

DMIP - Exercise:

Defect Pixel Interpolation

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Pattern Recognition Lab (CS 5)



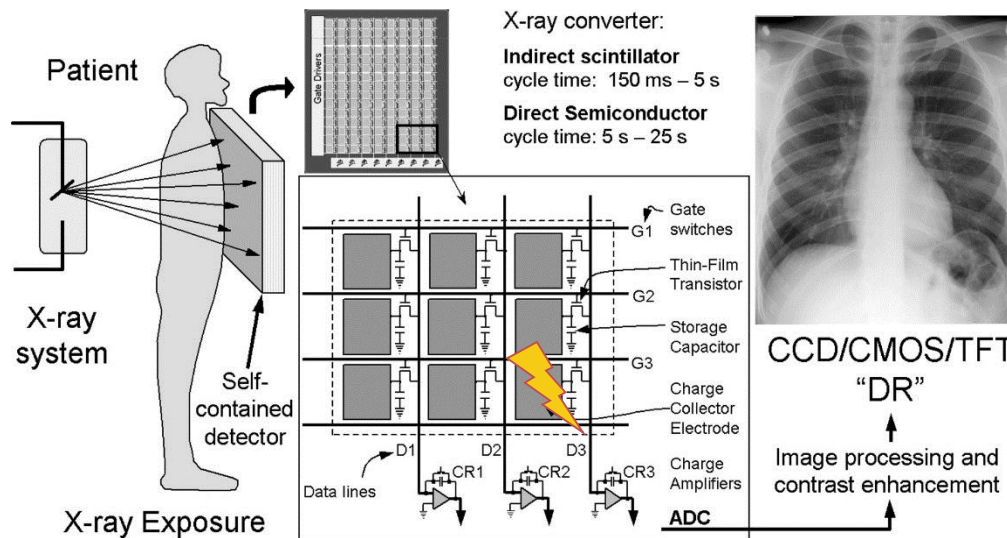
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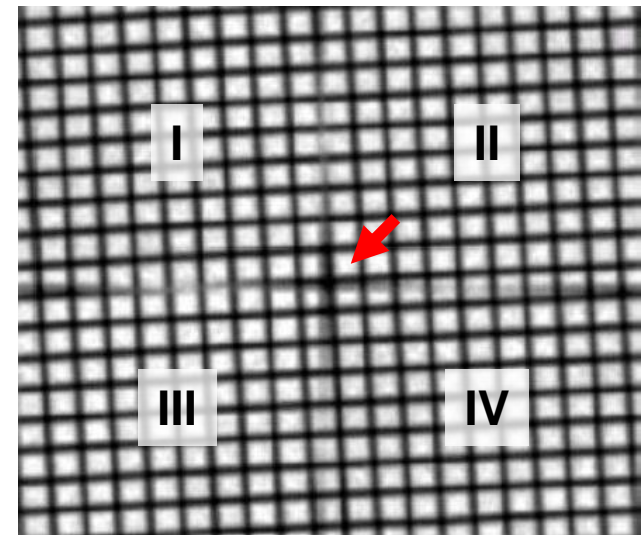
Flat Panel Detectors

Defect Pixels in Flat Panel Detectors

- Defect pixels are caused by defect detector cells.
- Small detectors are composed to generate a large one, which leads to butting cross effects.



Artifacts due to inactive pixels or rows



Butting Cross Artifact

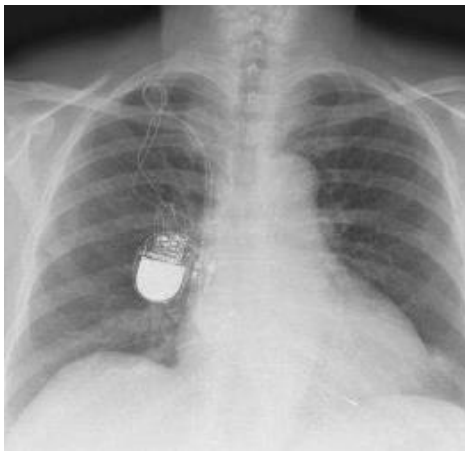
Flat Panel Detectors

Defect Pixels in Flat Panel Detectors

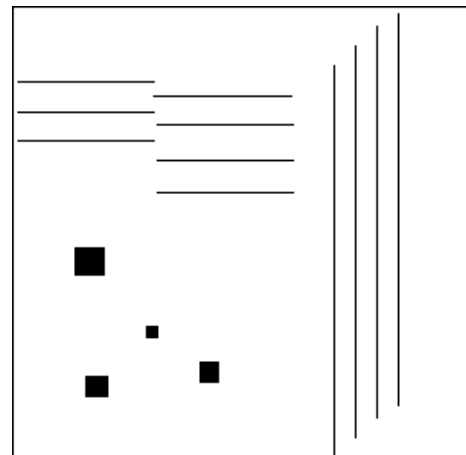
- Let $f_{i,j}$ denote the intensity value at grid point (i,j) of the **ideal image** f that has no defect pixels.
- Let $w_{i,j}$ denote the indicator value at (i,j) where w is **mask image** that indicates defect and uncorrupted pixels:

$$w_{i,j} = \begin{cases} 0 & \text{if pixel is defect} \\ 1 & \text{otherwise} \end{cases}$$

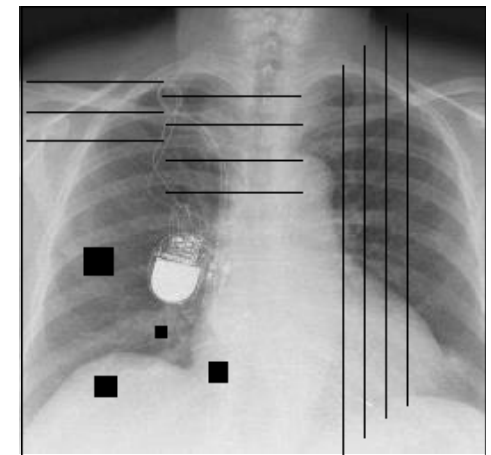
- Let $g_{i,j}$ denote the intensity value at grid point (i,j) of the **observed image** g that is acquired with the flat panel detector that has defect pixels.



Ideal image f



Mask image w



Observed image g

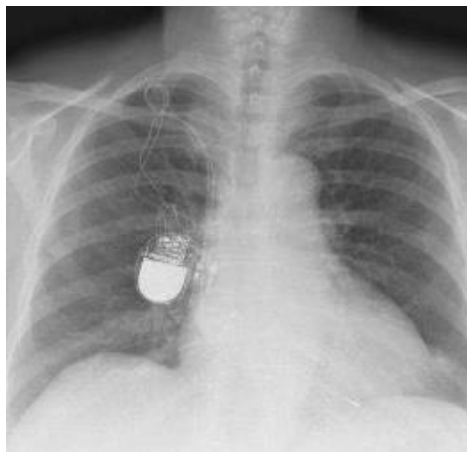
Flat Panel Detectors

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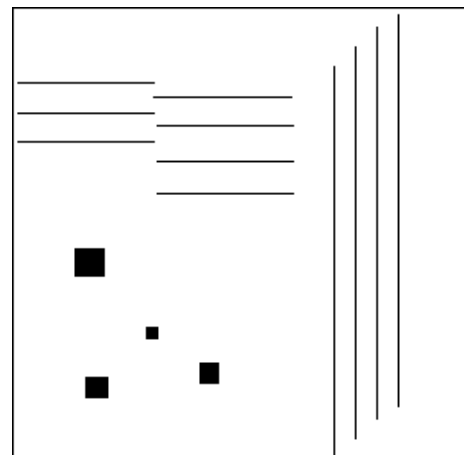
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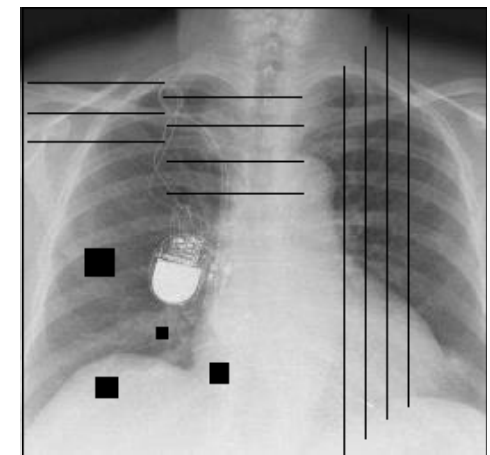
Ideal image f

•



Mask image w

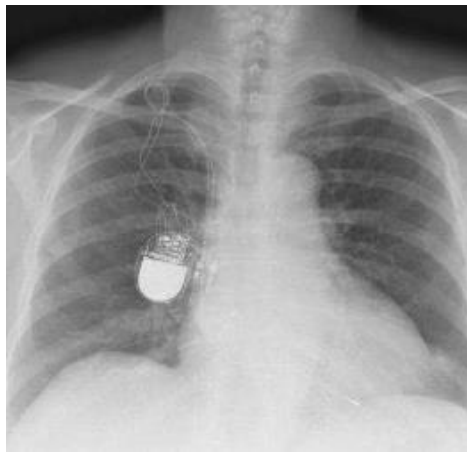
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Observed image g

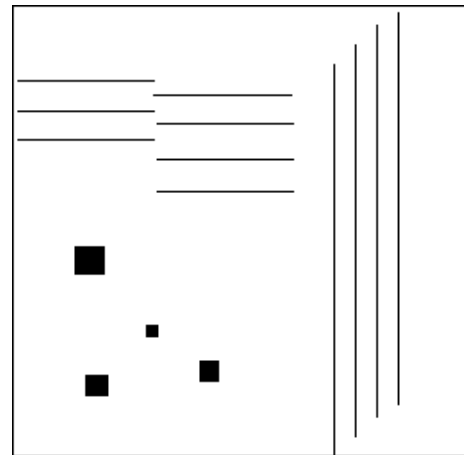
Flat Panel Detectors

Defect Pixels in Flat Panel Detectors



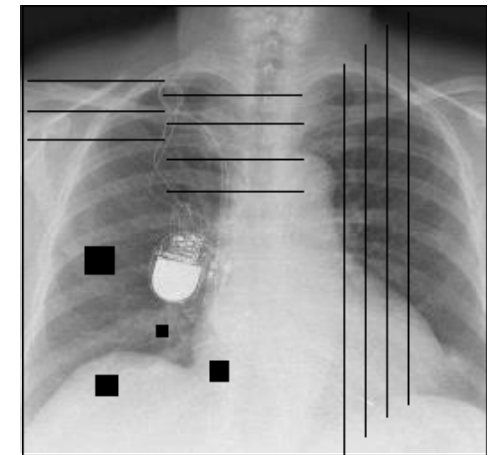
Ideal image f

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Mask image w

=



Observed image g

- Defect pixel problem: $f(n) \cdot w(n) = g(n)$
- Goal: Find $f(n)$, given the observed image $g(n)$ and the defect mask $w(n)$

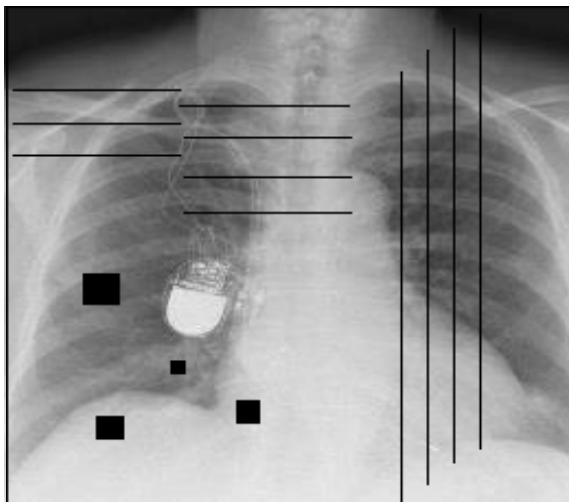


Defect Pixel Correction

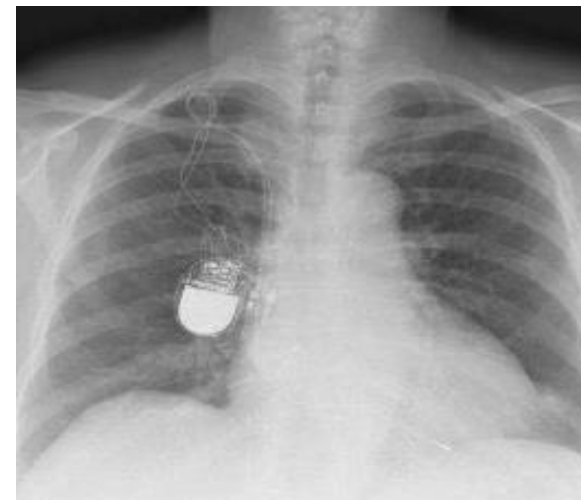
Defect Pixel Correction

Problem Statement

- Restore the **ideal image** based on the **observed image** and the known defect **pixel mask**.
- Defect pixel correction:
 - *Spatial Domain:* Interpolation
 - *Frequency Domain:* Band Limitation
 - *Frequency Domain:* Iterative Deconvolution



Observed image g

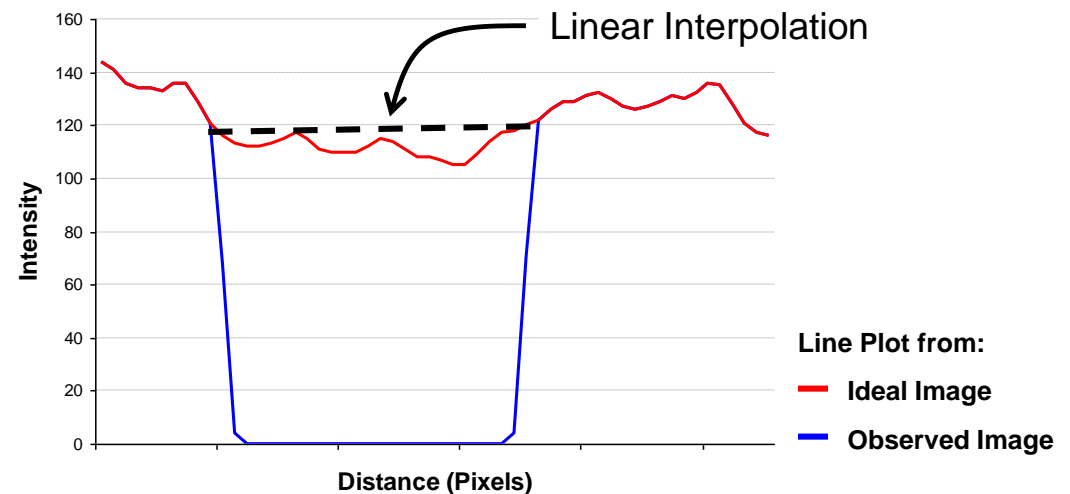
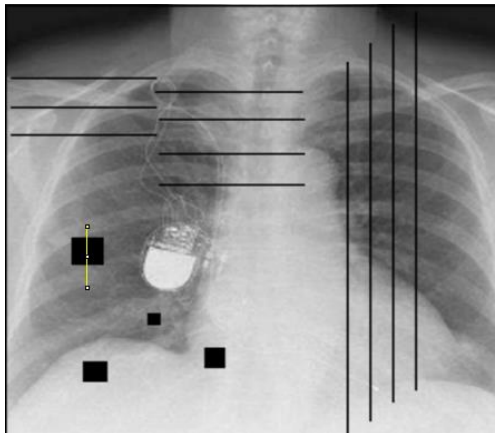


Ideal image f

Defect Pixel Correction

Defect Pixel Correction by Spatial Interpolation

- Interpolate between active pixels to recover the inactive ones:
 - Bilinear interpolation
 - Median
- **However, this is only suitable for small defect areas!**





Defect Pixel Correction

Fourier Transform Revisited

- Convolution theorem
- Symmetry property of Fourier transform of real signals



Defect Pixel Correction

Fourier Transform Revisited

- **Convolution theorem**

$$\begin{aligned} FT(f \star h)(\xi) &= \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} f(k)h(n-k)e^{\frac{-2\pi in\xi}{N}} \\ &= \sum_{k=0}^{N-1} f(k) \sum_{n=0}^{N-1} h(n-k)e^{\frac{-2\pi in\xi}{N}} \\ &= \sum_{k=0}^{N-1} f(k)e^{\frac{-2\pi ik\xi}{N}} H(\xi) = F(\xi)H(\xi) = G(\xi) \end{aligned}$$

The **convolution** of two signals in the **time domain**, corresponds to a **multiplication** in the **frequency domain**.

- **Symmetry property of Fourier transform of real signals**



Defect Pixel Correction

Fourier Transform Revisited

- Convolution theorem
- Symmetry property of Fourier transform of real signals

If $f(n)$ is a real valued discrete signal of length N , the Fourier transform $F(\xi)$ fulfills the symmetry property:

$$F(\xi) = F^*(N - \xi)$$

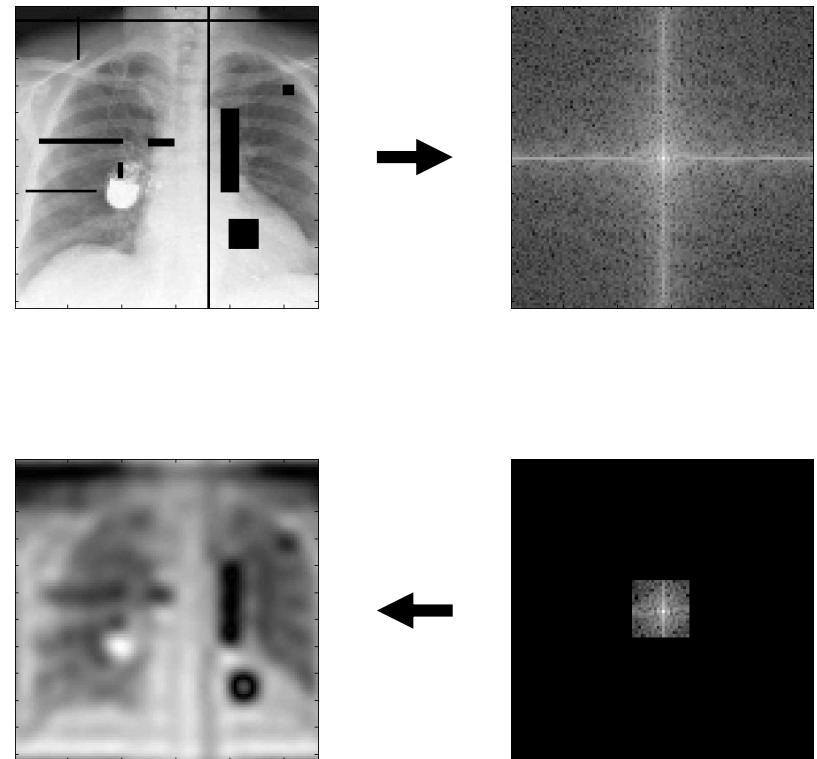
where "*" denotes the conjugate complex.



Defect Pixel Correction

Iterative Band Limitation

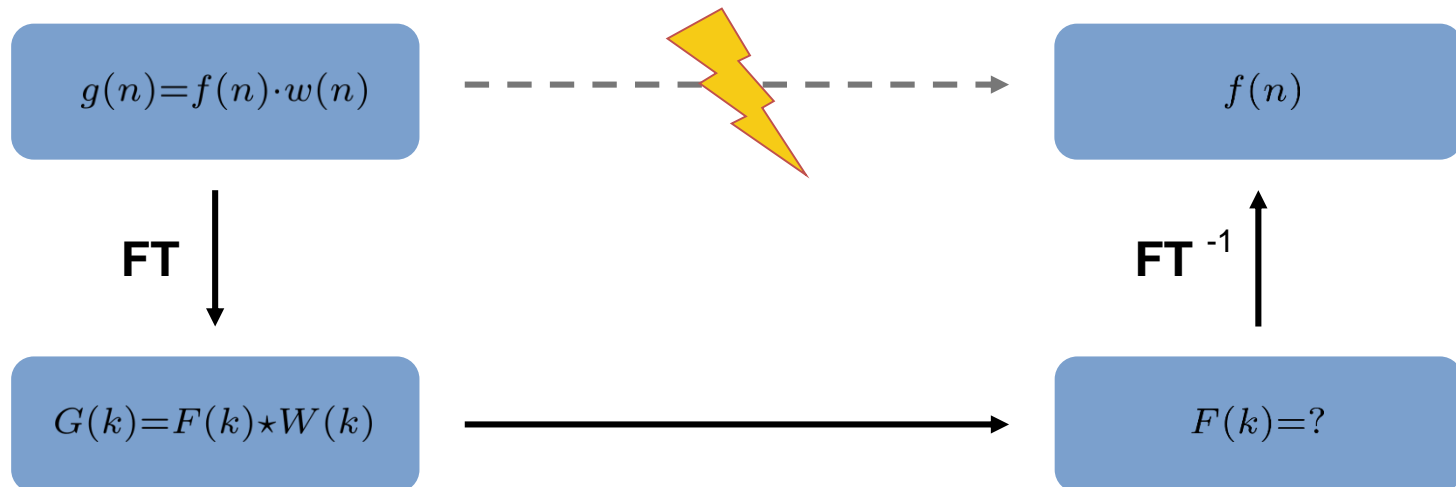
1. **Fourier transform**
2. **Cut off high frequencies**
3. **Inverse Fourier transform**
4. **Replace only defect areas**
5. **Repeat from 1.**



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- Application of Fourier convolution theorem:



$$G(k) = \frac{1}{N} F(k) \star W(k) = \frac{1}{N} \sum_{l=0}^{N-1} F(l) \cdot W(k - l), \quad 0 \leq n, k < N$$



Defect Pixel Correction

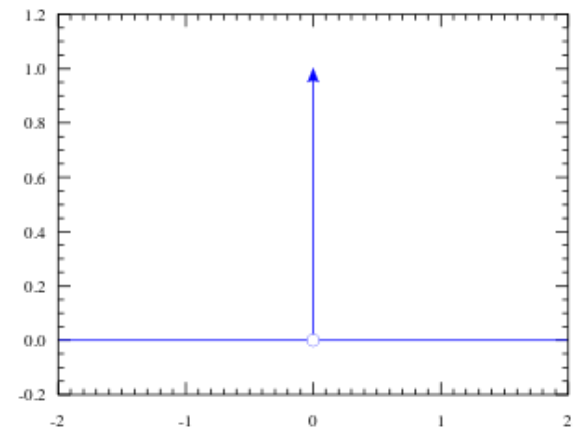
Defect Pixel Correction by Symmetry Properties

- **Application of Fourier symmetry properties:**
- Use Dirac's δ -function to select a line pair $F(s)$ and $F(N-s)$:

$$\hat{F}(k) = \hat{F}(s)\delta(k - s) + \hat{F}(N - s)\delta(k - N + s)$$

where \hat{F} denotes an estimate of F , and δ -function is defined by:

$$\delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$



Dirac δ -function

Image taken from: Wikipedia



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- After frequency selection, we convolve with the mask

$$\hat{G}(k) = \frac{1}{N} \hat{F}(k) \star W(k) = \frac{1}{N} \sum_{l=0}^{N-1} \hat{F}(l) \cdot W(k-l)$$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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- $\hat{F}(l)$ is only non-zero if $l = s \vee l = N - s$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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- $\hat{F}(l)$ is only non-zero if $l = s \vee l = N - s$

$$\hat{G}(k) = \frac{1}{N} \left(\hat{F}(s)W(k-s) + \hat{F}(N-s)W(k-(N-s)) \right)$$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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- $\hat{F}(l)$ is only non-zero if $l = s \vee l = N - s$

$$\begin{aligned} \hat{G}(k) &= \frac{1}{N} \left(\hat{F}(s)W(k-s) + \underbrace{\hat{F}(N-s)W(k-(N-s))}_{= \hat{F}^*(s)} \right) \\ &= \hat{F}^*(s) \end{aligned}$$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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- We are only interested in $\hat{G}(k)$ at position $k = s$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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$$\hat{G}(s) = \frac{1}{N} \left(\hat{F}(s)W(0) + \hat{F}^*(s)W(2s-N) \right)$$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- After frequency selection, we convolve with the mask

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$$\hat{G}(k) = \frac{1}{N} \left(\hat{F}(s)W(k-s) + \hat{F}^*(s)W(k-(N-s)) \right)$$

- We are only interested in $\hat{G}(k)$ at position $k = s$

$$\hat{G}(s) = \frac{1}{N} \left(\hat{F}(s)W(0) + \hat{F}^*(s)W(2s) \right)$$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- **Application of Fourier symmetry properties:**

Select a line pair $G(s)$ and $G(N-s)$ of the Fourier transform of the observed image. The observed image is described by a convolution of the ideal image and the known defect pixel mask:

$$G(s) = \frac{1}{N} \left(\hat{F}(s)W(0) + \hat{F}^*(s)W(2s) \right)$$

- And for the conjugate complex:

$$G^*(s) = \frac{1}{N} \left(\hat{F}^*(s)W^*(0) + \hat{F}(s)W^*(2s) \right)$$

- Using these two equations, we can compute:

$$\hat{F}(s) = N \frac{G(s)W^*(0) - G^*(s)W(2s)}{|W(0)|^2 - |W(2s)|^2}$$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- Special case without symmetry property (for $s=0$ and $s = N/2$)

$$G(s) = \frac{1}{N} \left(\hat{F}(s)W(0) + \hat{F}^*(s)W(2s) \right)$$

$$G^*(s) = \frac{1}{N} \left(\hat{F}^*(s)W^*(0) + \hat{F}(s)W^*(2s) \right)$$



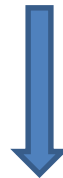
Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

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$$G(s) = \frac{1}{N} \left(\hat{F}(s)W(0) + \hat{F}^*(s)W(2s) \right)$$

~~$$G^*(s) = \frac{1}{N} \left(\hat{F}^*(s)W^*(0) + \hat{F}(s)W^*(2s) \right)$$~~



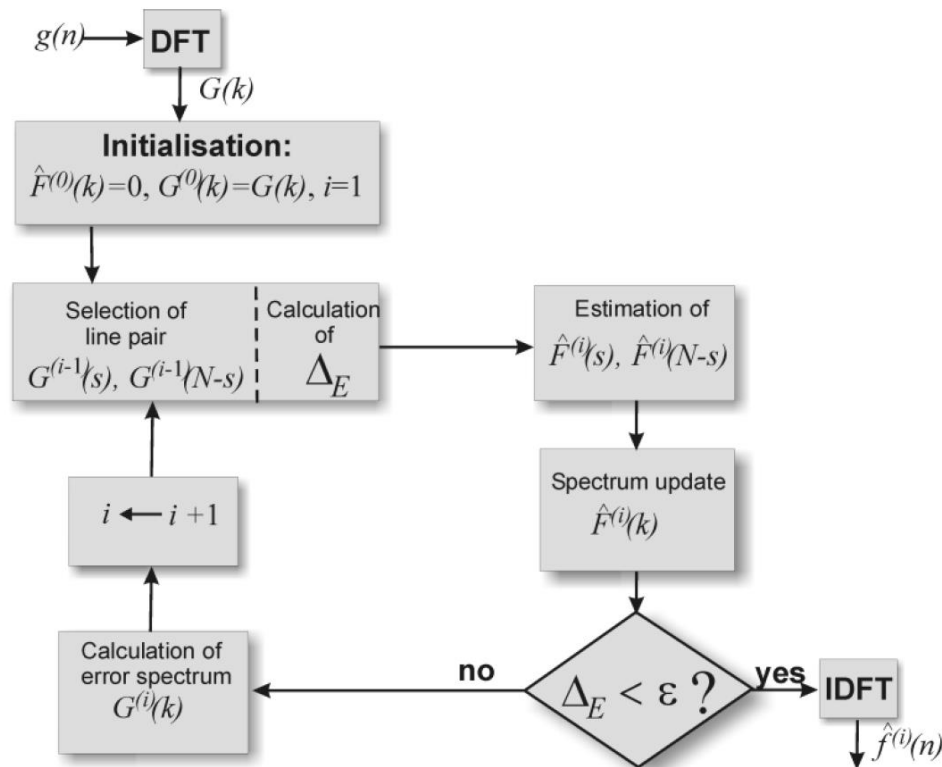
$$\hat{F}(s) = N \left(\frac{G(s)}{W(0)} \right)$$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

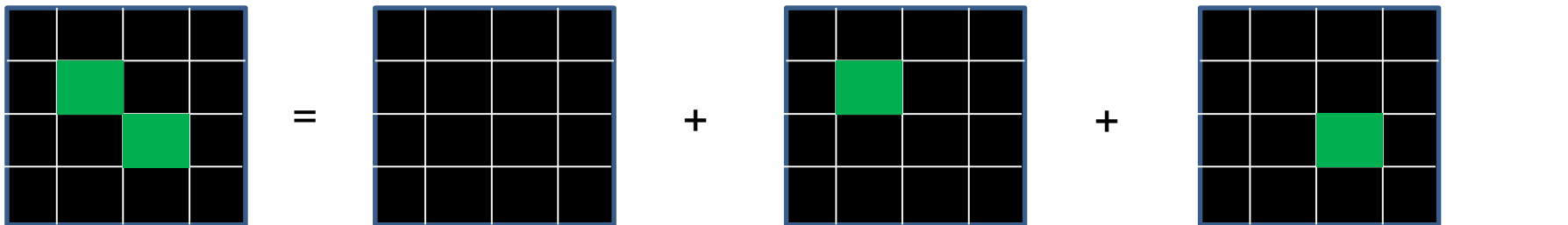
- So far only correction at line pair
 - ➔ Iterative correction of line pairs



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- So far we only found estimates for a single or a pair of selected lines
- We also need to update the global estimate of the spectrum after each linepair computation


$$\hat{F}^{(1)}(k) = \hat{F}^{(0)}(k) = 0 + \hat{F}(s^{(1)}) + \hat{F}(N - s^{(1)}) = \hat{F}^*(s^{(1)})$$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- How can we update the error spectrum G ?

$$G^{(i)}(k) = G^{(i-1)}(k) - \hat{G}(k) = G^{(i-1)}(k) - \frac{1}{N} \left(\hat{F}(k) * W(k) \right)$$



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- How can we update the error spectrum G ?

$$G^{(i)}(k) = G^{(i-1)}(k) - \hat{G}(k) = G^{(i-1)}(k) - \frac{1}{N} \left(\hat{F}(k) * W(k) \right)$$

- Just subtract the new estimate from the previous estimate
- Requires convolution!!! → Same complexity as FFT?
- $F(k)$ changed only at two positions → Convolution much easier to compute



Defect Pixel Correction

Defect Pixel Correction by Symmetry Properties

- How to select the line pairs each iteration?
- Just select the maximum of the error spectrum

$$s^{(i)} = \operatorname{argmax}_{\hat{s}^{(i)}} G^{(i)}(\hat{s}^{(i)})$$

- Detailed derivation can be found in the paper.
- It is based on Parseval's theorem.
- Approach minimizes the mean square error (MSE) in valid areas:

$$\text{MSE} = \sum_{n=0}^N (g(n) - \hat{f}(n)w(n))^2$$