Nonlinear Diffusion Noise Reduction in CT Using Correlation Analysis

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***Abstract*—Noise reduction in CT images gains more and more attention. It provides a possibility to increase signal-to-noise ratio, hence giving more space for a further reduction of radiation dose. Nevertheless a reduction of noise also bears the risk of suppressing medical relevant information. We propose a new noise reduction method that tries to minimize this risk by estimating the real image structure out of the correlations of two input data sets affected with uncorrelated noise. Such data sets can be achieved by reconstructing a CT scan with only the odd and the even numbered projections respectively. Furthermore, the method adapts itself to the spatially changing behavior of the noise on CT images by estimating the local noise variance out of the difference of the input images. It can be applied to 2D and 3D data, with the latter giving better results due to the fact that more pixels are available for correlation computation and variance estimation. Examples show that the new method easily surpasses standard approaches and leads to noise suppression rates of about 66%.**

***Index Terms*—Computed Tomography, Noise Reduction, Nonlinear Diffusion, Correlation Analysis**

I. Introduction

Common noise reduction methods often fail to produce convincing results when dealing with CT images. The reason for this lies in the unknown distribution of the noise in the reconstructed data. The intensity of the noise is spatially varying and directed noise structures appear. We present a new denoising method based on nonlinear isotropic diffusion that adapts itself to changing noise variance in different image regions and reduces oriented noise without noticeable loss of resolution by taking correlations of input images with uncorrelated noise into account. The approach is suitable for both 2D and 3D data.

II. Previous Work

In [1] A. Borsdorf proposed a Wavelet based denoising method for CT images. By separately reconstructing the odd and even numbered projections of a CT scan two sets of slices are obtained which include the same information but noise between the data is uncorrelated. By using a correlation analysis in the wavelet domain combined with a orientation and position dependent noise estimation [2] only the part of the wavelet coefficients which is supposed to contain image structure is kept for reconstruction of a noise suppressed image. In this work, the idea of this approach, e.g. using two data sets with uncorrelated noise, is picked up and transfered to the spatial domain and nonlinear diffusion methods.

III. Method

Noise is removed by minimizing an energy functional, which results in the following Euler-Lagrange equation:

$$u - u_0 = \tau \operatorname{div}(g(\|\nabla u\|)\nabla u)$$

This is equivalent to solving a Perona and Malik nonlinear isotropic diffusion equation [3] for a fixed artificial timestep τ. The initial image u_0 is set to the average of the input images u_1 and u_2. The sought-after solution is u. At the image boundary a homogeneous Neumann condition is applied. $g(\|\nabla u\|)$ is called an edge-stopping function regulating the diffusion process. In the proposed method the edge-stopping function has the form

$$g(\|\nabla u\|) = \tilde{g}(S(\|\nabla u_1\|, \|\nabla u_2\|), V).$$

$\tilde{g}$ is an edge-stopping function depending on two parameters. The Tukey egde-stopping [4] function was chosen because of its egde preserving properties:

$$\tilde{g}(a, \sigma) = \begin{cases} \left(1 - (\frac{a}{\sigma})^2\right)^2 & |a| \leq \sigma, \\ 0 & |a| > \sigma \end{cases}$$

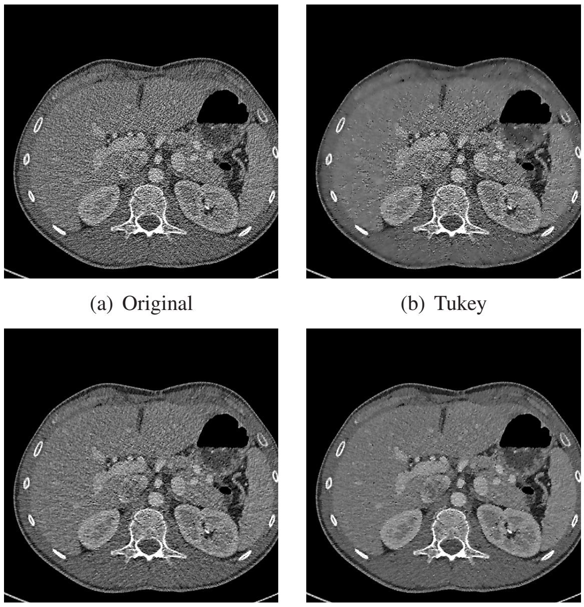
S is a function that weights the gradients of the two input images u_1 and u_2, taking into account the correlation of the images in a gaussian neighborhood window around the pixel currently in view. If there is a high correlation between the images, e.g. when looking at an egde, the value returned will be rather high, if the correlation is low, for example in constant image regions with uncorrelated noise, a small value will be returned. The idea is to slow down diffusion in image regions where both input images show the same structure. V is an estimate of the noise variance at the actual position calculated in the same gaussian neighborhood as S from the difference of the two noisy input images.

To maintain the nonlinearity of the Perona-Malik diffusion process, the images u_1 and u_2 are diffused with the same results of the egde-stopping function, too. After discretization by finite differences the partial differential equations are solved by a nonlinear multigrid solver [5]. The solver of the diffusion equation updates u, u_1 and u_2 simultaneously.

The application of the above formulas in 3D is straightforward and because of the possibility to shrink the gaussian neighborhoods used for correlation and variation calculation while the number of pixels involved in the calculation stays the same the results surpass the 2D ones, as is shown in Fig. 1.

IV. Results

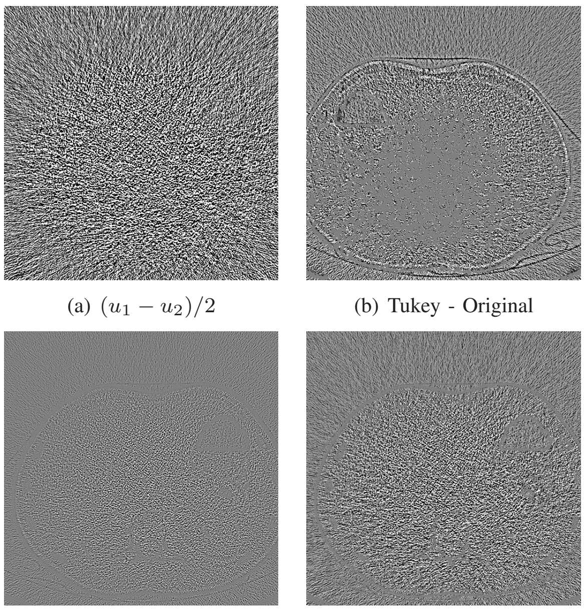
Fig. 1 shows results from the proposed method and one standard nonlinear diffusion method applied to a thin reconstructed slice (0.8 mm) compared to the average of the input images, which reflects the result of a reconstruction of all projections. It is referred to as the original image. In Fig. 2 the difference images to the original image are shown, providing an impression of the denoising behavior of the different approaches. Fig. 1(b) clearly shows, that a standard nonlinear diffusion method fails to denoise a CT image with spatially varying noise power in an adequate manner. While noise in the center of the image is nearly unchanged, the outer regions are already blurred. Using the proposed method in 2D, Fig. 1(c) shows that this method is capable of adapting itself to the local noise variance, thus removing noise more uniformly. A noise reduction of 38% is achieved throughout the image. To get a proper estimate of the correlation of the input images a gaussian window with a variance of 2 was used in a 9×9 neighbourhood. Because image structure like egdes have a greater influence on the correlation value of more distant pixels, unfortunately noise remains around such image structure. Hence, if a natural look of the image should not be sacrificed the noise suppression must be kept weak. Using 3D data reduces this problem, because pixels for estimating the noise variance and correlation can be taken from the neighbourhood in all three dimension. Fig. 1(d) shows the result using a gaussian window of variance 1.5 in a $5 \times 5 \times 5$ neighbourhood. It can be seen clearly that a strong noise suppression of about 66% is achieved while image structure nearly remains unharmed.



(a) Original (b) Tukey

(c) 2D Proposed (d) 3D Proposed

Fig. 1: Denoising results for a CTA of a liver, displayed with $c = 200$ and $w = 700$.



(a) $(u_1 - u_2)/2$ (b) Tukey - Original

(c) 2D Proposed - Original (d) 3D Proposed - Original

Fig. 2: Difference images, displayed with $c = 0$ and $w = 200$.

V. Conclusions

A modified Perona-Malik diffusion process was presented that is able to deal with the special noise characteristics of CT data. The method surpasses standard diffusion methods due to its adaption on local noise variations and its regularizing of the diffusion depending on an estimation of the real image structure by calculating correlations between two input images with uncorrelated noise. A noise suppression rate of about 66% can be achieved.

References

[1] A. Borsdorf, *et al.*, "Wavelet based Noise Reduction by Identification of Correlation," in *Pattern Recognition (DAGM 2006), Lecture Notes in Computer Science*, K. Franke, *et al.*, Eds., vol. 4174. Berlin: Springer, 2006, pp. 21–30.

[2] A. Borsdorf, *et al.*, "Separate CT-Reconstruction for Orientation and Position Adaptive Wavelet Denoising," in *Bildverarbeitung fr die Medizin 2007*, A. Horsch, *et al.*, Eds. Berlin: Springer, 2007, pp. 232–236.

[3] P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 12, no. 7, pp. 629–639, 1990.

[4] M. J. Black, *et al.*, "Robust anisotropic diffusion," *IEEE Trans. on Image Processing*, vol. 7, no. 3, pp. 421–432, 1998.

[5] U. Trottenberg, *et al.*, *Multigrid*. Academic Press, San Diego, CA, USA, 2001.