Noise, Filtering and Smoothing

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Noise Sources

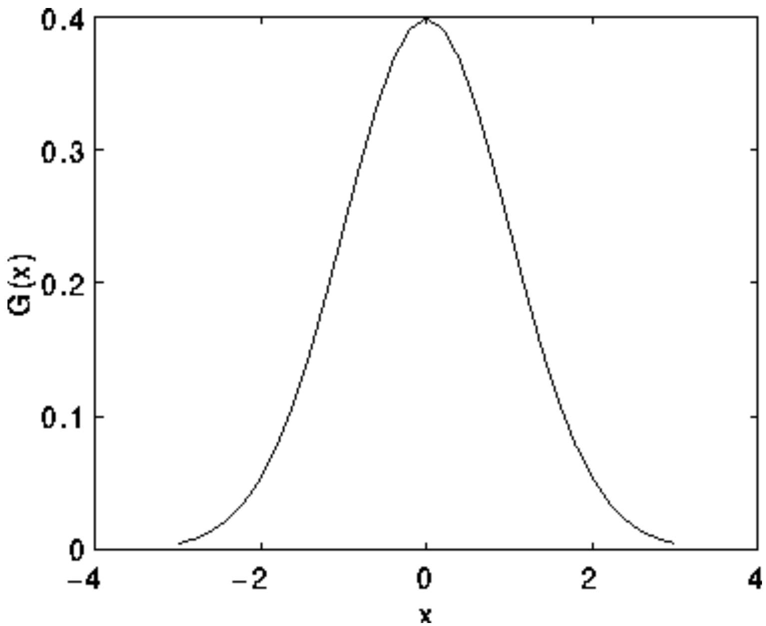
- Photon noise: variation in the #photons falling on a pixel per time interval T.
- Saturation: each pixel can only generate a limited amount of charge.
- Blooming: saturated pixel can overflow to neighboring pixels.

Noise Sources - continued

- Thermal noise: heat can free electrons and generate a response when there is no real signal.
- Electronic noise.
- Burned pixels.
- Black is not black.
- Keep in mind: Camera response may not be linear over the number of photons falling on a surface (camera gamma)

Detector Noise

- Source of noise: the discrete nature of radiation, i.e. the fact that each imaging system is recording an image by counting photons.
- Can be modeled as an independent additive noise which can be described by a zero-mean Gaussian.



Salt and Pepper Noise

- A common form of noise is caused by *data drop-out noise.*
- It is also known as commonly referred to as intensity spikes, speckle, or salt and pepper noise.
- Sources of error:
  - Errors in the data transmission.
  - Burned pixels: the corrupted pixels are either set to the maximum value (which looks like snow in the image) or are set to zero ("peppered" appearance), or a combination of the two.
  - Single bits are flipped over.
- Isolated/localized noise. It only affects individual pixels.

Filtering

- Most of the images we capture are noisy.
- Goal:

Noisy Image_{in} → Filter → Clean Image_{out}

- This notion of filtering is more general and can be used in a wide range of transformations that we may want to apply to images.

Image_{in} → Filter → Image_{out}

- Mathematically, a filter *H* can be treated as a function on an input image I:

$$H(I) = R$$

- Note: We use the terms *filter* and *transformation* interchangeably.

Linear Transformation

- A transformation H is **linear** if, for any inputs $I_1(x,y)$ and $I_2(x,y)$ (in our case input images), and for any constant scalar α we have:

$$H(\alpha I_1(x,y)) = \alpha H(I_1(x,y))$$

and

$$H(I_1(x,y) + I_2(x,y)) = H(I_1(x,y)) + H(I_2(x,y))$$

- This means:
  - Scaling of the input corresponds to scaling of the output.
  - Filtering an additive image is equivalent to filtering each image separately and then adding the results.

Shift-Invariant Transformation

- A transformation *H* is **shift-invariant** if for every pair (x_0, y_0) and for every input image $I(x,y)$, such that

$$H(I(x,y)) = R(x,y)$$

we get

$$H(I(x-x_0, y-y_0)) = R(x-x_0, y-y_0)$$

- This means that the filter *H* does not change as we shift it in the image (as we move it from one position to the next).

Convolution

- If a transformation (or filter) is linear shift-invariant (LSI) then one can apply it in a systematic manner over every pixel in the image.
- **Convolution** is the process through which we apply **linear shift-invariant filters** on an image.

- Convolution is defined as:

$$R(x,y)=\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}H(x-i,y-j)I(i,j)$$

and is denoted as:

$$R=H*I$$

Another Look at Convolution

- Filtering often involves replacing the value of a pixel in the input image *F* with the weighted sum of its neighbors.
- Represent these weights as an image, ***H***
- ***H*** is usually called the **kernel**
- The operation for computing this weighted sum is called **convolution.**

$$R = H * I$$

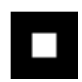
- Convolution is:
  - commutative, $H * I = I * H$
  - associative, $H_1 * (H_2 * I) = (H_1 * H_2) * I$
  - distributive, $(H_1 + H_2) * I = (H_1 * I) + (H_2 * I)$

Smoothing via Simple Averaging

- One of the simplest filters is the mean filter: $H = \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$
- In this case, $R(x,y) = \sum_{i=-1}^{1} \sum_{j=-1}^{1} I(x-i, y-j) H(i,j)$
- It is used for removing image noise, i.e. for smoothing.

Original image

$*$



$=$

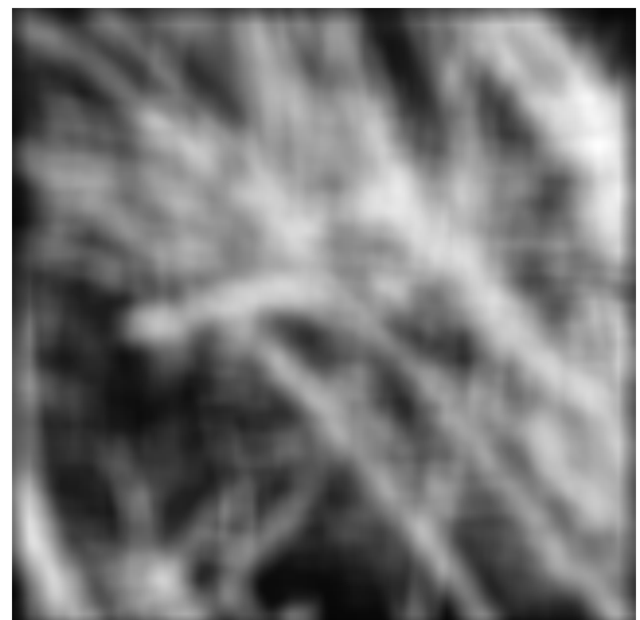
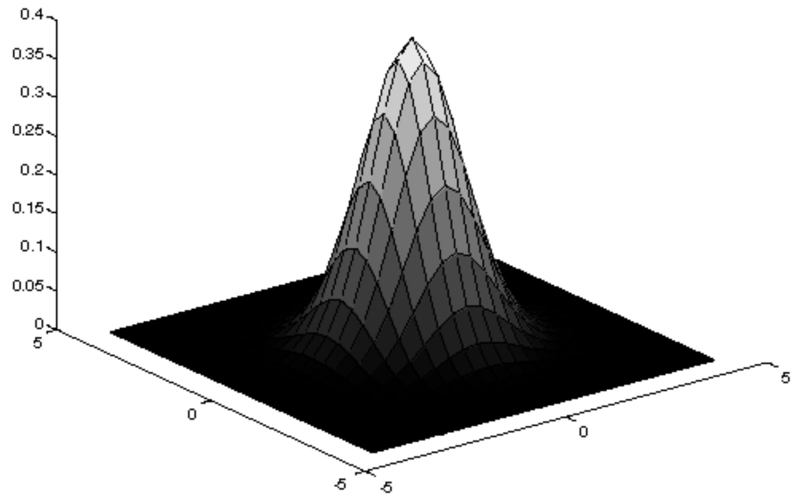


Image after mean filtering (25x25 kernel)

Gaussian Smoothing

- Idea: Use a weighted average. Pixels closest to the central pixel are more heavily weighted.
- The Gaussian function has exactly that profile.
- Gaussian also better approximates the behavior of a defocused lens.



Isotropic Gaussian Filter

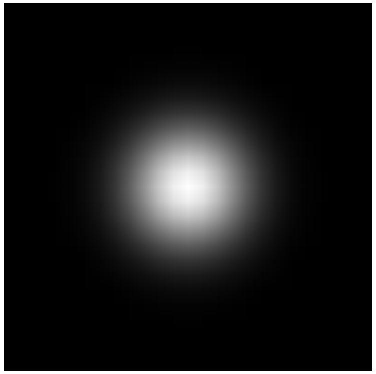
- To build a filter *H*, whose weights resemble the Gaussian distribution, assign the weight values on the matrix *H* according to the Gaussian function:

$$H(i,j) = e^{-(i^2 + j^2)/2\sigma^2}$$

$$H_{Gauss} = \begin{bmatrix} 1/16 & 1/8 & 1/16 \\ 1/8 & 1/4 & 1/8 \\ 1/16 & 1/8 & 1/16 \end{bmatrix}$$

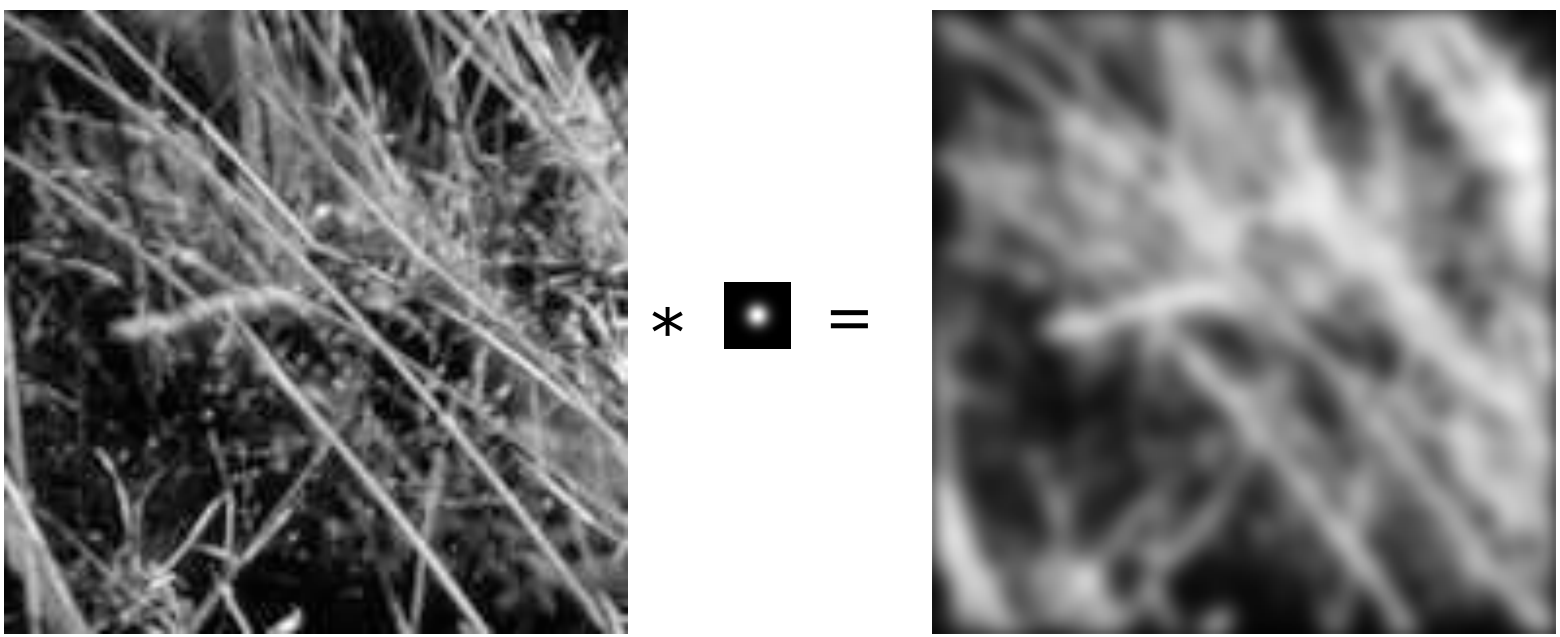
- Small σ, almost no effect, weights at neighboring points are negligible.
- Large σ, blurring, neighbors have almost the same weight as the central pixel.
- Commonly used σ values: Let w be the size of the kernel *H*. Then σ=w/5.

For example for a 3x3 kernel, σ=3/5=0.6



Gaussian Smoothing Example

- Compared to mean filtering, Gaussian filtering exhibits no “ringing” effect.



Original image

Image after Gaussian filtering (25x25 kernel)

"Ringing" effect

Original image

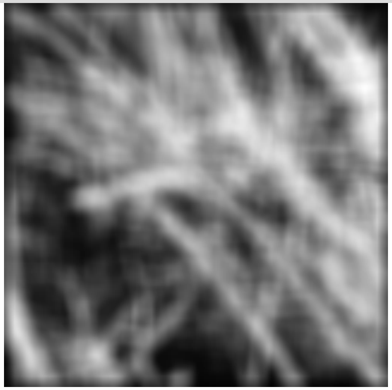


Image after Mean filtering (25x25 kernel)

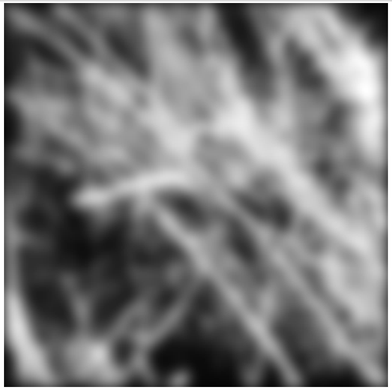
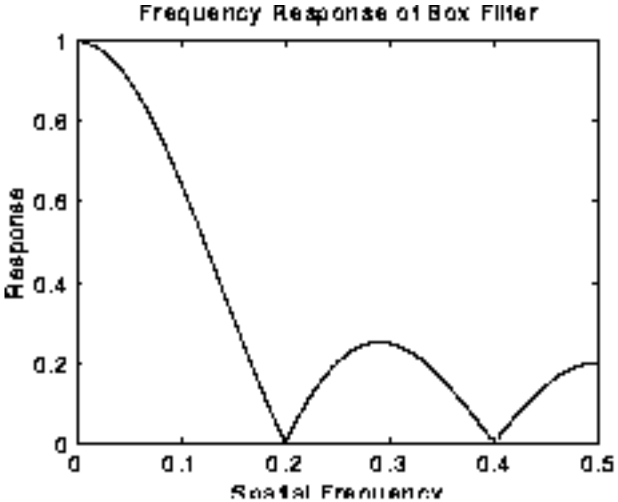
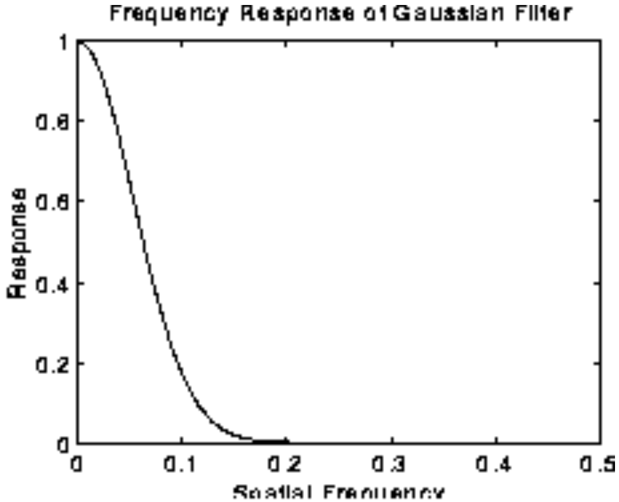


Image after Gaussian filtering (25x25 kernel)

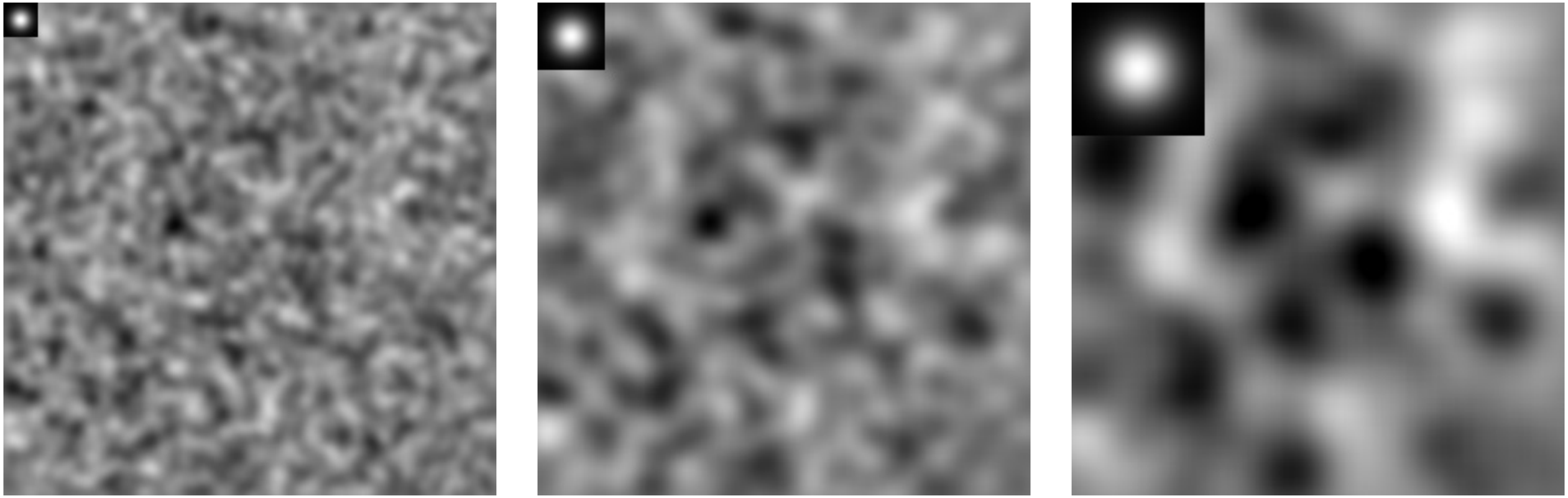




A close look at the frequency response of the two filters show that: compared to Gaussian filtering, mean filtering exhibits oscillations

The Effect of σ

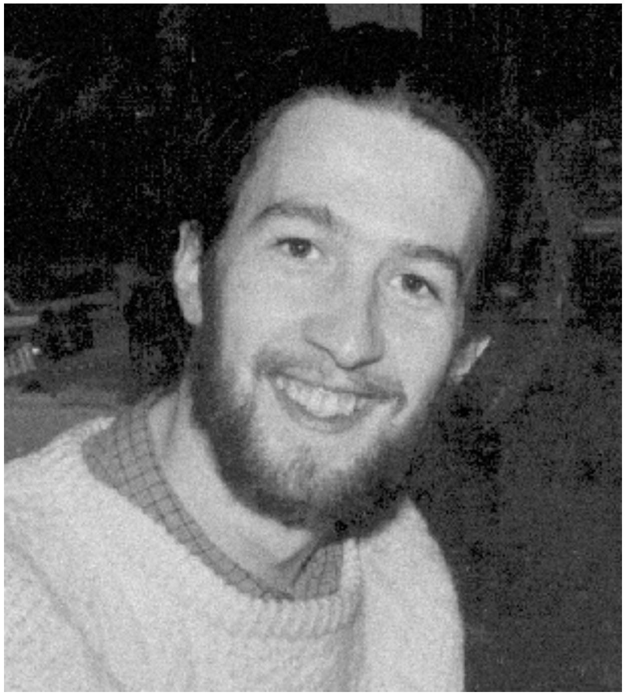
- Different σ values affect the amount of blurring, but also emphasize different characteristics of the image.



Non-Linear Smoothing

- The **median** filter considers each pixel in the image in turn and looks at its nearby neighbors to decide whether or not it is representative of its surroundings.
- It replaces a pixel value with the median of all pixel values in the neighborhood.
- It is a relatively slow filter, because it involves sorting.
- Can **not** be implemented via convolution.

Smoothing Examples



Original image corrupted by a zero mean Gaussian noise with σ=8.

Image after 5x5 Mean filtering

Image after 5x5 Gaussian filtering

Mean Filter

Original image

Image after 3x3 Mean filtering

Image after 7x7 Mean filtering

Image after applying 3 times 3x3 Mean filtering

- Mean filtering is sensitive to outliers.
- It typically blurs edges.
- It often causes a ringing effect.

Gaussian Filtering and Salt & Pepper Noise

Original image

Image with salt-pepper noise (1% prob. that a bit is flipped)

Image after 5x5 Gaussian filtering, σ=1.0

Image after 9x9 Gaussian filtering, σ=2.0

- Gaussian filtering works very well for images affected by Gaussian noise.
- It is not very effective in removing Salt and Pepper noise. Small σ values do not remove the Salt & Pepper noise, while large σ values blur the image too much.

Gaussian Filtering and Salt & Pepper Noise

Original image

Image with salt-pepper noise (1% prob. that a bit is flipped)

After 3x3 mean filtering

After 7x7 mean filtering

After 5x5 Gaussian filter, σ=1.0

After 9x9 Gaussian filter, σ=2.0

Median Filtering and Salt & Pepper Noise

Original image

Image with salt-pepper noise (5% prob. that a bit is flipped)

Image after 3x3 Median filtering

Image after 7x7 Median filtering

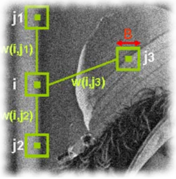
Image after applying 3 times 3x3 Median filtering

- Median filtering preserves high spatial frequency details.
- It works well when less than half of the pixels in the smoothing window have been affected by noise.
- It is not very effective in removing Gaussian noise.

Non-Local Means

- The output pixel is a weighted average of all the image pixels.

$$R(\vec{p}) = \sum_{\vec{q}\in N} w(\vec{p},\vec{q}) I(\vec{q})$$



where *N* is the set of all pixel positions, $\vec{p} = (x, y)$ is a pixel position, $0 \le w(\vec{p},\vec{q}) \le 1$ and $\sum_{\vec{q}\in N} w(\vec{p},\vec{q}) = 1$.

- The weight assigned to each pair of pixels depends on the similarity of the *grey values* in the neighborhood $\mathscr{N}(\vec{p})$ centered around each of the two pixels:

$$w(\vec{p},\vec{q}) = \frac{1}{Z(\vec{p})} e^{-\frac{\left\| I(\mathscr{N}(\vec{p})) - I(\mathscr{N}(\vec{q}))\right\|_{2,\sigma}^2}{h^2}} \qquad Z(\vec{p}) = \sum_{\vec{q}} e^{-\frac{\left\| I(\mathscr{N}(\vec{p})) - I(\mathscr{N}(\vec{q}))\right\|_{2,\sigma}^2}{h^2}}$$

where $\left\| I(\mathscr{N}(\vec{p})) - I(\mathscr{N}(\vec{q}))\right\|_{2,\sigma}^2$ is the Euclidean distance weighted by a Gaussian function of standard deviation σ.

Examples of Weight Values

- In this figure, the original image is on the left and the weights for the central pixel (white dot) are shown on the right.

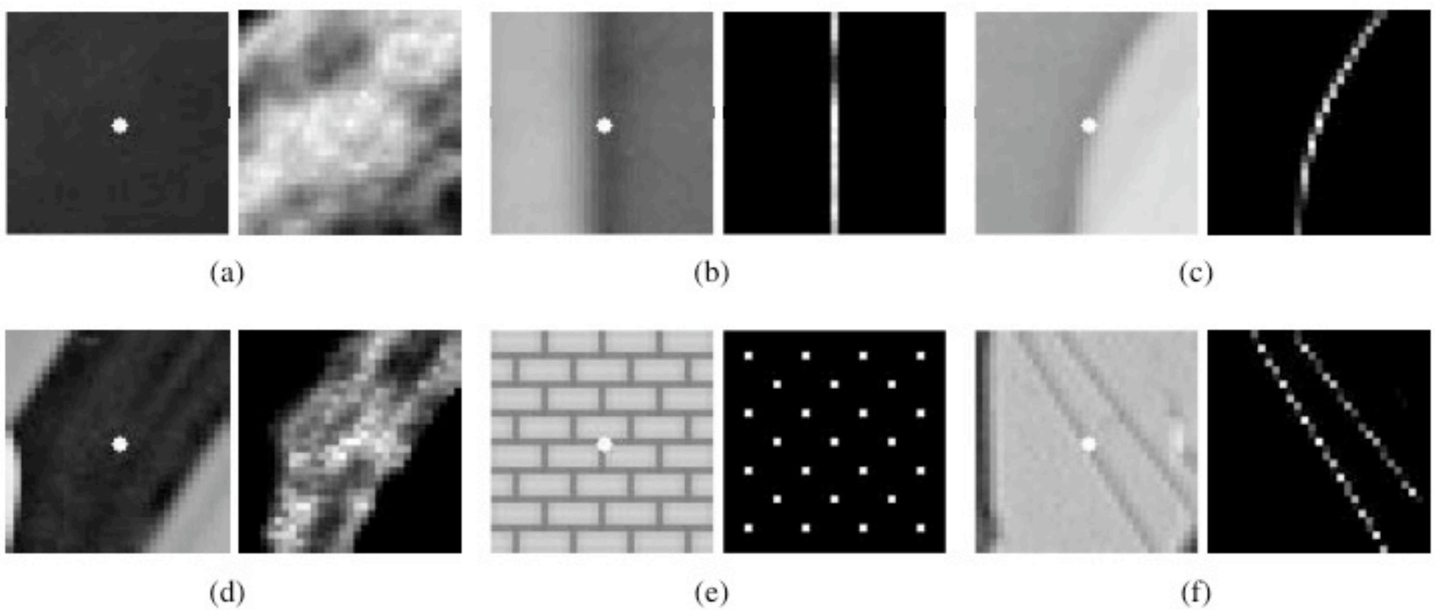
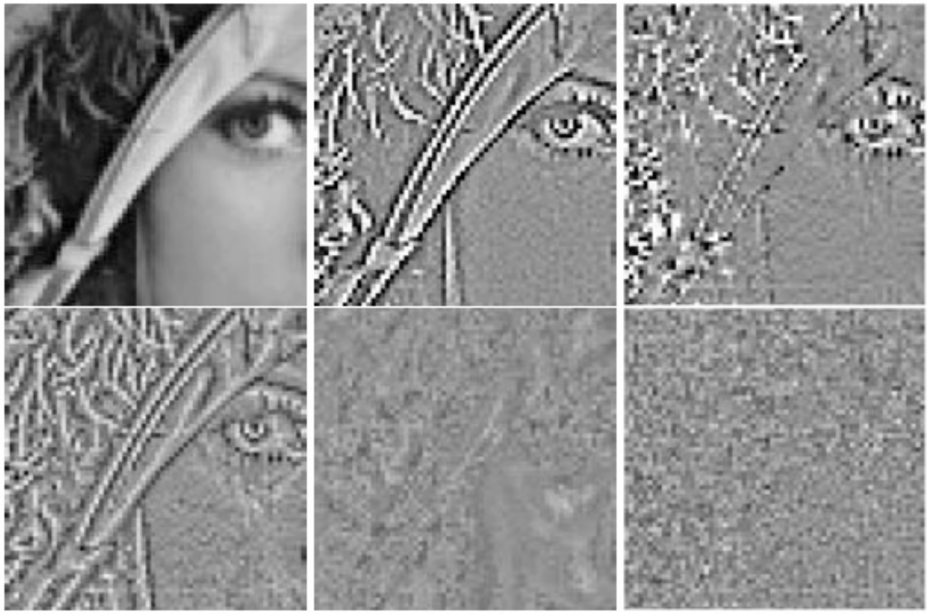


Figure 2. Display of the NL-means weight distribution used to estimate the central pixel of every image. The weights go from 1(white) to zero(black).

Example of Non-Local Means



Left figure: Denoising using from left to right and from top to bottom Gaussian filter, anisotropic filter, total variation denoising, neighborhood filtering and non-local means.

Right figure: The original picture an the differences between the denoised and the original image for each of the methods.

Non-Local Means Example 2

noisy

denoised

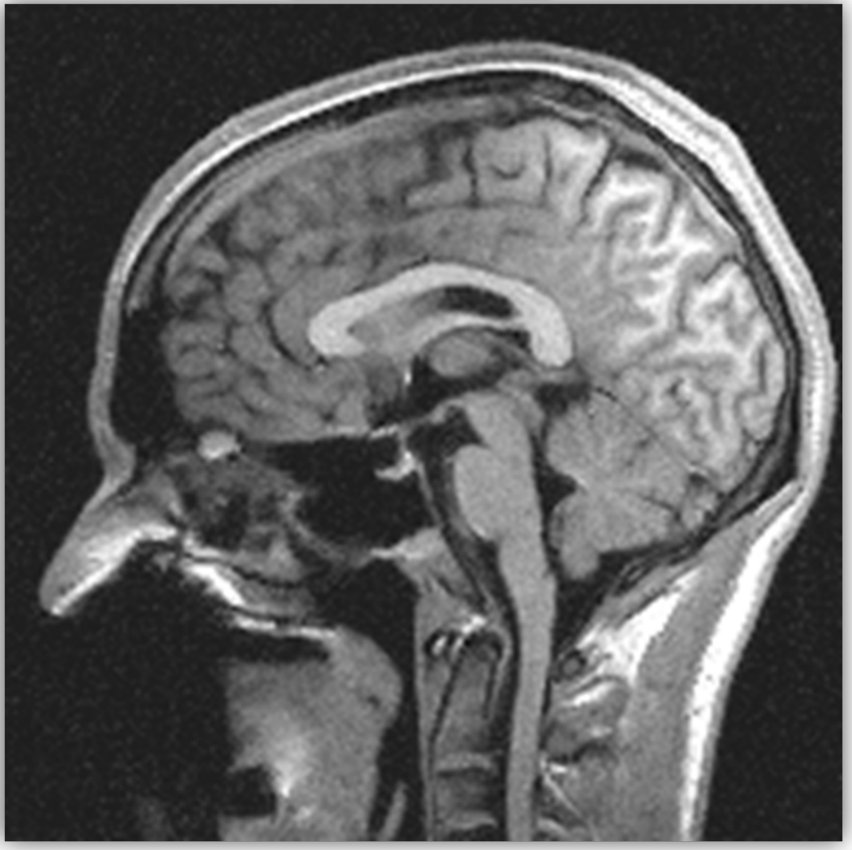


Image Sources

1. "Image with salt & pepper noise", Marko Meza.
2. Many of the smoothing and edge detection images are from the slides by D.A. Forsyth, University of Illinois at Urbana-Champaign.
3. The examples in slides 18-21 are courtesy of R. Fisher, S. Perkins, A. Walker and E. Wolfart
4. Non-Local Means figures are from the paper A. Buades, B. Coll and J.-M. Morel, "A Non-Local Algorithm for Image Denoising," *Computer Vision and Pattern Recognition*, 2005.