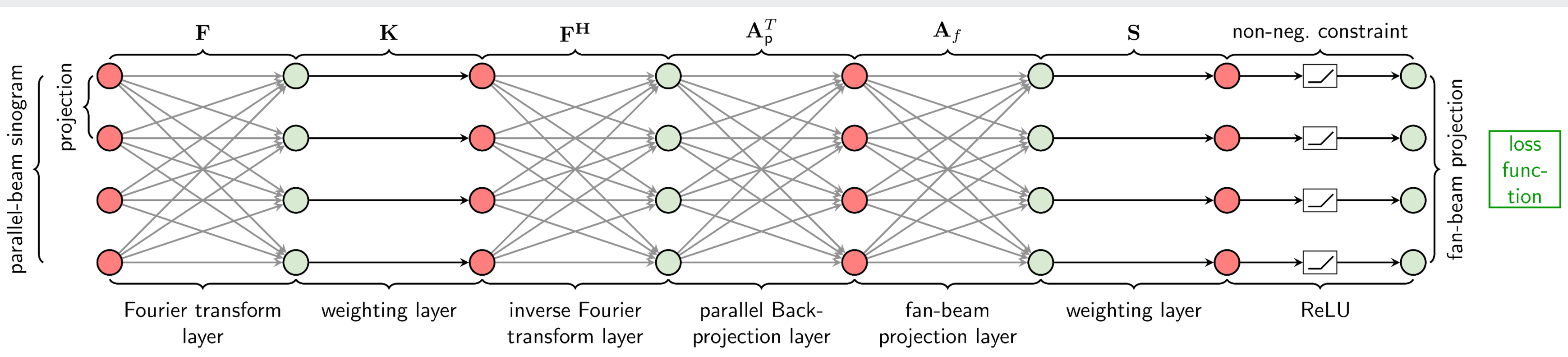

Figure 3: Proposed network topology based on mathematical model. Only layer **K** and **S** are trained.

Results

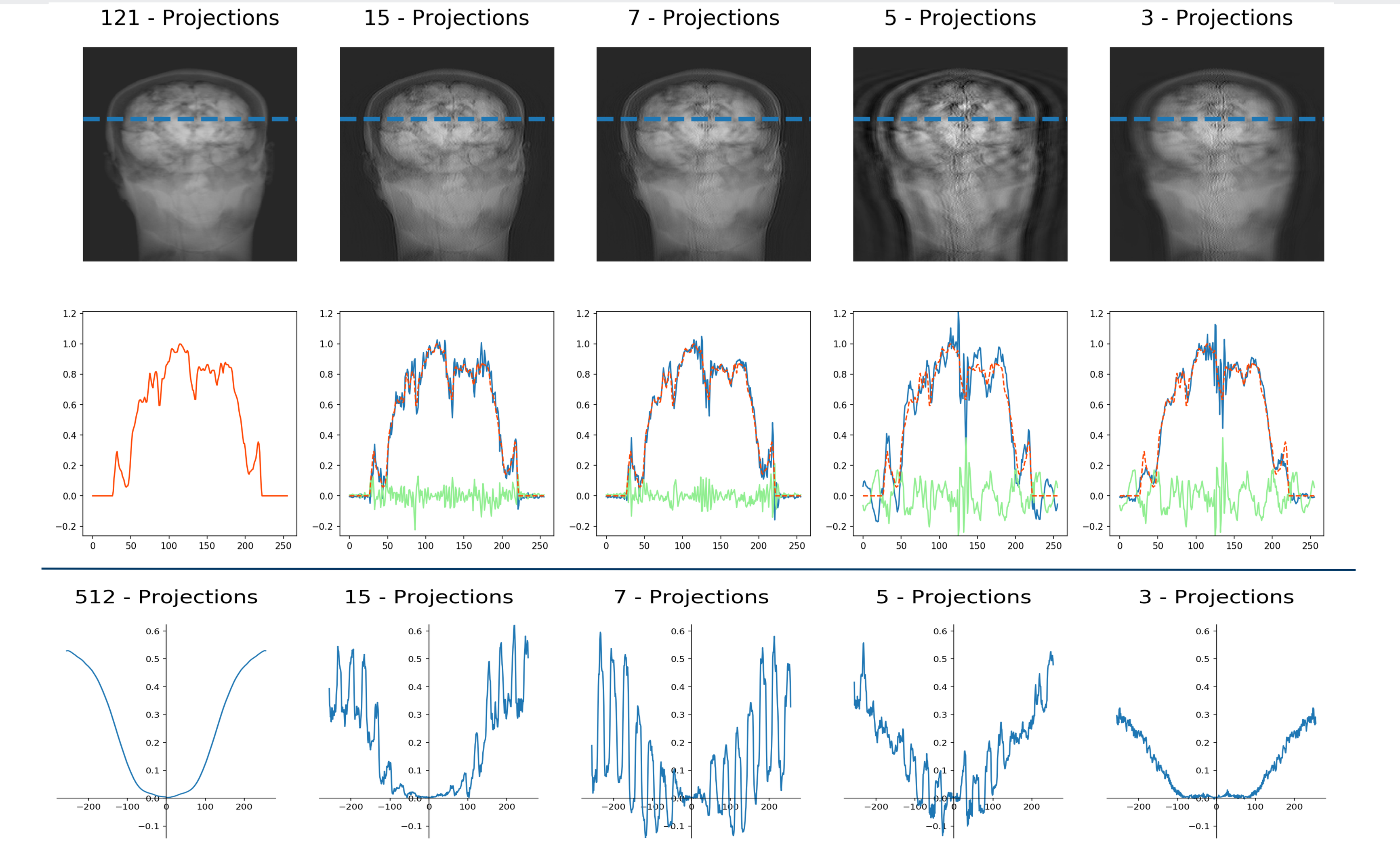

Figure 4: Sub-sampling comparison projection-independent filter. The plot colors are red for the reference, blue for the respective line profile and green for the difference. The bottom row shows the respective filters.

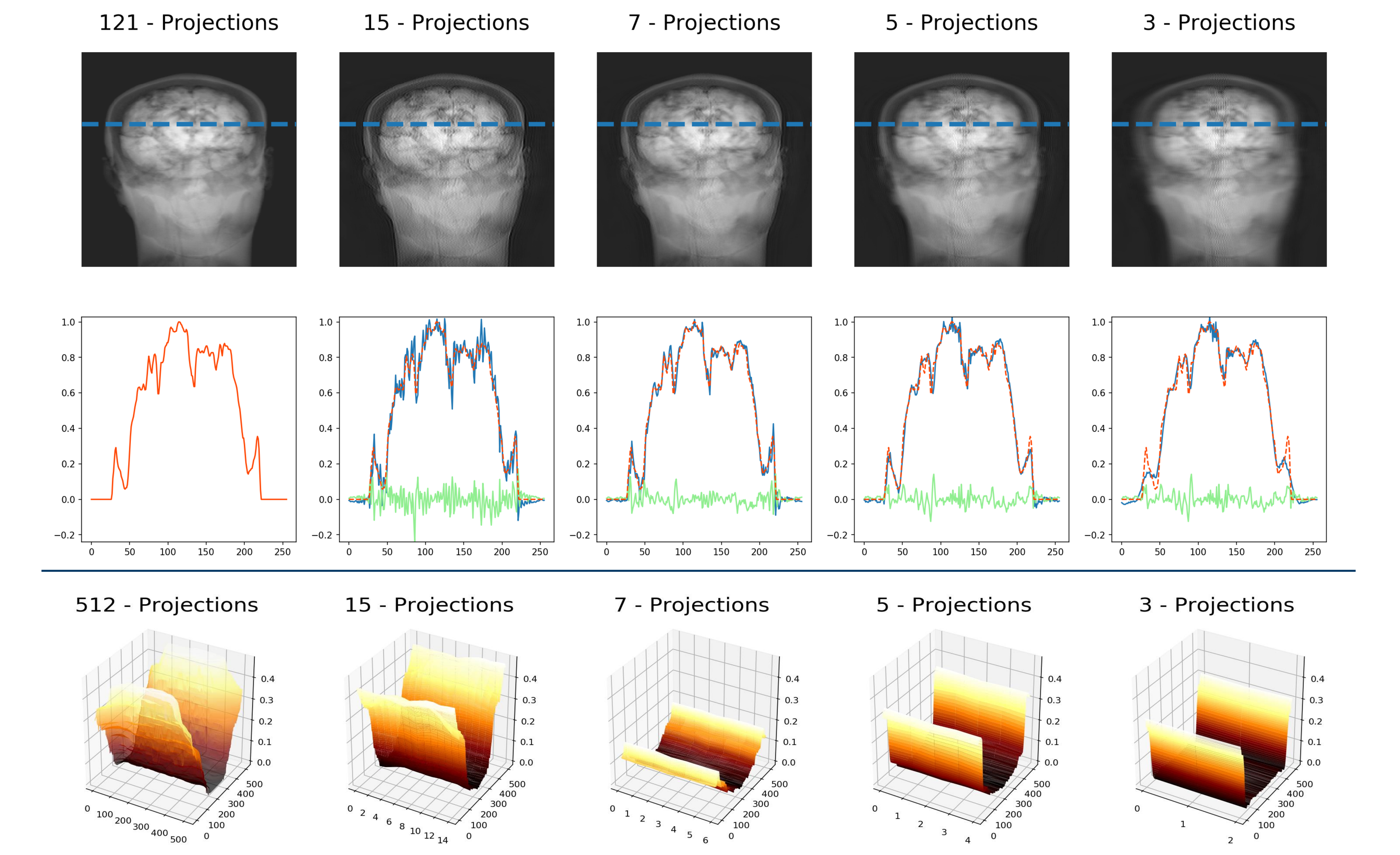

Figure 5: Sub-sampling comparison projection-dependent filter. The plot colors are red for the reference, blue for the respective line profile and green for the difference. The bottom row shows the respective filters.

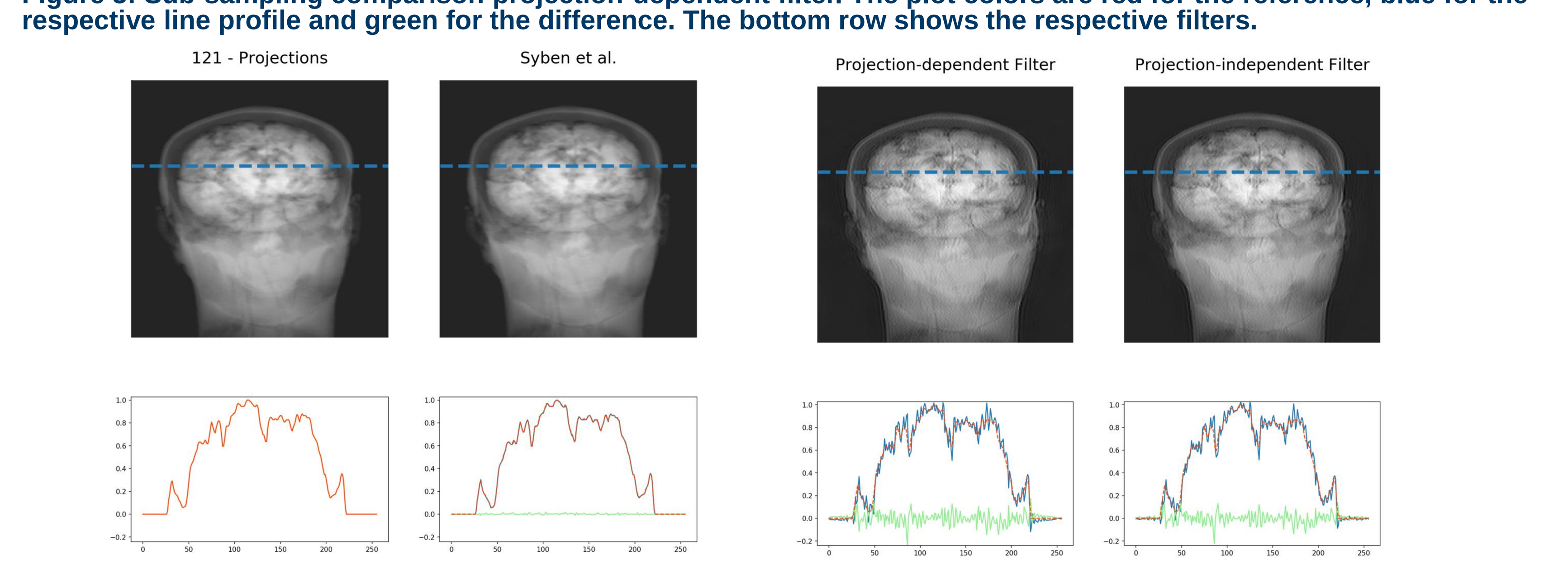

Figure 6: Comparison of the geometrical and the proposed convolution-based rebinning method with projection-dependent and –independent filter. 121 projections are used for the geometrical method to calculate the difference.

Conclusion

- An efficient convolution-based algorithm could be found by learning the unknown operator in the mathematical model
- The convolution-based algorithm achieve sharper results that the interpolation-based pendant
- Learning process has to be better understood for a more substantiated regularization

References

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Acknowledgements

EIT Health is supported by the EIT, a body of the European Union