About bit-approximation

An integer matrix

In[209]:=
```
C3 = {62, 34, -31, 24, 18, 17, -15, 14, 42, 29, -12, 15, -35, 10, -9, 15, -4, 1, -11,
    0, -2, 6, 4, -1, 29, 10, -1, 9, 6, 9, 5, 13, 4, 45, 3, 0, 13, -1, -2, 21, 0,
    13, -1, 7, 4, 5, -11, 3, 26, -21, -1, 0, 3, 1, 7, 9, 4, 5, 0, 8, 6, 0, 2, 7};
```

In[210]:=
```
M3 = Table[C3[[M2L[i, j] + 1]], {i, 0, 7}, {j, 0, 7}]; M3 // MatrixForm
```

Out[210]//MatrixForm=

$$\begin{pmatrix} 62 & 34 & 18 & 17 & -4 & 1 & -2 & 6 \\ -31 & 24 & -15 & 14 & -11 & 0 & 4 & -1 \\ 42 & 29 & -35 & 10 & 29 & 10 & 6 & 9 \\ -12 & 15 & -9 & 15 & -1 & 9 & 5 & 13 \\ 4 & 45 & 13 & -1 & 26 & -21 & 3 & 1 \\ 3 & 0 & -2 & 21 & -1 & 0 & 7 & 9 \\ 0 & 13 & 4 & 5 & 4 & 5 & 6 & 0 \\ -1 & 7 & -11 & 3 & 0 & 8 & 2 & 7 \end{pmatrix}$$

Converting the coefficients to signed binary representation

In[211]:=
```
M3sb = Map[tosignedbinary[#, 6] &, M3, {2}]; M3sb[[1, 1]]
```

Out[211]=
```
{0, 1, 1, 1, 1, 1, 0}
```

Defining the bit-planes

In[212]:=
```
bp[k_] := M3sb[[All, All, k + 1]]
```

The sign-plane

In[213]:=
```
bp[0] // MatrixForm
```

Out[213]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The bit-planes

In[214]:=
```
Grid[Partition[Table[bp[k] // MatrixForm, {k, 1, 6}], 2]]
```

Out[214]=

$$\begin{pmatrix} 1&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 1&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix} \quad \begin{pmatrix} 1&0&1&1&0&0&0&0\\ 1&1&0&0&0&0&0&0\\ 0&1&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&1&1&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 1&1&1&1&1&0&0&0\\ 1&1&0&1&1&1&0&1\\ 1&1&1&1&0&1&0&1\\ 0&1&1&0&1&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&1&0&0 \end{pmatrix} \quad \begin{pmatrix} 1&0&0&0&1&0&0&1\\ 1&0&1&1&0&0&1&0\\ 0&1&0&0&1&0&1&0\\ 1&1&0&1&0&0&1&1\\ 1&1&1&0&0&1&0&0\\ 0&0&0&1&0&0&1&0\\ 0&1&1&1&1&1&1&0\\ 0&1&0&0&0&0&0&1 \end{pmatrix}$$

$$\begin{pmatrix} 1&1&1&0&0&0&1&1\\ 1&0&1&1&1&0&0&0\\ 1&0&1&1&0&1&1&0\\ 0&1&0&1&0&0&0&0\\ 0&0&0&0&1&0&1&0\\ 1&0&1&0&0&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&1&1&1&0&0&1&1 \end{pmatrix} \quad \begin{pmatrix} 0&0&0&1&0&1&0&0\\ 1&0&1&0&1&0&0&1\\ 0&1&1&0&1&0&0&1\\ 0&1&1&1&1&1&1&1\\ 0&1&1&1&0&1&1&1\\ 1&0&0&1&1&0&1&1\\ 0&1&0&1&0&1&0&0\\ 1&1&1&1&0&0&0&1 \end{pmatrix}$$

The approximations based on the first k bit-planes

In[215]:=
```
Grid[Partition[Table[approx[M3, k] // MatrixForm, {k, 1, 6}], 2]]
```

```
number of bits needed: 6

number of bits needed: 6

number of bits needed: 6

number of bits needed: 6

number of bits needed: 6

number of bits needed: 6
```

Out[215]=

$$\begin{pmatrix} 47 & 47 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 47 & 0 & -47 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 47 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 55 & 39 & 23 & 23 & 0 & 0 & 0 & 0 \\ -23 & 23 & 0 & 0 & 0 & 0 & 0 & 0 \\ 39 & 23 & -39 & 0 & 23 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 39 & 0 & 0 & 23 & -23 & 0 & 0 \\ 0 & 0 & 0 & 23 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 59 & 35 & 19 & 19 & 0 & 0 & 0 & 0 \\ -27 & 27 & -11 & 11 & -11 & 0 & 0 & 0 \\ 43 & 27 & -35 & 11 & 27 & 11 & 0 & 11 \\ -11 & 11 & -11 & 11 & 0 & 11 & 0 & 11 \\ 0 & 43 & 11 & 0 & 27 & -19 & 0 & 0 \\ 0 & 0 & 0 & 19 & 0 & 0 & 0 & 11 \\ 0 & 11 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -11 & 0 & 0 & 11 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 61 & 33 & 17 & 17 & -5 & 0 & 0 & 5 \\ -29 & 25 & -13 & 13 & -9 & 0 & 5 & 0 \\ 41 & 29 & -33 & 9 & 29 & 9 & 5 & 9 \\ -13 & 13 & -9 & 13 & 0 & 9 & 5 & 13 \\ 5 & 45 & 13 & 0 & 25 & -21 & 0 & 0 \\ 0 & 0 & 0 & 21 & 0 & 0 & 5 & 9 \\ 0 & 13 & 5 & 5 & 5 & 5 & 5 & 0 \\ 0 & 5 & -9 & 0 & 0 & 9 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 62 & 34 & 18 & 16 & -4 & 0 & -2 & 6 \\ -30 & 24 & -14 & 14 & -10 & 0 & 4 & 0 \\ 42 & 28 & -34 & 10 & 28 & 10 & 6 & 8 \\ -12 & 14 & -8 & 14 & 0 & 8 & 4 & 12 \\ 4 & 44 & 12 & 0 & 26 & -20 & 2 & 0 \\ 2 & 0 & -2 & 20 & 0 & 0 & 6 & 8 \\ 0 & 12 & 4 & 4 & 4 & 4 & 6 & 0 \\ 0 & 6 & -10 & 2 & 0 & 8 & 2 & 6 \end{pmatrix} \qquad \begin{pmatrix} 62 & 34 & 18 & 17 & -4 & 1 & -2 & 6 \\ -31 & 24 & -15 & 14 & -11 & 0 & 4 & -1 \\ 42 & 29 & -35 & 10 & 29 & 10 & 6 & 9 \\ -12 & 15 & -9 & 15 & -1 & 9 & 5 & 13 \\ 4 & 45 & 13 & -1 & 26 & -21 & 3 & 1 \\ 3 & 0 & -2 & 21 & -1 & 0 & 7 & 9 \\ 0 & 13 & 4 & 5 & 4 & 5 & 6 & 0 \\ -1 & 7 & -11 & 3 & 0 & 8 & 2 & 7 \end{pmatrix}$$

The mean squared approximation error

In[216]:=
```
Table[NNorm[approx[M3, k] - M3] / 64, {k, 1, 6}]
```

```
number of bits needed: 6
number of bits needed: 6
number of bits needed: 6
number of bits needed: 6
number of bits needed: 6
number of bits needed: 6
```

Out[216]=
```
{1.46284, 0.878064, 0.387487, 0.160109, 0.09375, 0.}
```

In[217]:=
```
ListPlot[%, Filling → Axis]
```

Out[217]=