Seminar New Iterative Reconstruction Methods in Medical Imaging

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Pattern Recognition Lab (CS 5)

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Requirements & Information

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Course Assessment: 2 Options

- **<u>Option „Implementation“</u>**
  - Implementation of a conference paper (C++ MR framework)
  - Appointments by requirement
  - Short report (~2 pages)
    - Implementation overview, results
  - Presentation (30 min)

- **<u>Option „Review“</u>**
  - Review of a journal paper (~10-15 pages)
  - Presentation (30 min)
  - Attendance of the reconstruction colloquium (Thursday 9:00 – 10:30)
  - Long report (~10 pages):
    - Introduction/ Context
    - Content of the paper
    - Problems for the choosen application

Iterative Magnetic Resonance Reconstruction - Basics

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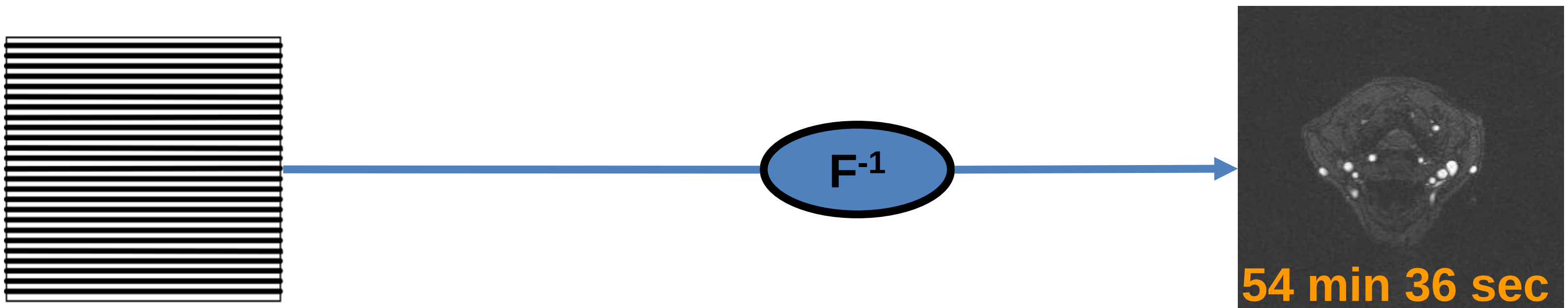
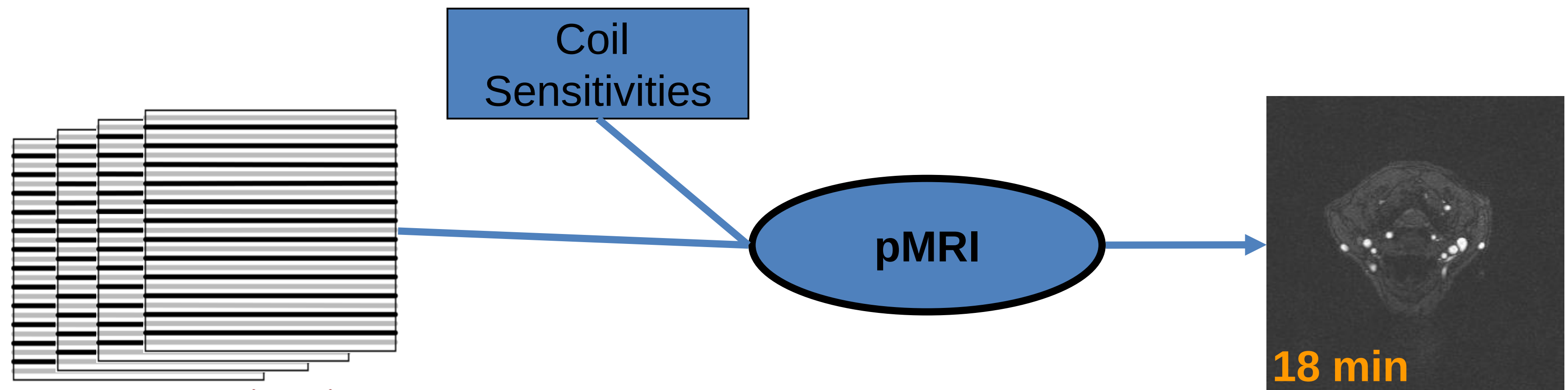
Acquisition details – Reconstruction of undersampled data



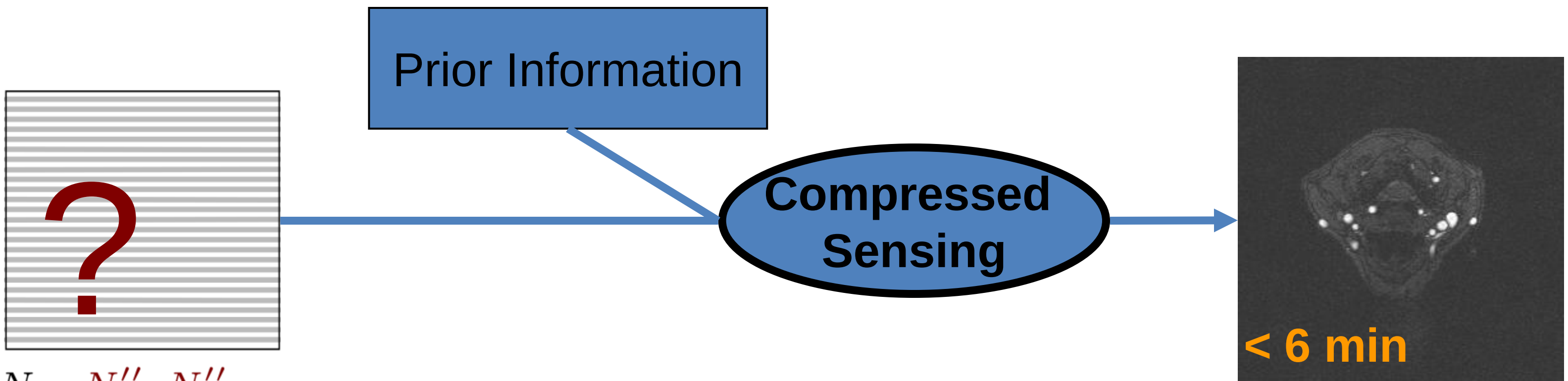
$\mathbf{Y} \in \mathbb{C}^{N_{kx}\, N_{ky}\, N_{kz}}$

Acquisition details – Reconstruction of undersampled data



$$\mathbf{Y} \in \mathbb{C}^{N_{kx} N'_{ky} N'_{kz} N_c}$$

Acquisition details – Reconstruction of undersampled data



$\mathbf{Y} \in \mathbb{C}^{N_{kx} N''_{ky} N''_{kz}}$ $\mathbf{X} \in \mathbb{C}^{N_x N_y N_z}$

Iterative Reconstruction

$$f(\mathbf{I}) = D(\mathbf{I}) + \alpha R(\mathbf{I})$$

- The target function $f(\mathbf{I})$ is composed of the data fidelity term $D(\mathbf{I})$ and the regularization function $R(\mathbf{I})$

- The solution is found iteratively by solving the minimization problem

$$\mathbf{I}^{k+1} = \underset{\mathbf{I}^k}{\operatorname{argmin}}\, f(\mathbf{I}^k)$$

Iterative Reconstruction

$$f(\mathbf{I}) = D(\mathbf{I}) + \alpha R(\mathbf{I}) = \sum_{j}^{Y} \left|\left|\mathbf{PFC^jI} - \mathbf{M^j}\right|\right|_2^2 + \alpha R(\mathbf{I})$$

- The data fidelity term includes the pMRI reconstruction algorithm switching between
  - Multiple coil k-space data $\mathbf{M^j} \in \mathbb{C}^{N_k \times 4N_t}$
  - The actual image estimate $\mathbf{I} \in \mathbb{C}^{N \times 4N_t}$
- The data fidelity term contains
  - Coil Sensitivity maps $\mathbf{C^j} \in \mathbb{C}^{N \times N}$
  - Fourier Coefficient Matrix $\mathbf{F} \in \mathbb{C}^{N_k \times N}$
  - Sampling Pattern matrix $\mathbf{P} \in \mathbb{C}^{N_k \times 4N_t}$

MR Options

- Option „Review“

  Kim D., Dyvorne H., Otazo R., Feng L., Sodickson D., Lee V.: „Accelerated Phase-Contrast Cine MRI Using k-t SPARSE-SENSE“, MRM 67:1054-1064 (2012)

  - Phase Contrast MRI
  - 2 step iterative reconstruction
  - Special Focus on Temporal FFT + Temporal PCA

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MR Options

Framework Introduction
Monday 06.05. 15:00

- Option „Implementation“

  Löcher: „L1-SPIRiTphase for Separate Magnitude and Phase Reconstruction with a Divergence Penalty for 3D Phase-Contrast Flow Measurements“, ISMRM (2012)

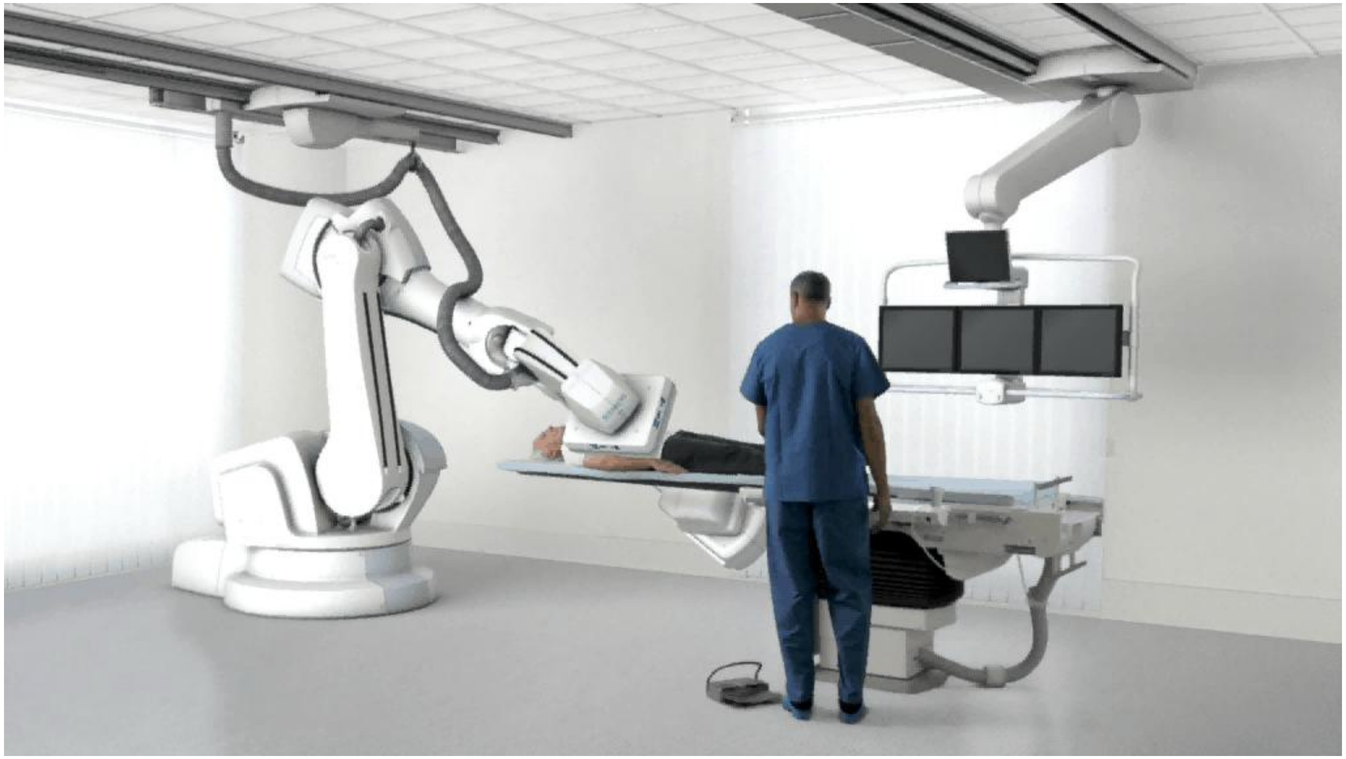
  - Implementation of the divergence-free constraint into the MR framework
  - Phase Contrast MRI (blood flow data)

$$f(\mathbf{I}) = D(\mathbf{I}) + \alpha R(\mathbf{I}) = \sum_{j}^{Y} \left|\left|\mathbf{PFC^{j}I} - \mathbf{M^{j}}\right|\right|_2^2 + \alpha R(\mathbf{I})$$

Wavelet ✓	PCA ✓
Total Variation ✓	Divergence-free

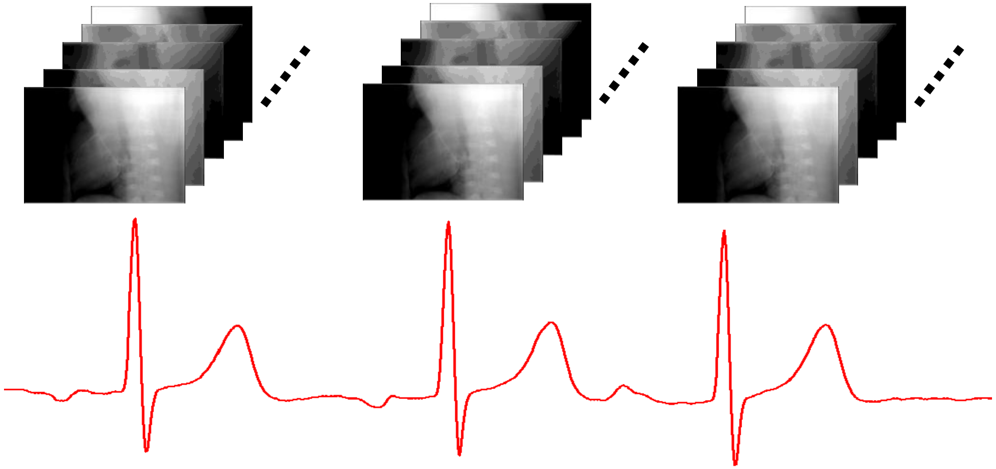
Iterative Reconstruction to improve Cardiac Imaging with C-arm CT

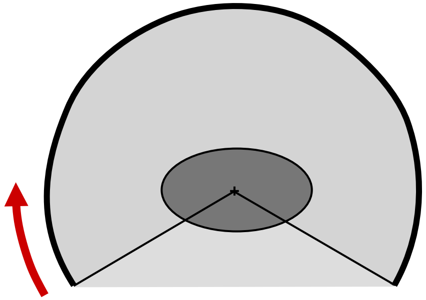
Cardiac C-arm CT imaging

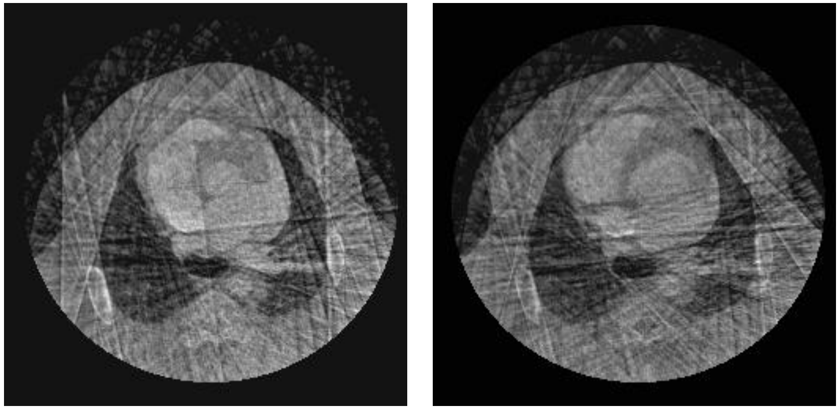


Artis zeego C-arm system. Image taken from Siemens AG, Healthcare Sector.

ECG-gating







FDK reconstruction of a relative heart phase of 30% and 80%. Image courtesy of Fahrig Lab, RSL, Department of Radiology, Stanford University (W 1530 C 650 HU).

Geometry: Cone-beam geometry

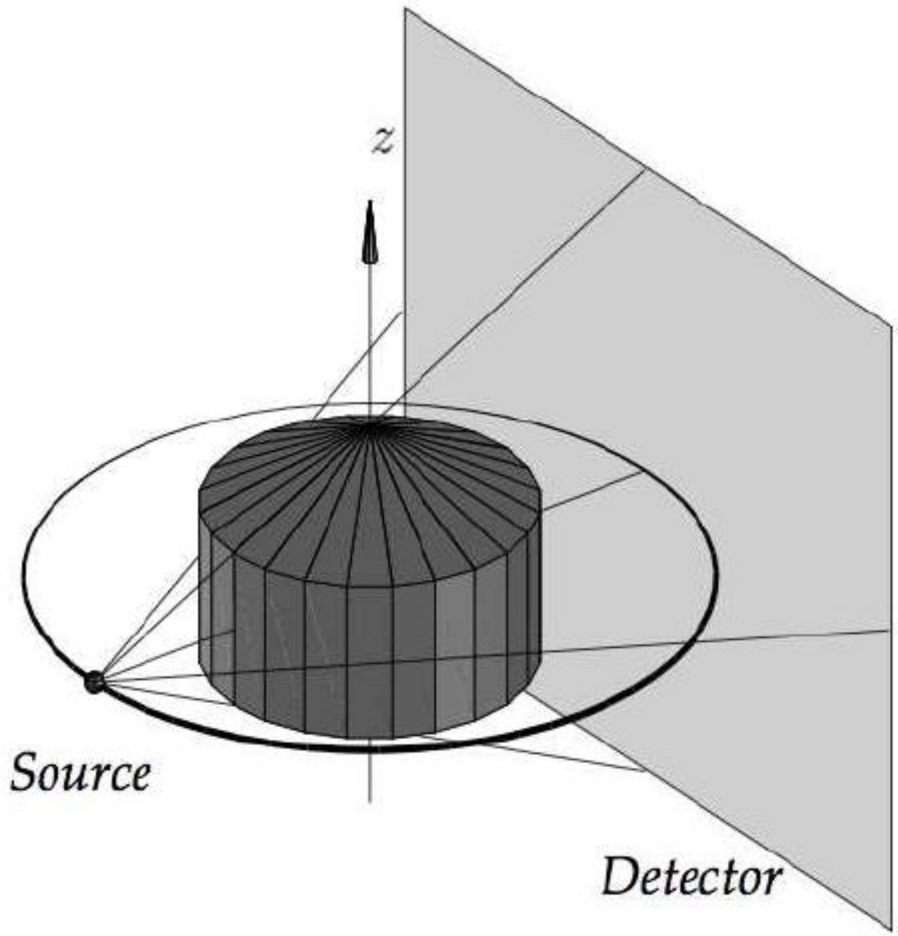
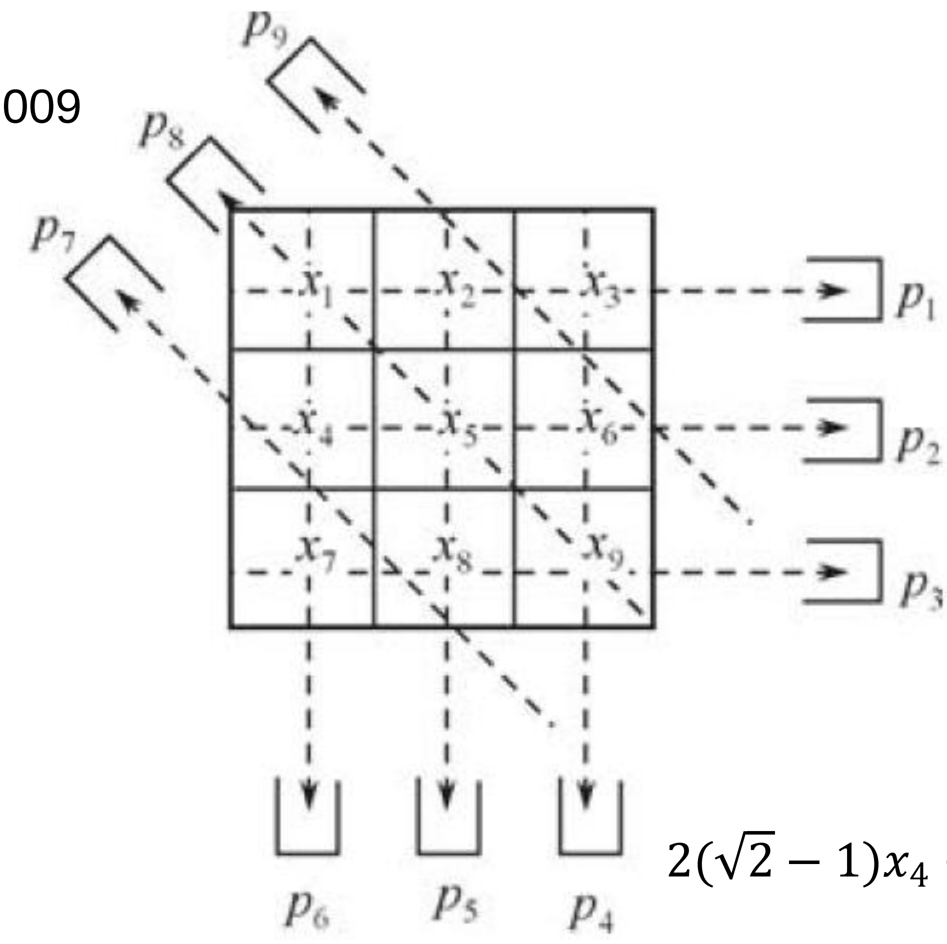


Image:
Zeng, 2009



$$x_1 + x_2 + x_3 = p_1$$

$$x_4 + x_5 + x_6 = p_2$$

$$x_7 + x_8 + x_9 = p_3$$

$$x_3 + x_6 + x_9 = p_4$$

$$x_2 + x_5 + x_8 = p_5$$

$$x_1 + x_4 + x_7 = p_6$$

$$2(\sqrt{2} - 1)x_4 + (2 - \sqrt{2})x_4 + 2(\sqrt{2} - 1)x_8 = p_7$$

$$(\sqrt{2})x_1 + (\sqrt{2})x_5 + (\sqrt{2})x_9 = p_8$$

$$2(\sqrt{2} - 1)x_2 + (2 - \sqrt{2})x_3 + 2(\sqrt{2} - 1)x_6 = p_9$$

Linear Equation System

- Rewrite to

$$\boldsymbol{AX} = \boldsymbol{P}$$

with

$$\boldsymbol{X} = (x_1, x_2, \dots, x_9)^\top$$

$$\boldsymbol{P} = (p_1, p_2, \dots, p_9)^\top$$

- $\boldsymbol{A}$ is the system matrix with elements a_{ij}
- The a_{ij} describe the contribution of each voxel to each ray

Linear Equation System

- For larger systems solutions of

$$\boldsymbol{A}\boldsymbol{X} = \boldsymbol{P}$$

with

$$\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{P}$$

$$\boldsymbol{X} = (\boldsymbol{A}^\top \boldsymbol{A})^{-1}\boldsymbol{A}^\top \boldsymbol{P}$$

$$\boldsymbol{X} = \boldsymbol{A}^\top (\boldsymbol{A}\boldsymbol{A}^\top)^{-1}\boldsymbol{P}$$

are infeasible (Gauss-Seidel, SVD, etc.)

⇨ Solution that does not require the inversion of $\boldsymbol{A}$ or a product of $\boldsymbol{A}$ is desirable

Algebraic Reconstruction Technique (ART)

- Idea: Find an iterative solution of

$$\boldsymbol{AX} = \boldsymbol{P}$$

using Kaczmarz' method:

- For each pixel p_i and each row $\boldsymbol{A_i}$ of $\boldsymbol{A}$ perform the following update:

$$\boldsymbol{X}^{k+1} = \boldsymbol{X}^{k} + \frac{p_i - \boldsymbol{A_i}\,\boldsymbol{X}^{k}}{\boldsymbol{A_i}\,\boldsymbol{A_i}^{\top}}\boldsymbol{A_i}^{\top}$$

- Repeat until convergence

ART Extensions

- Slow convergence is the main drawback of ART
- Extensions aim at improving convergence speed
- Compute update using more than one projected pixel:
  - Simultaneous ART (SART): Multiple updates at the same time and combine the result
  - Simultaneous Iterative Reconstruction Technique (SIRT): Compute update once per iteration
- Use intelligent methods to select the order of the update equations (Ordered Subsets)
- Use more realistic models within the system matrix

Regularizations

- Introduction of additional information into the reconstruction process to enforce a certain property of the solution
- Advantageous, if the problem is underdetermined
- Additional weighting terms used to suppress noise or artifacts

Papers

- Option „Review“

  Jia et al.„4D Tomography Reconstruction from Few-Projection Data via Temporal Non-local Regularization“, MICCAI (2010)

  - 4D-CT reconstruction
  - Temporal Non-local Means (TNLM)
  - Alternating two step algorithm

Papers

- Option „Review“

  Jia et al.„GPU-based Iterative Cone Beam CT Reconstruction Using Tight Frame Regularization“, PMB (2011)

  - Cone-beam CT (CBCT)
  - Iterative tight frame (TF) based algorithm

Papers

- Option „Review“

  Sidky et al.„A constrained, total-variation minimization algotihm for low-intensity x-ray CT“, MP (2011)

  - Low intensity x-ray CT
  - Based on steepest-descent and projection onto convex sets (SD-POCS)