Exercises for
Pattern Recognition
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General Information:

Exercises (1 SWS):	Tue 12:15 – 13:45 (0.154-115) and Fri 08:15 – 09:45 (0.151-115)
Certificate:	Oral exam at the end of the semester
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Fisher Transform and Norms

Exercise 1 In this exercise, we derive the Fisher transform for dimensionality reduction of feature vectors. The Fisher transform is a formulation of the LDA. In the training step, the class label for each pattern is known. The optimal projection axis $\boldsymbol{a}^*$ for the Fisher transform is calculated according to the Rayleigh ratio:

$$\boldsymbol{a}^* = \underset{\boldsymbol{a}}{\operatorname{argmax}} \frac{\boldsymbol{a}^T \boldsymbol{\Sigma}_{\texttt{inter}} \boldsymbol{a}}{\boldsymbol{a}^T \boldsymbol{\Sigma}_{\texttt{intra}} \boldsymbol{a}}$$

The definitions for $\boldsymbol{\Sigma}_{\texttt{inter}}$ and $\boldsymbol{\Sigma}_{\texttt{intra}}$ can be found in the lecture slides. We are only interested in the 2 class problem.

(a) Describe the different quantities in the Rayleigh ratio.

(b) Reformulate the given (unconstrained) optimization problem to a constrained optimization problem using Lagrange multipliers.

(c) Solve the constrained problem to determine $\boldsymbol{a}^*$.
Hint: derivatives w.r.t. $\boldsymbol{a}$ must be 0. Use the Matrix Cookbook.

Exercise 2 Implement a classification algorithm which classifies a new feature only by calculating the distance to the class prototypes (i.e., the mean of a class). Use the option to pass parameters to the classification function, and implement three approaches to calculate the distance:

(a) the L1-Norm

(b) the L2-Norm (= euclidean distance)

(c) the Mahalanobis distance, which incorporates the covariance matrices.