Diagnostic Medical Image Processing (DMIP)
WS 2015/16
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Exercise 7: Rigid Registration

1 Theory

1.1 Rigid registration in 2-D

Rigid registration only allows rotations and translations. That means that the objects maintain their shape and size. In 2-D rigid transformations can be described as

$$\mathbf{p}_k = \mathbf{R}\mathbf{q}_k + \mathbf{t} \tag{1}$$

where $\mathbf{R}$ is the rotation matrix

$$\mathbf{R} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \tag{2}$$

$\mathbf{t}$ the translation vector $\mathbf{t} = (\mathbf{t}_1, \mathbf{t}_2)$.

1.2 Solving the Correspondence Problem

Given k point correspondences $(\mathbf{p}_k, \mathbf{q}_k) \in \mathbb{R}^2$ in two images, the corresponding optimization problem is

$$\arg\min_{\varphi,\mathbf{t}_1,\mathbf{t}_2} \sum_{k=1}^{N} \left\|\mathbf{p}_k - \mathbf{R}\mathbf{q}_k - \mathbf{t}\right\|^2 . \tag{3}$$

To avoid solving this nonlinear problem, one can make use of complex numbers.

1.3 Distance measures

With distance measures (aka similarity measures) you can calculate the similarity between objects. Let F be the reference image and M be the moving image.

1.3.1 Sum of Squared Differences - SSD

The Sum of Squared Differences is defined as

$$\mathcal{D}_{SSD} = \sum_{i,j} (F_{ij} - M_{ij})^2 \tag{4}$$

SSD is based on the assumption that the intensities of the same tissue within 2 datasets are equal. The SSD should be minimized.

1.3.2 Mutual Information - MI

The Mutual Information is defined as

$$\mathcal{D}_{MI} = H(F) + H(M) - H(F, M) \tag{5}$$

where $H(F), H(M)$ are the entropies of the images F and M and $H(F, G)$ is the entropy in the bivariate case (joint entropy). MI evaluates how much information is shared in both pictures. The mutual entropy should be maximized.

2 Exercises

2.1 Part 1: Solving the correspondence problem

Two point clouds are given. Calculate the translation and rotation according to the optimization formula given above. Formulate the linear optimization problem which you can solve by using SVD.

2.2 Part 2: Rigid Registration with SSD

At first, we generate a phantom image and transform it:

1. Generate a phantom image
2. Transform the image as follows:
   (a) Rotate the image by 45°
   (b) Translate the image with $\mathbf{t} = (20, 1)$.

Then, the images are registered:

1. Apply a Gaussian filter on both images
2. Implement the Error Measure SSD
3. Use the optimizer to find the parameters for the rotation and translation

2.3 Part 3: Rigid Registration with Mutual Information

Load the images *Proton.png* and *T1.png*.

1. Apply a Gaussian filter on both images
2. Calculate the joint histogram and a histogram for each image
3. Calculate the joint and the marginal entropy
4. Calculate the Mutual Information