Exercises for
Pattern Analysis
Thomas Köhler
Assignment 4, 07.05.2014

General Information:

Lecture (3 SWS):	Mon 08.15 – 09:45 (H16) and Tue 08.15 – 09.45 (H16)
Exercises (1 SWS):	Wed 12.15 – 13.15 (00.151-113) and Thu 12.30 – 13.30 (09.150)
Certificate:	Oral exam at the end of the semester
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Probability Density Estimation - Part I

Exercise 1 Probability density estimation based on the Parzen Window approach can be utilized for the Bayes classifier to determine a decision boundary. Let us assume we have n training samples available whereas n_i samples belong to class y_i.

(a) Write down the class conditional probability density $p(\boldsymbol{x}|y_i)$ for class y_i in terms of Parzen window estimation.

(b) Approximate the prior $P(y_i)$ probability for class y_i.

(c) Write down the decision rule for the Bayes classifier using the approximated class conditional density as well as the estimated prior.

Exercise 2 Let $p(x)$ be an exponential density with parameter $\lambda > 0$:

$$p(x) = \begin{cases} \lambda \exp(-\lambda x), & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Use the window function

$$\varphi(x) = \begin{cases} \exp(-x), & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

to approximate $p(x)$ and calculate the mean of the Parzen window estimation $\bar{p}(x)$.

Hint: $\bar{p}(x)$ is a probability density function since the samples used for Parzen window estimation are drawn out of an exponential distribution. This density is equal to the expected value of all suitable Parzen window estimates.

(b) Calculate the expected value $\bar{\mu}$ of the mean density $\bar{p}(x)$ and prove that $\bar{\mu}$ converges to the expected value $\mu = 1/\lambda$ if the kernel width h approaches infinity.